THE

PRACTICAL BUSINESS

ARITHMETIC

FOR

Common Schools and Academies,

[illegible]

PROMISCUOUS EXAMPLES

With and without Answers.

BY

WHITMAN PECK, A. M.

NEW YORK

J. W. SCHERMERHORN & CO.

WITHOUT ANSWERS

A PRACTICAL BUSINESS ARITHMETIC,

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INCLUDING A GREAT VARIETY OF

PROMISCUOUS EXAMPLES.

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WHITMAN PECK, A.M.,

AUTHOR OF THE PROMISCUOUS EXERCISES IN ANDREW'S LATIN LESSONS

(REVISED EDITION.)

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PREFACE.

The distinguishing feature of this Arithmetic, that which has chiefly led to its publication, is its containing, in addition to the examples under each rule, a large number of "*Promiscuous Examples*" *under several different rules.* No two of these together being alike, pupils need to think how each one is to be done independently of another, instead of only doing all like one already done in the book, or by their teacher. They can often do page after page of examples as commonly arranged under their respective rules, though they could not do much simpler examples as they are apt to occur in practical business. Hence men often say, that their knowledge of Arithmetic, when they commenced business, consisted in little more than knowing how to add, subtract, multiply and divide, when directed to do so in an arithmetic, or by their teacher. This defect, it is believed, will be remedied by the repeated use of the *promiscuous examples* in this book. They are so classified and arranged that each "Exercise" requires the application of what has been previously studied in some portion of the book. The author having found such exercises almost indispensable in teaching arithmetic, has thought it would be a great convenience to teachers, to have an arithmetic containing a large number of promiscuous examples. Many experienced teachers, also, all with whom he has consulted, have confirmed him in this opinion. Hence, though there is perhaps too great a variety of arithmetics already in use, this is offered to the public.

It is believed, too, that this arithmetic contains in one book, all the most important matter usually found in an arithmetical series, in which much the same matter is repeated in different books, thus greatly increasing the expense, without any real advantage. The first part, including the Fundamental Rules, is adapted to children beginning to study arithmetic after having received a little oral instruction; and they are advanced so gradually, that they will be apt to learn this part thoroughly before they reach Compound Numbers and Fractions

Most of the examples in this arithmetic are designedly short, that less time may be consumed in the operations, and more be devoted to learning the principles and their applications. They are, also, so simple that most pupils may be expected to do them, with a little assistance in some cases, without requiring too much of the teacher's time in explaining what they seldom understand or remember. Some more difficult examples, designed for advanced pupils, will be found at the end of the book, and it is designed to publish in another book many more such examples, and some principles of Higher Arithmetic omitted in this, which, however, is sufficient to fit persons for the practical business of life.

The author, also, thinks that he has greatly simplified the study of arithmetic by reducing the number of its rules. He applies the Rules for Reduction of Compound Numbers to Reduction of Fractional Compound Numbers (common and decimal,) and the rules of Percentage to all its various applications, such as Commission, Brokerage, Stocks, Profit and Loss, etc., etc.

Suggestions to Teachers.

Pupils should be required to explain fully the examples in arithmetic, at least enough of them to show that they thoroughly understand them. At first, they will need to use the blackboard or their slates, but they should also learn to give the explanations mentally, omitting the numbers if too large to be thus calculated, but naming them at each step as they proceed. If they can do this beforehand, they need not be required to perform operations with which they are already perfectly familiar. In this way they will study mental as well as written arithmetic.

Though the Promiscuous Examples are numerous, some pupils may need to do them repeatedly, in order to become as familiar as they ought to be with the practical application of what they have previously studied. Others may not need to do them all. One or two exercises at a time may be sufficient. After a few days, give them one or two more similar exercises, and continue to do this from time to time till the principles and rules are permanently fixed in their minds.

The rules are designed to aid pupils in making their own rules, rather than to be verbally committed to memory. They should learn to perform all arithmetical operations, and explain them, independently of the rules in books.

CONTENTS.

ARITHMETIC.

Article 1.—Arithmetic is the science of numbers. It teaches their nature and use.

Number is one or more things, or **Units** ; as *one, two, three ;* the number of pupils in a class is four, five, &c.

Abstract numbers are numbers not applied to any particular thing ; as *one, two, five,* &c. **Concrete** numbers are numbers applied to particular things ; as *five men, ten cents.*

NOTATION.

Art 2.—Notation is the method of writing numbers.

There are two methods, the **Roman,** introduced by the ancient Romans, and the **Arabic,** introduced by the Arabians, which is chiefly used in Arithmetic.

Art. 3.—The Roman method uses letters for numbers ; as, I, one; V, five; X, ten; L, fifty; C, one hundred; D, five hundred; M, one thousand.

These seven letters repeated or united express all other numbers.

If a letter is repeated, its value is multiplied as many times ; as, II, (two times one,) two; XX, twenty; XXX, thirty.

If a letter is written before another of greater value, its value is subtracted from that of the greater; but if written after another of greater value it is added; as, IV, four; VI, six; IX, nine; XI, eleven.

A small line (—) over a letter multiplies its value a thousand times; as, $\overline{\text{V}}$, five thousand.

TABLE OF ROMAN LETTERS USED FOR NUMBERS.

I.	One.	IX.	Nine.	LXXX.	Eighty.
II.	Two.	X.	Ten.	XC.	Ninety.
III.	Three.	XX.	Twenty.	C.	One hundred.
IV.	Four.	XXX.	Thirty.	CC.	Two hundred.
V.	Five.	XL.	Forty.	D.	Five hundred.
VI.	Six.	L.	Fifty.	M.	One thousand
VII.	Seven.	LX.	Sixty.	$\overline{\text{V}}$.	Five thousand.
VIII.	Eight.	LXX.	Seventy.		

Art. 4.—The **Arabic Notation** uses the following ten figures for numbers:

(*Written*)	*0.*	*1.*	*2.*	*3.*	*4.*	*5.*	*6.*	*7.*	*8.*	*9.*
	Naught or Cipher.	one.	two.	three.	four.	five.	six.	seven.	eight.	nine.
(*Printed*)	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.

These figures, except the cipher, are called **Digits.**

A figure written alone, or on the right hand of a number, has only its *simple* value; as, 1, one; 2, two; 5, five, &c.

A figure written before another has *ten* times its simple value; also when prefixed to two others it has *one hundred* times its simple value. Hence figures increase in value ten fold from right to left.

10 (ten and naught) ten.
11 (ten and one) eleven.
12 (ten and two) twelve.
13 (ten and three) thirteen.
14 (ten and four) fourteen.
15 (ten and five) fifteen.
16 (ten and six) sixteen.
17 (ten and seven) seventeen.
18 (ten and eight) eighteen.
19 (ten and nine) nineteen.

20 (two tens) twenty.
21 (2 tens and 1) twenty-one.
30 (three tens) thirty. &c.
40 (four tens) forty, &c.
50 (five tens) fifty, &c.
60 (six tens) sixty, &c.
70 (seven tens) seventy, &c.
80 (eight tens) eighty, &c.
90 (nine tens) ninety, &c.
100 (ten tens) one hundred, &c.

In all the numbers from 10—19 the figure 1 is used for *ten*. In the number 11 the figure 1 is used for ten and one ; and in 111 it is used for one hundred, ten and one.

Next to hundreds are thousands, tens of thousands, hundreds of thousands, millions, &c., as in the following French method, which is chiefly used.

FRENCH NOTATION AND NUMERATION TABLE.

	&c.	**Trillions.**	Hundreds of billions.	Tens of billions.	**Billions.**	Hundreds of millions.	Tens of millions.	**Millions.**	Hundreds of thousands.	Tens of thousands.	**Thousands.**	Hundreds.	Tens.	**Units.**	
0	0	3	2	1	0	9	8	7	6	5	4	3	2	1	
	5			4			3			2			1		Periods.

Next to trillions are quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions, &c.

In this table numbers are divided into periods of three figures each, beginning at the right hand, the 1st *units*, the 2d *thousands*, the 3d *millions*, &c.

ENGLISH NOTATION AND NUMERATION TABLE.

Billions.	Hundreds of thousands of millions.	Tens of thousands of millions.	Thousands of millions.	Hundreds of millions.	Tens of millions.	**Millions.**	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds	Tens.	**Units.**	
3	2	2	0	9	8	7	6	5	4	3	2	1	
3			2						1				Periods.

Rule for Notation.—*Leaving space enough on the right for as many periods, of three figures each, as the number will contain, begin at the left hand, and write the number belonging to each period, filling the vacant places with ciphers.*

Example.—Write two millions, seventy-five thousand, three hundred and five.

There will be two periods on the right of millions. Write 2 in the millions' period, 075 in the thousands' period, and 305 in the last or units' period ; thus, 2,075,305.

NUMERATION.

Art. 5.—Numeration is reading numbers.

Small numbers are easily read by repeating the name of each figure as it is written. In reading a large number observe the following

Rule.—*Consider the number as divided into periods of three figures each, beginning at the right hand ; then, begin-*

ning at the left hand, read each period as if it stood alone, adding its name, except that of the last; thus,

The number 1,230,987,654,321, is read one trillion, two hundred and thirty billions, nine hundred and eighty-seven millions, six hundred and fifty-four thousand, three hundred and twenty-one.

EXERCISES IN NUMERATION.

Read the following numbers down and across the page. It will be best for pupils to write them first, if they have not learned to do so readily and plainly.

10	28	30	48	50	61	72	80	91
13	25	33	45	56	68	76	88	98
16	22	36	42	59	65	73	85	95
19	27	39	47	51	62	79	82	92
11	24	31	44	54	67	70	87	97
14	21	34	41	57	64	74	84	94
17	29	37	49	52	60	77	81	99
12	26	32	46	55	69	75	89	96
15	23	35	43	58	66	78	86	93
18	20	38	40	53	63	71	83	90

210	328	430	550	672	761	891	980
228	333	445	678	789	890	901	999
234	456	543	785	876	983	779	985
389	598	671	872	963	753	861	742
465	550	655	744	833	922	766	888

1000	10000	100000	1000000
1234	23456	345678	4567890
2345	34567	456789	5678901
3456	45678	567890	6789012
4567	56789	678901	7890123
5678	67890	789012	8901234

10000000	1000000000
123456789	12345678901
2345678901	345678901234
34567890123	4567890123456

12,345,678,908,765,432,102,468.

EXERCISES IN NOTATION.

Write all the numbers from—

Ten to twenty-five.
Twenty-five to fifty.
Fifty to seventy-five.
Seventy-five to one hundred.

Write—

One hundred and ten.
Two hundred and eleven.
Three hundred and one.
Four hundred and twenty.
Five hundred and sixty-seven.
Six hundred and seventy-nine.
Eight hundred and ninety.
Nine hundred and thirty-four.
Ten hundred and eleven.
Eleven hundred and twenty.
Twelve hundred and fifty-five.
Fifteen hundred and sixty-two.
Nine hundred and eighty-six.
Six hundred and fifty-four.
Three hundred and twenty-one.
One hundred and twenty-three.
Four hundred and fifty-six.
Seven hundred and eight.
Nine hundred and ten.
Two hundred and eleven.
Three hundred and forty-five.
Four hundred and fifty-six.
Five hundred and sixty-seven.
Six hundred and seventy-eight.
Seven hundred and eighty-nine.
Eight hundred and eight.
Nine hundred and ninety.
Ten hundred and twenty.
Twelve hundred and eleven.
Sixteen hundred and seventeen.
Eighteen hundred and ninety.
Nine hundred and seventy-five.
Eight hundred and sixty-four.
Seven hundred and fifty-three.
Six hundred and forty-two.
Five hundred and thirty one.
Four hundred and twenty.
Three hundred and one.
Two hundred and three.
Three hundred and fourteen.
Four hundred and twenty-four.
Five hundred and fifteen.
Six hundred and ten.
Seven hundred and twelve.

One hundred.
Two thousand.
Thirty thousand.
Four hundred thousand.
Five millions.
Six hundred and six.
Seven thousand eight hundred and nine.

Eighty thousand and ninety.
Nine hundred thousand and one hundred.
Ten million, eleven thousand and twelve.
Thirteen hundred and fourteen.
Fifteen thousand, one hundred and two.
Three hundred thousand and four.
Sixty million, seventy thousand and eight hundred.
One hundred and ten millions, two hundred and thirty-four thousand, four hundred and five.

Two hundred and thirty-four.
Five thousand, six hundred and seventy-eight.
Ninety thousand and seventeen.
Three hundred thousand, five hundred and seven.
Eleven millions, one hundred and five thousand.
Five hundred millions, seven thousand and eighty-one.
Seventy-five thousand, three hundred and forty.
Eight hundred thousand, two hundred and five.
Nine thousand, seven hundred and fifty-three.
Three millions, four hundred and thirty-two.
Twelve millions, eleven thousand and nine hundred.
One hundred and twenty millions, seventeen thousand, six hundred and seven.
Six thousand, seven hundred and thirty-one.
Seven hundred and forty-eight.
Sixty-eight thousand, four hundred and fifty-one.
Thirty-nine millions, nine hundred and twelve thousand, three hundred and ninety-six.
Seven hundred and fifty thousand, five hundred and sixty-three.
Forty-six thousand, five hundred and four.
Twelve hundred and ninety-seven.
Two thousand, five hundred and sixty-six.
Four millions, five hundred and four thousand, three hundred and twenty-two.
Twenty-five thousand, seven hundred and thirty-eight.
One thousand, four hundred and thirty-three.

Five millions, three hundred and one thousand, seven hundred and ninety-five.

The following are not designed for very young pupils.

Write—

One billion, two hundred and thirty-four millions, five thousand and seven hundred.

Three trillions, twenty-five billions, three hundred and four millions, forty-five thousand, six hundred and seventy-four.

Twenty billions, four hundred and twelve millions, sixty-five thousand and thirty-two.

Four hundred trillions, seventy-seven billions, seven hundred and seven millions, nine thousand, five hundred and sixty-three.

Four quadrillions and five hundred trillions.

Five quintillions and sixty-eight trillions.

Six sextillions and five hundred quintillions.

Seventy billions.

Eighty trillions.

Ninety quadrillions.

One hundred quintillions, two hundred and ten quadrillions, thirty-five trillions, seven hundred billions and sixty-four millions.

Fifteen sextillions, five hundred and sixty quintillions, four hundred and twenty-five trillions.

FUNDAMENTAL RULES.

Art. 6.—Arithmetic teaches the use of numbers in four principal ways, viz: **Addition, Subtraction, Multiplication,** and **Division,** called the **Fundamental Rules** of Arithmetic.

ADDITION.

Art. 7.—Addition is uniting two or more numbers in one. The number thus found, or the answer, is called the **Sum** or **Amount.**

Simple Addition is uniting *like* numbers, or numbers of the same name, in one; as, 3 apples added to 4 apples are 7 apples.

Unlike numbers cannot be added; as 3 apples and 4 pears are neither 7 apples nor 7 pears.

Addition is often expressed by the sign (+) **Plus,** placed between numbers to be added.

[The sign (=) of equality placed between numbers, shows that they are equal.]

ILLUSTRATION.—The *sum* or *amount* of 2 added to 3 is equal to 5; $2+3=5$.

ADDITION TABLE.

[This table is promiscuously arranged. The answers are not given, because it is better that pupils should learn them by thinking for themselves, and not have them for reference. They should be able to recite them perfectly and promptly.]

Add	0	1	1	0	2	2	1	2	0	3	2	3
	1	1	0	2	1	2	2	0	3	1	3	0
Sums	1	2										
	1	3	3	0	4	2	4	1	4	3	4	4
	3	2	3	4	1	4	3	4	0	4	2	4
	0	5	2	5	4	5	1	5	3	5	5	0
	5	1	5	3	5	0	5	2	5	4	5	6
	6	2	6	4	6	1	6	3	6	5	6	6
	1	6	3	6	5	6	2	6	7	6	0	6
	1	7	3	7	5	7	7	2	7	4	7	6
	7	2	7	4	7	6	7	7	3	5	7	7

Add	2	8	4	8	6	8	8	3	8	5	8	7
	8	3	8	5	8	7	8	8	4	8	6	8
	3	9	5	9	7	9	9	4	9	6	9	8
	9	4	9	6	9	8	9	9	5	9	7	9

ADDITION OF UNITS.

MENTAL EXERCISES.

How many boys are 2 boys and 1 more? 1+2? 2+2? 3+1? 2+3? 3+4? 4+2? 3+3? 4+3? 4+4?

How many girls are 5 girls and 2 more? 2+5? 3+5? 5+4? 5+3? 5+5? 6+3? 4+6? 6+5? 6+6?

How many men are 7 men and 3 more? 3+7? 7+4? 5+7? 7+6? 7+7? 8+3? 4+9? 5+8? 9+5? 6+8? 8+8? 7+9? 9+9?

How many women are 8 women and 2 more? 3+8? 9+4? 8+5? 6+9? 8+7? 9+6?

EXAMPLES FOR THE SLATE OR BLACKBOARD.

Example 1.—Add 4, 3, 2, 3, 5, and 2.

1 *Process.*—2 and 5 are 7, and 3 are 10, and 2 are 12, and 3 are 15, and 4 are 19. Name only the result of each addition; as 7, 10, 12, 15, 19.

	4
	3
	2
	3
	5
	2
Ans.	19

Rule.—*Write the numbers under one another; draw a line underneath, and under it write the sum or amount.*

EXAMPLES.

(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
2	3	4	5	4	3	2	4	5	4	2	3
3	4	5	1	3	3	2	4	5	5	3	4
4	3	3	5	1	3	2	4	5	3	4	3
3	4	4	2	2	3	2	4	5	4	4	2
4	2	2	5	3	3	2	4	5	5	3	3
2	1	4	3	4	3	2	4	5	3	2	4
3	1	1	5	4	2	3	5	5	4	3	5

(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)
4	5	1	2	3	1	2	3	4	5	6	7
3	1	5	4	4	2	3	2	1	1	1	2
4	5	1	2	5	3	4	3	4	5	6	7
2	2	4	3	2	4	5	4	2	2	2	1
4	5	1	2	2	5	6	3	4	5	6	7
1	3	3	4	1	6	7	5	3	3	3	3
4	5	5	4	5	7	8	3	4	5	6	7
					6	7	7	4	4	4	5

(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)
1	2	3	4	5	6	7	1	2	3	7	1
7	6	4	4	6	6	6	2	3	7	7	2
2	3	5	5	7	6	7	3	4	5	7	3
7	6	6	5	5	6	6	4	4	3	7	4
3	4	7	6	6	6	7	5	6	7	7	5
7	6	6	6	7	6	6	6	7	3	7	6
4	5	4	5	4	6	7	7	0	6	7	7
7	7	2	5	3	6	6	4	6	7	7	8

(38)	(39)	(40)	(41)	(42)	(43)	(44)	(45)	(46)	(47)	(48)	(49)	(50)
2	3	4	5	6	7	8	9	7	8	9	6	5
3	4	5	4	7	8	9	0	8	6	4	3	8
4	5	6	6	8	9	0	8	9	4	8	9	3
5	6	7	8	9	0	8	1	7	7	3	6	7
6	7	8	9	0	7	9	9	0	3	5	3	9
7	8	9	0	2	8	1	8	6	2	7	9	8
8	0	0	8	3	9	8	2	4	6	8	6	0
9	9	8	7	4	7	9	8	9	8	7	3	8

ADDITION OF UNITS, TENS, HUNDREDS, &c.

MENTAL EXERCISES.

How many lambs are 10 lambs and 1 more? 10+3? 10+5? 10+7? 10+9? 10+2? 10+4? 10+6? 10+8? 11+1? 11+2? 11+4? 11+6? 11+8? 11+3? 11+5? 11+7? 11+9?

How many sheep are 12 sheep and 1 more? 12+3? 12+5? 12+7? 12+9? 12+2? 12+4? 12+6? 12+8? 13+3? 13+4? 13+6? 13+8? 13+5? 13+7? 13+9?

How many horses are 14 horses and 2 more? 14+4? 14+6? 14+8? 14+3? 14+5? 14+7? 14+9? 15+2? 15+4? 15+6? 15+8? 15+3? 15+5? 15+7? 15+9? 16+2? 16+5? 16+8?

How many cows are 17 cows and 3 more? 17+5? 17+7? 17+9? 17+4? 17+6? 17+8? 18+4? 18+6? 18+8? 18+9? 18+7? 13+5? 19+3? 19+5? 19+7? 19+9? 19+6? 19+8?

EXAMPLES FOR THE SLATE.

Example 51.—Add 123, 234, 345, and 456.

Process.—Write the numbers thus,

123
234
345
456
———
Ans. 1158

Add the right hand column 6+5+4+3=18 units, or 1 ten and 8 units. Write 8 under *units* and add 1 to the tens. Add the second column 1+5+4+3+2=15 tens, 1 hundred and 5 tens. Write 5 under *tens* and add 1 to hundreds. Thus proceed.

Rule.—*Write the numbers under one another, so that all the right-hand figures shall be in the same column, and the others in proper order, tens next to units, &c.*

Beginning at the right hand, add each column separately. If the sum consists of only one figure, write it under the column; but if it consists of two or more, write only the right hand figure and carry or add the others to the next column if there is any; otherwise write both figures.

Proof.—*Add the same columns downward.*

Figures of different local value cannot be added; 2 tens and 3 units are neither 5 tens (50) nor 5, but 23.

EXAMPLES.

(52)	(53)	(54)	(55)	(56)	(57)	(58)	(59)	(60)	(61)	(62)	(63)	(64)
23	34	45	50	01	12	23	32	45	33	44	55	34
34	51	01	12	23	45	44	34	54	44	44	55	53
45	02	23	34	34	30	54	22	45	55	44	55	45
23	34	45	51	50	44	32	10	54	22	44	55	34
34	51	04	23	12	32	45	54	45	11	44	55	53

(65)	(66)	(67)	(68)	(69)	(70)	(71)	(72)	(73)	(74)	(75)	(76)	(77)	(78)
12	23	34	45	56	67	78	89	90	66	77	88	99	89
34	45	56	67	78	89	90	01	12	66	77	88	99	98
56	67	78	89	90	01	12	23	34	66	77	88	99	79
78	89	90	01	12	23	34	45	56	66	77	88	99	96
90	09	18	23	34	45	56	67	78	66	77	88	99	45

(79)	(80)	(81)	(82)	(83)	(84)	(85)	(86)	(87)	(88)
123	423	345	456	567	678	789	890	901	910
456	456	678	789	890	901	012	123	234	345
789	789	901	012	123	234	345	456	567	678
012	045	234	345	456	567	678	789	890	910
345	678	567	678	789	890	901	012	123	234
678	997	891	901	012	123	234	345	456	567

(89)	(90)	(91)	(92)	(93)	(94)	(95)	(96)	(97)
123	234	345	431	543	654	764	876	987
456	567	678	098	210	321	325	543	654
789	890	901	765	987	098	109	210	321
123	123	234	321	654	765	876	987	098
456	564	567	098	321	432	543	654	765
789	897	890	765	098	109	210	321	431
123	135	123	432	765	876	987	098	098
456	678	456	109	431	543	632	765	726

(98)	(99)	(100)	(101)	(102)	(103)	(104)	(105)	(106)
789	890	901	678	567	456	345	231	123
012	123	234	901	890	789	678	456	345
345	456	567	234	123	012	901	678	567
678	789	890	567	456	334	223	890	789
901	012	123	890	789	455	344	012	901
234	345	456	123	011	667	556	234	123
567	678	789	456	223	788	677	456	345
890	901	012	789	334	990	889	678	567
123	234	345	012	455	011	901	890	789
456	567	678	345	667	223	334	123	901

MENTAL EXERCISES.

How many chickens are 20 chickens and 2 more? 20 + 4? 20+7? 20+9? 21+3? 21+5? 21+7? 21+9? 22+4? 22+6? 22+8? 23+3? 23+4? 23+7? 23+9? 23+6?

How many ducks are 24 ducks and 3 more? 24+6? 24+9? 25+4? 25+7? 25+9? 26+8? 27+4? 28+7? 29+5? 30+7? 31+8? 32+6? 33+5? 36+9? 39+8? 41+5? 43+7? 46+8? 47+7? 49+8? 51+7? 55+5? 57+6? 58+7?

How many pigeons are 63 pigeons and 9 more? 63+7? 64+5? 65+7? 66+6? 67+7? 68+8? 69+9? 70+7? 75+6? 76+8? 77+8? 79+4? 79+6? 79+7? 81+9? 83+8? 84+9? 86+7? 88+8? 90+9? 91+7? 93+8? 97+5? 98+7? 99+9?

How many quails are 10 quails and 10 more? 10 + 12? 10+17? 10+19? 11+13? 11+15? 12+11? 13+12? 14+13? 15+12? 16+11? 17+13? 18+15? 19+11? 19+14? 19+16? 19+19?

EXAMPLES FOR THE SLATE.

(107)	(108)	(109)	(110)	(111)	(112)
12345	23456	34567	45678	56789	67899
67890	78901	89012	90987	98765	12344
11223	23456	34567	65432	56789	45677
34455	78901	89012	10123	91023	78900
66778	23456	34567	45678	34567	01234
89900	78901	89012	90876	78901	56789
11223	23456	34567	54321	12345	98765
34455	78901	89012	09876	67890	54321

(113)	(114)	(115)	(116)	(117)	(118)
34682	46820	65431	76432	61234	5064
57931	06842	76543	67675	45678	785
24680	3697	87654	56567	54321	9543
13564	568	98764	78789	76428	6748
2805	7634	09876	97973	85947	97054
3579	86420	1234	46467	64758	7865
46820	13579	765	33590	81927	753
97531	24680	12076	45678	79635	39

119. 65340 + 6731 + 748 + 68451+396+7503+46075 + 1290+ 25738+46803.

120. 54268+405+1708 + 43671+72049 + 492+1760+25357+ 1434+84162.

121. 246768 + 21380 + 4075 + 126849 + 257 + 1305 + 24350+ 439871+40306+601734.

122. 3947+73845+300901+499091+45131+564429+484292+ 178737+58072+65344+194532+758+14.

Find the sum of the following numbers:

EXAMPLE (123.)

One hundred and eighteen thousand, nine hundred and forty-eight.

One thousand, one hundred and ninety-two.

Two millions, eight hundred and sixteen thousand, seven hundred and sixty.

Ninety thousand, four hundred and forty-five.

One hundred and twenty-eight thousand.

One million, one hundred and forty-three thousand, eight hundred and twelve.

Twenty-four thousand, six hundred and sixty-four.

Three millions, two hundred and forty-three thousand.

EXAMPLE (124.)

Twenty-seven millions, six hundred and nineteen thousand, eight hundred and sixty-six.

Three hundred and fifty-four thousand, seven hundred and ninety-seven.

Two millions, two hundred and ninety thousand, three hundred and sixty-three.

Nine hundred and thirty thousand, four hundred and thirty.

Four hundred thousand.

One hundred and twenty-seven millions, seven hundred and seventy-eight thousand, nine hundred and eighty-one.

One million, four hundred and twenty-one thousand, six hundred and sixty-one.

Eight hundred and sixty-nine thousand.

EXAMPLE (125.)

Sixty millions, seven hundred and eight thousand, five hundred and two.

Two millions, nine hundred and thirty-seven thousand and sixty-six.

Sixty-one thousand.

One million, six hundred and twenty-five thousand.

EXAMPLE (126.)

Two millions, one hundred and twenty thousand, three hundred and ninety-seven.

Eighty-six thousand.

Two millions and six hundred thousand.

Four hundred and fifty thousand.

PRACTICAL EXAMPLES.

127. A farmer has sold at different times 30, 45, 48, 50, 56, and 63 bushels of oats; how many bushels altogether?

128. A family consumed in a year 6 loads of coal, weighing as follows: 1250, 1168, 987, 1076, 879, and 1275 pounds; how many pounds in all?

129. A merchant has bought 5 cases of muslin, containing respectively the following number of yards: 963, 897, 985, 1005, and 889; how many yards in all?

130. The population of the New England States in 1850 was respectively 583,000, 318,000, 314,000, 995,000, 148,000, and 371,000, what was the whole population of New England?

SUBTRACTION.

Art. 8.—Subtraction is taking a less number from a greater. The number thus found is called the **Difference** or **Remainder.**

The number to be subtracted is called the **Subtra-**

hend, and the number from which it is to be taken the **Minuend.**

Simple Subtraction is taking one number from another of the same name.

Subtraction is sometimes expressed by the sign (—) **minus.** The number written after the sign is to be subtracted from the one before it; as 5—3=2; three subtracted from 5 leaves 2.

SUBTRACTION TABLE.

[It is promiscuously arranged and without answers.]

From......	1	2	3	4	1	2	3	4	2	3	4	2
Take......	0	1	2	0	1	1	0	2	1	1	2	1
Remainder	1	1	&c.									
	4	5	4	5	6	4	5	6	5	6	7	6
	1	2	2	3	1	3	4	1	1	2	3	4
	7	8	7	8	7	8	7	8	7	8	7	8
	1	3	2	5	3	7	4	2	5	4	6	6
	9	10	9	10	9	10	9	10	9	10	9	10
	2	3	4	5	6	7	8	2	3	4	5	6
	11	12	11	12	11	12	11	12	11	12	11	12
	6	9	4	7	2	5	8	3	9	8	7	6
	13	14	13	14	13	14	13	14	13	14	13	14
	9	8	7	6	5	4	3	2	8	7	6	9
	15	16	15	16	15	16	15	16	15	16	15	16
	8	9	6	7	9	8	7	5	4	3	4	6

From......	17	18	17	18	17	18	17	18	17	18	17	17
Take......	9	6	3	8	5	7	8	9	6	5	4	3
	—	—	—	—	—	—	—	—	—	—	—	—
	19	18	17	16	19	18	17	16	15	14	18	19
	9	7	8	9	8	6	9	8	9	8	9	7
	—	—	—	—	—	—	—	—	—	—	—	—

MENTAL EXERCISES.

In a class of 2 girls how many will be left if 1 girl leave it? 2 girls? How many are 2—1? 2—2?

In a class of 3 boys how many will be left if 2 boys leave it? 1 boy? 3 boys?

How many are 3—2? 3—1? 3—3?

Edward has 5 apples and his sister 3, how many more has he than his sister?

How many are 5—4? 5—2? 5—5? 5—3? 5—1?

Mary had 6 cakes and gave 3 to her brother, how many had she left?

How many are 6—5? 6—3? 6—1? 6—6? 6—4? 6—2?

Anna has 7 good marks at school and 4 bad ones, how many more good ones has she than bad ones?

How many are 7—3? 7—5? 7—1? 7—7? 7—6? 7—4?

John had 8 oranges and has given away all except 5, how many has he given away?

How many are 8—5? 8—7? 8—3? 8—6? 8—2? 8—4?

Thomas has 9 cents and his brother 7, how many more has he than his brother?

How many are 9—6? 9—3? 9—7? 9—4? 9—5?

EXAMPLES FOR THE SLATE OR BLACKBOARD.

EXAMPLE 1.—From 5698 subtract 3245.

Process.—5 units from 8 units leave 3 units; 4 tens from 9 tens leave 5 tens, &c.

$$\begin{array}{r} 5698 \\ 3245 \\ \hline \textit{Ans. } 2453 \end{array}$$

Ex. 2—From 7653 subtract 4865.

Process.—Since 5 cannot be taken from 3, add one of the 5 tens, or 10 to the 3; then 5 from 13 leave 8. For the same reason add one of the 6 hundreds, or 10 to the 4 tens left, and take 6 from 14, &c.

$$\begin{array}{r} 7653 \\ 4865 \\ \hline \textit{Ans. } 2988 \end{array}$$

Ex. 3.—From 6004 subtract 3425.

	6004
	3425
Ans.	2579

Process.—Though there are no tens in the upper number, it contains at least 10 units; therefore add ten to the 4 and take 5 from 14, and consider that only 9 of the 10 tens which the number contains remain in the tens place; or the result is the same if 1 is added or carried to the next lower figure, the two numbers being equally increased.

Rule.—*Write the less number under the greater, so that the right hand figure of each shall be under each other, and draw a line beneath them.*

Begin at the right hand and subtract each figure of the lower number from the one directly above it, and write the difference beneath it.

If the upper figure is less than the lower, add ten to it, then subtract and carry one to the next lower, or subtract one from the next upper figure.

Proof.—*Add the remainder to the less number, and if the sum is equal to the greater, the subtraction is correct.*

EXAMPLES.

(4)	(5)	(6)	(7)	(8)
654321	765432	876543	987654	987653
331201	234201	145312	431212	534443

(9)	(10)	(11)	(12)	(13)
767676	878787	989898	888888	999999
423122	534231	675643	544332	497531

(14)	(15)	(16)	(17)
7532175321	8642086420	9753195713	1023678415
4826084356	3736393035	2846098648	897534706

(18)	(19)	(20)	(21)
2125374859	7685946354	8796059384	9281736471
958763465	5869370819	3948126893	2736453673

(22)	(23)	(24)	(25)
3435796354	4356534798	5768743276	6543987672
1527697616	2785374689	2378679863	5734689778
(26)	(27)	(28)	(29)
7569354978	8954978654	9872579846	7963864201
3678567889	5379889910	6743287938	3576383712
(30)	(31)	(32)	(33)
6743876549	9576854378	4759813548	8743645211
4371968732	2768907569	1964679473	1170927802
(34)	(35)	(36)	(37)
5478698472	6802468024	7791357915	8642042864
3789589756	3579137951	4749454347	7894013498
(38)	(39)	(40)	(41)
6547328964	5473143256	8321731214	9543682410
2785679546	1758916407	5732157321	5437691607
(42)	(43)	(44)	(45)
4002007100	5002070022	6070005000	8573000012
870345076	3754327819	3700072001	4781973607

46. 75694 — 3590 = what number?
47. 87532 — 7615 = "
48. 987563 — 43215 = "
49. 107923 — 36001 = "
50. 100000 — 999 = "
51. 845067 — 71389 = "
52. 1789543 — 76508 = "
53. 200100 — 54321 = "
54. 500000 — 99887 = "
55. 6000000 — 887766 = "

56. From ten thousand and five, subtract seven thousand five hundred and seven.

57. From twenty-five thousand, one hundred and six, subtract ten thousand, six hundred and seventy-nine.

58. From one hundred thousand and fifty-eight, subtract fifteen thousand, three hundred and ninety.

59. From one million, subtract seven thousand and five.

60. From ten millions, ten thousand and ten, subtract five hundred thousand, five hundred and five.

PRACTICAL EXAMPLES.

61. A man purchased a farm for 6750 dollars, and sold it for 9000 dollars, how much did he gain?

62. Mt. Washington, N. H., is 6285 feet high; Mt. Mansfield, Vt., 4,279 feet; how much higher is Mt. Washington than Mt. Mansfield?

63. The Missouri River is 3100 miles long, and the Mississippi 2500; how much longer is the former than the latter?

64. The population of London in 1850 was two millions, three hundred and sixty-three thousand; that of Paris one million, fifty-three thousand and two hundred; what was the difference?

65. A man was born in the year 1780 and died in 1859, how old was he when he died?

66. North America contains eight millions of square miles; Europe, three millions five hundred thousand; how many more square miles are there in North America than in Europe?

MULTIPLICATION.

Art. 9.—Multiplication is finding a number, equal to a given number repeated by addition as many times as there are units in another given number; as, 4 times 6 are 24, or $6+6+6+6=24$.

The number to be multiplied is called the **Multiplicand.** The number by which the other is multiplied is called the **Multiplier.** The answer, or number found by multiplication, is called the **Product.**

Both the multiplicand and multiplier are called **Factors,** because they make the product.

Simple multiplication is that in which the multiplicand is of only one name.

The sign of multiplication is (×) an oblique cross.

ILLUSTRATION.—5 times 6 are 30 : 6×5=30. The multiplicand is 6 ; the multiplier 5 ; and the product 30. 5 and 6 are also factors.

The multiplicand must properly be of the same name as the product or answer required, though it is often more convenient to make the larger number the multiplicand, while the result is the same. The number of marks below is the same, whether they are arranged in five groups of six each, or six groups of five each, or all in one group ; thus

|||||| |||||| |||||| |||||| |||||| or ||||| ||||| ||||| ||||| ||||| |||||

or ||||||||||||||||||||||||||||||

Both 6×5 and 5×6=30. But if 6 men can reap a field in 5 days, and it be required to find how many men could reap it in one day, the multiplicand will be 6 men, and the product 30 men ; while if it be required to find in how many days one man could reap it, the multiplicand will be 5 days, and the product 30 days.

The multiplier, though often used as a concrete number applied to some particular things, is properly only an abstract number. In the illustration of the preceding remark, 6 men multiplied by 5 days, or 5 days multiplied by 6 men, would be absurd ; but 6 men multiplied by 5, which is the same as the number of days, are 30 men ; and 5 days multiplied by 6, which is the same as the number of men, are 30 days.

MULTIPLICATION TABLE.

2 times			3 times			4 times			5 times			6 times			7 times		
1	are	2	1	are	3	1	are	4	1	are	5	1	are	6	1	are	7
2	"	4	2	"	6	2	"	8	2	"	10	2	"	12	2	"	14
3	"	6	3	"	9	3	"	12	3	"	15	3	"	18	3	"	21
4	"	8	4	"	12	4	"	16	4	"	20	4	"	24	4	"	28
5	"	10	5	"	15	5	"	20	5	"	25	5	"	30	5	"	35
6	"	12	6	"	18	6	"	24	6	"	30	6	"	36	6	"	42
7	"	14	7	"	21	7	"	28	7	"	35	7	"	42	7	"	49
8	"	16	8	"	24	8	"	32	8	"	40	8	"	48	8	"	56
9	"	18	9	"	27	9	"	36	9	"	45	9	"	54	9	"	63
10	"	20	10	"	30	10	"	40	10	"	50	10	"	60	10	"	70
11	"	22	11	"	33	11	"	44	11	"	55	11	"	66	11	"	77
12	"	24	12	"	36	12	"	48	12	"	60	12	"	72	12	"	84

8 times			9 times			10 times			11 times			12 times		
1	are	8	1	are	9	1	are	10	1	are	11	1	are	12
2	"	16	2	"	18	2	"	20	2	"	22	2	"	24
3	"	24	3	"	27	3	"	30	3	"	33	3	"	36
4	"	32	4	"	36	4	"	40	4	"	44	4	"	48
5	"	40	5	"	45	5	"	50	5	"	55	5	"	60
6	"	48	6	"	54	6	"	60	6	"	66	6	"	72
7	"	56	7	"	63	7	"	70	7	"	77	7	"	84
8	"	64	8	"	72	8	"	80	8	"	88	8	"	96
9	"	72	9	"	81	9	"	90	9	"	99	9	"	108
10	"	80	10	"	90	10	"	100	10	"	110	10	"	120
11	"	88	11	"	99	11	"	110	11	"	121	11	"	132
12	"	96	12	"	108	12	"	120	12	"	132	12	"	144

The multiplication table promiscuously arranged without the products :

Multiplicands	12	9	11	8	10	7	4	6	3	5	2
Multipliers	12	11	10	9	8	7	6	5	4	3	2

ANOTHER FORM.

Multipli'ds	4	7	10	2	5	8	11	0	3	6	9	12
Multipliers	3	2	3	2	3	2	3	2	3	2	3	2

1	4	7	10	2	5	8	11	0	3	6	9	12
3	2	3	2	3	2	3	2	3	2	3	2	3
1	4	7	10	2	5	8	11	0	3	6	9	12
4	5	4	5	4	5	4	5	4	5	4	5	4
1	4	7	10	2	5	8	11	0	3	6	9	12
5	4	5	4	5	4	5	4	5	4	5	4	5
1	4	7	10	2	5	8	11	0	3	6	9	12
6	7	6	7	6	7	6	7	6	7	6	7	6
1	4	7	10	2	5	8	11	0	3	6	9	12
7	6	7	6	7	6	7	6	7	6	7	6	7
1	4	7	10	2	5	8	11	0	3	6	9	12
8	9	8	9	8	9	8	9	8	9	8	9	8

Multpl'd	1	4	7	10	2	5	8	11	0	3	6	9	12
Mulp'rs	9	8	9	8	9	8	9	8	9	8	9	8	9
	1	4	7	10	2	5	8	11	0	3	6	9	12
	10	11	12	10	11	12	10	11	12	10	11	12	10
	1	4	7	10	2	5	8	11	0	3	6	9	12
	11	12	10	11	12	10	11	12	10	11	12	10	11
	1	4	7	10	2	5	8	11	0	3	6	9	12
	12	10	11	12	10	11	12	10	11	12	10	11	12

[Let pupils write the above table and the respective products on their slates; also recite the products without being written till they can do so promptly without mistakes.

Art. 10.—Multiplication by one figure.

MENTAL EXERCISES.

At 3 cents each, how much will 2 oranges cost? 4? 6? 3? 5?

Answer.—If 1 orange cost 3 cents, 2 oranges will cost 2 times or twice 3 cents, which are 6 cents.

At 4 cents each, how much will 3 lemons cost? 5? 2? 4? 6? 3? 5?

At 5 cents each, how much will 4 melons cost? 3? 5? 2? 4? 6?

At 6 cents each, how much will 5 pine apples cost? 3? 2? 4? 6? 5?

At 7 cents a pint, how much will 2 pints of cherries cost? 4? 6? 3? 5?

At 8 cents a pint, how much will 3 pints of strawberries cost? 5? 2? 4? 6?

At 9 cents a quart, how much will 4 quarts of chestnuts cost? 2? 6? 3? 5?

At 10 cents a pound, how much will 5 pounds of sugar cost? 3? 2? 4? 6?

At 11 cents a pound, how much will 6 pounds of cheese cost? 4? 3? 5? 2?

At 12 cents a pound, how much will 3 pounds of ginger cost? 5? 2? 4? 6?

EXAMPLES FOR THE SLATE.

Example 1.—Multiply 4326 by 5.

Process.—5 times 6 are 30 ; write 0 units and carry the three tens as in addition ; 5 times 2 tens are 10 tens, and three carried are 13 tens ; write the 3 tens and carry the 1. Thus proceed.

written: 4326 × 5 = *Ans.* 21630

Proof.—4 times+1 time the number are 5 times the number. Therefore multiply by 4, and to the product add the multiplicand ; the result will be same as before if correct. Or repeat the process, inverting the order of the figures, as 6 times 4, 2 times 4, &c.

PROOF.

4326
4
17304
21630

Rule.—*Write the multiplier under the right-hand figure of the multiplicand, and draw a line under it. Begin at the right hand and multiply each figure of the multiplicand by the multiplier, carrying as in addition.*

Proof.—*Multiply by a number* 1 *less than before, and to the product add the multiplicand. If the result is the same, it is correct.*

(2.) 135024	135024	135024
2	3	4

Write thus and multiply each number by 2, 3, and 4 :

(3.) 246135	(11.) 416305	(19.) 165320
(4.) 350246	(12.) 520416	(20.) 510342
(5.) 461350	(13.) 631502	(21.) 621435
(6.) 502461	(14.) 310542	(22.) 205614
(7.) 613502	(15.) 421653	(23.) 136205
(8.) 153042	(16.) 532064	(24.) 103245
(9.) 264153	(17.) 643105	(25.) 124356
(10.) 305264	(18.) 520641	(26.) 235460

Multiply by 5 and 6 :

(27.) 679867	(33.) 689769	(39.) 486979
(28.) 579689	(34.) 786975	(40.) 579687
(29.) 497867	(35.) 879789	(41.) 397869
(30.) 938796	(36.) 787980	(42.) 278697
(31.) 479685	(37.) 979899	(43.) 697987
(32.) 596879	(38.) 676869	(44.) 708090

MENTAL EXERCISES.

At 2 shillings a yard, how much will 7 yards of calico cost? 8? 9? 5?

At 3 shillings a yard, how much will 8 yards of ribbon cost? 7? 9? 6?

At 4 shillings a yard, how much will 9 yards of silk cost? 7? 4? 8?

At 5 dollars a yard, how much will 7 yards of cloth cost? 9? 6? 8?

At 6 cents a skein, how much will 8 skeins of sewing silk cost? 6? 9? 7? 5?

At 7 shillings a pound, how much will 9 pounds of wool cost? 7? 8? 4?

At 8 dollars each, how much will 7 coats cost? 9? 6? 8? 5?

At 9 cents a yard, how much will 8 yards of muslin cost? 8? 7? 9? 4?

At 10 cents a spool, how much will 9 spools of cotton cost? 7? 5? 8? 6?

At 11 dollars each, how much will 7 shawls cost? 5? 3? 9? 8?

At 12 shillings a pair, how much will 8 pairs of gloves cost? 6? 9? 7? 4?

EXAMPLES FOR THE SLATE.

(45.)	14725803	14725803	14725803
	7	8	9

Write thus and multiply each number by 7, 8, and 9:

(46.)	69142580	(50.)	17452083	(54.)	12740853
(47.)	36914758	(51.)	61794250	(55.)	20859631
(48.)	69147280	(52.)	39641278	(56.)	31962745
(49.)	47258069	(53.)	50863194	(57.)	45270836

Art. 11.—Multiplication by two or more figures.

MENTAL EXERCISES.

At 2 dollars each, how much will 10 caps cost? 12? 11? 9?

At 3 dollars each, how much will 11 hats cost? 10? 12? 8?

At 4 dollars each, how much will 12 bonnets cost? 10? 7? 11?

At 5 shillings a pair, how much will 10 pairs of socks cost? 12? 9? 11?

At 6 shillings a yard, how much will 11 yards of linen cost? 9? 12? 10?

At 7 cents each, how much will 12 papers of pins cost? 10? 8? 11?

At 8 cents each, how much will 10 papers of needles cost? 12? 8? 11?

At 9 cents a dozen, how much will 11 dozen of buttons cost? 9? 12? 10?

At 10 cents a yard, how much will 12 yards of cambric cost? 10? 7? 11?

At 11 dollars each, how much will 11 shawls cost? 8? 12? 10?

At 12 dollars each, how much will 12 pieces of oilcloth cost? 10? 6? 11? 9? 8? 7?

EXAMPLES FOR THE SLATE.

EXAMPLE 58.—Multiply 172829 by 12.

Process.—The same as multiplying by units or one figure; 12 times 9, &c.

```
      172829
          12
Ans. 2073948
```

Write thus and multiply each number by 10, 11, and 12:

(59.)	96308527	(62.)	30852741	(65.)	30691472
(60.)	85274196	(63.)	27419630	(66.)	25803691
(61.)	74196308	(64.)	19630852	(67.)	50642839

Art. 12.—Multiplication by numbers greater than 12.

EXAMPLE 68.—Multiply 28357 by 234.

Process.—Multiplying by 234 is the same as multiplying by 2 hundreds, 3 tens and 4. 4 times 28357 =113428 by the same process as before. 3 times 28357=85071, and since the multiplier is tens the product will be tens; therefore write the right hand figure under tens. For a similar reason write the right hand figure of the product of 28357×2 under hundreds. Then the products of the number multiplied by 2 hundreds, 3 tens and 4 added together will be its product multiplied by 234.

```
       28357
         234
      113428
      85071
     56714
Ans. 6635538
```

RULE.—*Write the multiplier under the multiplicand. Begin at the right hand, and multiply by each figure of the multiplier separately, writing under each the right-hand figure of its product, with the other figures in their proper order.*

Add the products of each figure together, and the product of the two numbers will be found.

PROOF.—*Repeat the multiplication, only inverting the order of the figures, as in the above example; instead of* 4 *times* 7 *say* 7 *times* 4.

When the multiplier contains a cipher it may be passed over, but the right hand figure of the next product must be written, not under the cipher, but its own multiplier.

EXAMPLES.

(69.)	144×13	(80.)	1426×31	(90.)	15789×125
(70.)	245×18	(81.)	2536×42	(91.)	21478×234
(71.)	356×24	(82.)	3675×54	(92.)	34890×345
(72.)	567×45	(83.)	4879×65	(93.)	41256×456
(73.)	789×56	(84.)	5098×76	(94.)	54675×567
(74.)	890×67	(85.)	6109×87	(95.)	67812×678
(75.)	901×78	(86.)	7432×98	(96.)	78569×789
(76.)	234×89	(87.)	8987×89	(97.)	86453×897
(77.)	987×99	(88.)	9786×78	(98.)	95387×901
(78.)	876×98	(89.)	1067×99	(99.)	92896×802
(79.)	765×87				

Special Rules.

Art. 13.—Multiplication of numbers having ciphers on the right hand.

Ex. 100.—Multiply 245 by 100.

$$\begin{array}{r} 245 \\ 100 \\ \hline 000 \\ 000 \\ 245 \\ \hline \textit{Ans. } 24500 \end{array}$$

Process.—Write 245 with two ciphers (24500) on the right hand. This changes the local value of the figures the same as multiplying by 100.

Ex. 101.—Multiply 256 by 300.

Process.—3 times 256 are 768, and two ciphers are to be written after this product for the same reason as before.

$$\begin{array}{r} 256 \\ 300 \\ \hline \textit{Ans. } 76800 \end{array}$$

Ex. 102.—Multiply 4600 by 32000.

Process.—$46 \times 32 = 1472$. Write five ciphers at the right hand, for the product of hundreds by thousands is hundreds of thousands by the general rule,

$$\begin{array}{r} 4600 \\ 32000 \\ \hline 9200000 \\ 138 \\ \hline \textit{Ans. } 147200000 \end{array}$$

RULE.—*When the multiplier is* 10, 100, 1000, *&c., write as many ciphers as it contains on the right hand of the multiplicand.*

In other cases, write and multiply the other figures as if they had no ciphers on the right hand, and annex to the product as many ciphers as were not used in both numbers.

EXAMPLES.

(103.)	2250×10	(109.)	1020000×500
(104.)	3500×1200	(110.)	276000×3
(105.)	4732×1000	(111.)	375×30100
(106.)	130×51	(112.)	17020×1000
(107.)	356×100	(113.)	1000000×10000
(108.)	7000×700	(114.)	1004000×10500

MULTIPLICATION BY COMPOSITE NUMBERS.

Art. 14.—A **Composite** number is one which is the product of two other numbers ; as, 30 composed of 6×5.

Ex. 115.—Multiply 29 by 24.

Process.—Since 4 times 6=24, 4 times 6 times 29=24 times 29.

$$\begin{array}{r} 29 \\ 6 \\ \hline 174 \\ 4 \\ \hline \textit{Ans. } 696 \end{array}$$

RULE.—*Find two or more numbers which being multiplied together will produce the given multiplier. Multiply the* *multiplicand by one of them, and its product by another, till all the factors are used.*

EXAMPLES.

(116.)	115×15	(120.)	126×27	(123.)	324×48
(117.)	123×16	(121.)	99×32	(124.)	87×63
(118.)	39×36	(122.)	265×35	(125.)	405×108
(119.)	162×18				

(126.)	3456789× 1019	(135.)	871496×2468
(127.)	9830291× 7305	(136.)	397684×6005
(128.)	5006284× 6635	(137.)	469537×3708
(129.)	4000059× 7239	(138.)	873576×8764
(130.)	873000× 1000	(139.)	468937×7056
(131.)	257000× 4000	(140.)	798600×8750
(132.)	5749362× 3827	(141.)	750000×9000
(133.)	4327000× 3500	(142.)	596875×9678
(134.)	13786926×85043	(143.)	658907×7869

(144.) 9 6 3 8 5 2 7 1 4—Multiplicand.
4 7 5 1 8 0 6 3 9—Multiplier.

---

145. Multiply seventy-six thousand by sixty-eight hundred and four.

146. Multiply nine million and eight thousand by five hundred thousand and sixty.

147. Multiply eighty-seven thousand, six hundred and three, by nine thousand, eight hundred and sixty-five.

148. Multiply eighty-three thousand, four hundred and fifty seven, by six thousand, eight hundred and thirty-five.

149. Multiply nine hundred and four thousand, by ten thousand and two hundred.

150. Multiply eighty thousand and six hundred, by seven thousand and two.

151. Multiply three million, two hundred and forty thousand, by three hundred and twenty-four thousand.

152. Multiply three hundred and four thousand and seven hundred, by ninety-seven thousand, six hundred and three.

153. Multiply eight million, six hundred and forty-three thousand, by nine thousand, two hundred and thirty.

PRACTICAL EXAMPLES.

154. At 4 dollars a yard, how much will 25 yards of cloth cost ?

155. How many yards in seven pieces of muslin if each piece contain 28 yards ?

156. At 9 dollars a barrel, how much will 124 barrels of flour cost ?

157. At 112 dollars an acre, how much will a farm containing 270 acres cost ?

158. If a man travel 32 miles a day, how far will he travel in 24 days.

159. At 50 cents a bushel, how much will 136 bushels of apples cost ?

160. At 100 cents a day, how much will a laborer earn in 110 days ?

161. At 12 cents a pound, how much will 120 pounds of beef cost ?

162. At $97 each, how much will 15 horses cost ?

163. At $10 a barrel, how much will 225 barrels of flour cost ?

164. At $56 a head, how much will 25 cows cost ?

165. At $23 an acre, how much will 99 acres of land cost ?

166. In an orchard there are 84 rows of trees and 63 trees in each row ; how many trees are there ?

167. In a bale of sheeting there are 22 pieces, and in each piece 27 yards ; how many yards in all ?

168. In a hogshead there are 63 gallons ; how many gallons in 23 hhds.

169. In a box of calico there are 94 pieces, and each piece contains 35 yards ; how many yards in all ?

170. At $17 a barrel, how much will 211 barrels of molasses cost.

Many more such examples will be found among the Promiscuous Examples, after Division, and U. S. Money.

DIVISION.

Art. 15.—Division is finding either how often one number is contained in another; or one of as many equal parts of a number as are expressed by another number; thus,

We find either that 5 is contained in 30 6 times; or that 6 is one of 5 equal parts of 30.

The number to be divided is called the **Dividend;** the dividing number, the **Divisor;** the number found or the answer, the **Quotient;** and that part of the dividend less than the divisor, which is sometimes left after division, is called the **Remainder**.

The sign of division is ($\div$) a horizontal line between two dots, written after the dividend and before the divisor.

Division may also be expressed by writing the divisor under the dividend. In this way the remainder becomes a part of the quotient at the right hand. Such an expression is called a fraction; as $\frac{1}{2}$ (read) one-half; $\frac{1}{3}$, one-third; $\frac{2}{3}$, two-thirds; $\frac{3}{4}$, three fourths or quarters; $\frac{4}{5}$, four-fifths, &c., &c.

Illustration.—Divide 27 by 4. $27 \div 4$ or $\frac{27}{4} = 6$ and 3 left, or $6\frac{3}{4}$. 27 is the dividend, 4 the divisor, $6\frac{3}{4}$ the quotient, or 6 the quotient and 3 the remainder.

Remark 1.—Dividing a number by 2, divides it into 2 equal parts each of which is called one-half (written $\frac{1}{2}$;) dividing it by 3 divides it into 3 equal parts, each called one-third ($\frac{1}{3}$.) So dividing by 4 gives one-fourth ($\frac{1}{4}$,) by 5 one-fifth ($\frac{1}{5}$,) by 6, 7, 10, 15, &c., gives one-sixth ($\frac{1}{6}$,) one-seventh ($\frac{1}{7}$,) one-tenth ($\frac{1}{10}$,) one fifteenth ($\frac{1}{15}$,) &c.

2. The divisor and quotient correspond with the factors in multiplication, and the dividend with the product. Hence

3. When one factor and the product is given, the other factor may be found by division.

4. One of the factors must be of the same name as the dividend, and one of as many equal parts of it as are expressed by the other. If the divisor is one of the equal parts, the quotient is the number of

parts ; but if the divisor is the number of parts, the quotient is one of the parts ; thus,

20 cts. ÷ 5 cts. = 4 (equal parts); but 20 cts. ÷ 4 (equal parts) = 5 cts. (one of the equal parts of 20).

Art. 16.—There are two methods of division, **Short** and **Long**. In **short** division the process is mental, and only the result written. This is used when the divisor is less than 12. In **long** division the process and result are both written.

In dividing the divisor is usually written before the dividend, and the quotient in short division under it, thus,

$$3)\underline{15}$$
$$5$$

In long division after it, thus:

$$13)26(2$$
$$26$$

Short Division.

DIVISION TABLE WITHOUT THE QUOTIENTS.

2)	2	4	6	8	10	12	14	16	18	20	22	24
3)	3	6	9	12	15	18	21	24	27	30	33	36
4)	4	8	12	16	20	24	28	32	36	40	44	48
5)	5	10	15	20	25	30	35	40	45	50	55	60
6)	6	12	18	24	30	36	42	48	54	60	66	72
7)	7	14	21	28	35	42	49	56	63	70	77	84
8)	8	16	24	32	40	48	56	64	72	80	88	96
9)	9	18	27	36	45	54	63	72	81	90	99	108
10)	10	20	30	40	50	60	70	80	90	100	110	120
11)	11	22	33	44	55	66	77	88	99	110	121	132
12)	12	24	36	48	60	72	84	96	108	120	132	144

The same table promiscuously arranged:

2)	4	10	16	22	2	8	14	20	6	12	18	24
3)	6	15	24	33	3	12	21	30	9	18	27	36
4)	8	20	32	44	4	16	28	40	12	24	36	48
5)	10	25	40	55	5	20	35	50	15	30	45	60
6)	12	30	48	66	6	24	42	60	18	36	54	72
7)	14	35	56	77	7	28	49	70	21	42	63	84
8)	16	40	64	88	8	32	56	80	24	48	72	96
9)	18	45	72	99	9	36	63	90	27	54	81	108
10)	20	50	80	100	10	40	70	100	30	60	90	120
11)	22	55	88	121	11	44	77	110	33	66	99	132
12)	24	60	96	132	12	48	84	120	36	72	108	144

MENTAL EXERCISES.

At 2 cents each, how many peaches can be bought for 8 cents? 4? 12? 16? 20? 24? 28? 6? 10? 14? 18? 24?

Process.—If 2 cents will buy 1 peach, 8 cents will buy as many peaches as there are times 2 cts. in 8 cents, which are 4.

Or if 2 cents will buy 1 peach, half as many as the cents, 8 cents will buy half of 8 peaches, which are 4.

At 3 cents each, how many oranges can be bought for 6 cts? 12? 18? 24? 30? 36? 9? 15?

At 4 cents each, how many melons can be bought for 12 cents? 8? 16? 24? 32? 40? 48? 20? 28? 36? 44?

At 5 shillings a pound, how many pounds of tea can be bought for 15 shillings? 25? 35? 20? 30? 45? 55? 65? 40? 50? 60?

At 6 shillings a yard, how many yards of linen can be bought for 12 shillings? 24? 36? 48? 60? 18? 30? 42? 66? 72?

Divide the following numbers, and write the remainders over the divisors at the right hand of the quotient; as, 6)38
$6\frac{2}{6}$

Let the pupils also recite them without seeing the answers written:

2)	5	11	17	23	3	9	15	21	7	13	19	25
3)	8	17	27	35	5	14	23	32	11	20	28	37
4)	11	21	36	45	7	17	31	41	13	26	37	49
5)	13	27	44	58	8	22	37	53	17	23	48	63
6)	14	31	50	70	9	27	44	62	20	25	56	74
7)	17	37	60	81	11	29	53	74	23	31	67	89
8)	19	45	70	93	13	35	59	85	27	35	75	101
9)	21	50	80	103	15	40	65	97	31	39	84	112
10)	23	55	89	113	17	43	73	104	34	43	93	125
11)	25	61	90	127	19	47	78	113	36	48	100	131
12)	27	70	100	136	21	54	89	122	38	53	112	150

MENTAL EXERCISES.

At 7 cents a quart, how many quarts of milk can be bought for 14 cents? 28? 35? 42? 56? 49? 63? 21? 70?

At 8 shillings a bushel, how many bushels of corn can be bought for 24 shillings? 40? 16? 32? 48? 64? 80? 56?

At 9 dollars a barrel, how many barrels of flour can be bought for 18 dollars? 36? 27? 45? 63? 54? 72? 99? 81?

At 10 shillings a bushel, how many bushels of wheat can be bought for 30 shillings? 20? 50? 40? 70? 60? 90? 80? 100?

At 11 cents a pound, how many pounds of rice can be bought for 33 cents? 22? 44? 66? 55? 77? 99? 88? 110?

At 12 cents a pound, how many pounds of coffee can be bought for 24 cents? 48? 60? 36? 72? 96? 84? 120?

If three apples cost 6 cents, how much will one apple cost?

Process.—If three apples cost 6 cents 1 apple will cost as many cents as 3 is contained times in 6; or ⅓ of 6 cents, which is 2 cents.

If 5 peaches cost 15 cts., how much will 1 peach cost?

If 7 melons cost 42 cts., how much will 1 melon cost?

If 9 bushels of chestnuts cost 36 dollars, how much will 1 bushel cost?

If 8 barrels of flour cost 72 dollars, how much will one bbl. cost?

If 12 yards of cloth cost 84 dollars, how much will 1 yard cost?

EXAMPLES FOR THE SLATE.

EXAMPLE 1.—Divide 4325 by 5.

Process.—Since it is not easy to divide so large a number at once, it is considered as separated into parts; as 4000+300+20+5.

If 5 were contained in 4, the first figure at the left of the quotient would be thousands; but 5 is not contained in 4, therefore add 4000 to 300 and divide the hundreds. 5 is contained in 43 hundred (4300) 8 hundred (800) times, and 3 hundred (300) remainder. Add the remainder to the 2 tens (20) and divide the tens. 5 is contained in 32 tens (320) 6 tens times and 2 tens remainder, which add to 5 and divide. The last part of the quotient will be 5. The sum of all the parts is the quotient or answer required.

$$\left.\begin{array}{r} 5)4000 \\ 300 \\ 20 \\ 5 \end{array}\right\}\text{Parts of the dividend.}$$

$$\left.\begin{array}{r} 800 \\ 60 \\ 5 \end{array}\right\}\text{Parts of the quotient.}$$

Comm'nly written

$$\begin{array}{r} 5)\underline{4325} \\ \textit{Ans.}\ \ 865 \end{array}$$

The same result is obtained by the following process, commonly used:

5 into 43=8 and 3 remaining. Write the 8 under the last figure

divided, and prefixing the 3 remaining to the 2, divide 5 into 32=6 and 2 remaining. Write the 6 under the last figure divided, and thus proceed.

RULE FOR SHORT DIVISION.—*Write the divisor at the left of the dividend, and draw a line under the latter.*

Begin at the left, and divide each figure separately. Write each figure of the quotient under the last figure divided, and prefix the remainder, if there is any, to the next figure of the dividend and divide again.

If there is a remainder after dividing all the figures of the dividend, write it, with the divisor under it, at the right hand of the quotient.

If there is no other figure, a cipher must be written in any place except at the left hand.

PROOF.—*Multiply the divisor by the quotient and add the remainder. The product should be like the dividend.*

(2.) $2\overline{)135024}$ $3\overline{)135024}$ $4\overline{)135024}$

Write thus, and divide each number by 2, 3, and 4.

(3.)	246135	(11.)	416305	(19.)	165320
(4.)	350246	(12.)	520416	(20.)	510342
(5.)	461350	(13.)	631502	(21.)	621435
(6.)	502461	(14.)	310542	(22.)	205614
(7.)	613502	(15.)	421653	(23.)	136205
(8.)	153042	(16.)	532064	(24.)	103245
(9.)	264153	(17.)	643105	(25.)	124356
(10.)	305264	(18.)	520641	(26.)	235460

Divide each number by 5 and 6.

(27.)	679867	(33.)	689769	(39.)	486979
(28.)	579689	(34.)	786975	(40.)	579687
(29.)	497867	(35.)	879789	(41.)	397869
(30.)	938796	(36.)	787980	(42.)	278697
(31.)	479685	(37.)	979899	(43.)	697987
(32.)	596879	(38.)	676869	(44.)	708090

MENTAL EXERCISES.

If 6 pounds of sugar cost 75 cents, how much will 1 pound cost?

At 7 cents a pound how many pounds of fish can be bought for 25 cts.? 31 cts.? 37 cts.? 45 cts.? 65 cts.? 72 cts.? 90 cts.?

At 8 dollars a barrel, how many barrels of flour can be bought for $28? $18? $35? $50? $42? $60? $75? $83? $90? $100?

If 12 pounds of rice cost 100 cts., how much will 1 lb. cost?

At 12 cents a pound, how many pounds of starch can be bought for 30 cts.? 40 cts.? 50 cts.? 64 cts.? 70 cts.? 80 cts.? 90 cts.? 100 cts.? 110 cts.? 125 cts.? 150 cts.?

At 9 dollars a ton, how many tons of coal can be bought for $100? $75? $67? $55? $48? $42? $37? $35? $94?

If 11 tons of hay cost 125 dollars, what will 1 ton cost.

At 10 dollars a yard, how many yards of cloth can be bought for $34? $45? $56? $64? $75? $84? $96? $125?

EXAMPLES FOR THE SLATE.

(45.) 7)14725803 8)14725803 9)14725803

Write thus, and divide by 7, 8, and 9.

(46.)	69142580	(54.)	17452083	(62.)	12740853
(47.)	36914758	(55.)	61794250	(63.)	20859631
(48.)	69147280	(56.)	39641278	(64.)	31962745
(49.)	47258069	(57.)	50863194	(65.)	45270836
(50.)	58036914	(58.)	75208639	(66.)	53081964
(51.)	72580369	(59.)	42785301	(67.)	64195278
(52.)	80369147	(60.)	83096417	(68.)	78526309
(53.)	91472580	(61.)	94127528	(69.)	86304190

Divide the following numbers by 10, 11, and 12.

(70.)	96308527	(76.)	30852741	(82.)	30691472
(71.)	85274196	(77.)	27419630	(83.)	25803691
(72.)	74196308	(78.)	19630852	(84.)	50642839
(73.)	63085274	(79.)	93824605	(85.)	75039428
(74.)	52741963	(80.)	82493057	(86.)	64283917
(75.)	41963085	(81.)	71938246	(87.)	14725803

Art. 17.—Long Division.

Example 1. Divide 97836 by 18.

Process.—The same as in short division, except the quotient is placed at the right hand, and the products, with the remainders, are written under the parts of the dividend used.

Quotient figures must often be found by trial, and if the product of any trial figure and the divisor, is greater than the part of the dividend used, that figure is too great; on the other hand, if the remainder is greater than the divisor, the quotient figure is too small.

Very often a quotient figure may be found, or nearly so, by dividing the first figure or two in the dividend, by the first figure in the divisor, allowing more or less for carrying.

Divsr. Divid'd. Quoti'nt

$$
\begin{array}{l}
18)97386(5435\frac{6}{18} \\
\underline{90} \\
78 \\
\underline{72} \\
63 \\
\underline{54} \\
96 \\
\underline{90} \\
6
\end{array}
$$

Rule.—*Write the divisor at the left of the dividend, and leave a place for the quotient on the right.*

Find how many times the divisor is contained in the fewest left-hand figures of the dividend that contain it, and write the number in the quotient.

Multiply the divisor by this quotient figure and write the product under that part of the dividend which was found to contain the divisor. Subtract the product from the figures above it; to the remainder bring down the next figure in the dividend, and divide again.

Continue dividing till all the figures of the dividend are used, and if, after bringing down a figure, the number thus formed is less than the divisor, write a cipher (0) *in the quotient and bring down another figure.*

Proof.—*The same as in short division.*

EXAMPLES.

(2.)	(3.)	(4.)
13)147256(	14)152768(	15)175265(

Divide—

(5.) 3540 by 12	(17.) 5258 by 22	(29.) 158500 by 50
(6.) 8764 " 13	(18.) 10713 " 14	(30.) 140998 " 26
(7.) 5200 " 15	(19.) 11829 " 18	(31.) 116873 " 28
(8.) 5499 " 13	(20.) 11347 " 19	(32.) 127364 " 68
(9.) 7556 " 16	(21.) 39825 " 27	(33.) 295482 " 74
(10.) 4935 " 15	(22.) 48440 " 28	(34.) 316704 " 96
(11.) 9454 " 18	(23.) 37430 " 29	(35.) 190850 " 25
(12.) 6567 " 17	(24.) 13125 " 35	(36.) 151445 " 35
(13.) 5472 " 16	(25.) 30825 " 45	(37.) 135050 " 37
(14.) 6614 " 19	(26.) 32806 " 47	(38.) 55168 " 64
(15.) 7348 " 17	(27.) 19608 " 43	(39.) 5748435 " 63
(16.) 9182 " 21	(28.) 41088 " 52	(40.) 6480752 " 96

(41.) 215045924 by 86	(44.) 562752060 by 64
(42.) 405147456 " 56	(45.) 7254231 " 82
(43.) 459932616 " 66	(46.) 1053990 " 63

Art. 18.—Special Rules.

Division by numbers with ciphers on the right hand.

Ex. 47.—Divide 4720 by 100.

Process.—Cut off two figures from the right hand of the dividend; since this changes the value of the rest, the same as dividing by 100, 20 is the remainder unless regarded as a decimal fraction.

$$1,00)47,20=47\tfrac{20}{100}$$

Ex. 48.—Divide 76900 by 300.

Process.—The same as before, but divide also by 3, the other factor in the divisor.

$$\begin{array}{r} 3,00)769,00 \\ \hline 256\tfrac{100}{300} \end{array}$$

Rule.—*Cut off the ciphers on the right of the divisor, and and as many figures on the right of the dividend. Divide the rest of the dividend by the rest of the divisor, if greater than one, and if there is a remainder, annex to it the figures cut off from the dividend for the true remainder.*

EXAMPLES.

Divide—

(49.)	14100 by	600		(53.)	364000 by	6400
(50.)	365400 "	5000		(54.)	48000 "	1600
(51.)	138000 "	1000		(55.)	170000 "	400
(52.)	36009 "	1200		(56.)	1000009 "	300

Art. 19.—Division by Composite Numbers.

Ex. 57.—Divide 1732 by 24.

Process.—Dividing by 3 divides the number into one of three equal parts and 1 remainder; and dividing this by 8 divides the number itself into one of 3 times 8 equal parts, and one remaining to each of the three parts, which added to 1, the first remainder, makes 4 remainder.

$$\begin{array}{r} 3)\underline{1732} \\ 8)\underline{577+1} \\ 72+1=3 \\ 72+4 \end{array}$$

RULE.—*Divide by one of the factors, and the quotient thus found by the other factor. If there are remainders, multiply the second one by the first factor, and add the first remainder. Proceed in the same way if there are three or more factors.*

EXAMPLES.

Divide—

(58.)	2976 by	24	(70.)	186021 by	148
(59.)	134120 "	56	(71.)	119753 "	156
(60.)	155145 "	54	(72.)	246813 "	169
(61.)	105409 "	63	(73.)	2500000 "	200
(62.)	12148 "	35	(74.)	1435792 "	218
(63.)	1728 "	144	(75.)	1579248 "	227
(64.)	476345 "	100	(76.)	1681357 "	239
(65.)	567324 "	111	(77.)	1792460 "	244
(66.)	643142 "	115	(78.)	1864219 "	256
(67.)	792468 "	125	(79.)	2004312 "	260
(68.)	864219 "	131	(80.)	3600000 "	300
(69.)	975312 "	142	(81.)	1080000 "	1200

(82.)	700239	by	123	(90.)	470205	by	215
(83.)	1883187	"	249	(91.)	962984	"	276
(84.)	593583	"	241	(92.)	197776	"	376
(85.)	1886826	"	314	(93.)	255136	"	476
(86.)	478224	"	324	(94.)	124488	"	342
(87.)	854661	"	347	(95.)	5698546	"	829
(88.)	2539615	"	439	(96.)	5354320	"	635
(89.)	1200000	"	600	(97.)	2400000	"	800

(98.)	4874583	by	643	(109.)	7046606	by	898
(99.)	6079864	"	719	(110.)	7977489	"	923
(100.)	8264574	"	846	(111.)	3769248	"	948
(101.)	8095230	"	935	(112.)	3779008	"	548
(102.)	4674784	"	694	(113.)	4991875	"	625
(103.)	4663778	"	1246	(114.)	9691836	"	1234
(104.)	5332114	"	1234	(115.)	5237479	"	1823
(105.)	61142488	"	4136	(116.)	18219071	"	3001
(106.)	452491424	"	3143	(117.)	70287492	"	7117
(107.)	297396341	"	3047	(118.)	16736642	"	3497
(108.)	960000000	"	8000	(119.)	72000000	"	9000

(120.)	3013974002	by	3074
(121.)	25174363929	"	30243
(122.)	881137279449	"	90807
(123.)	153288487686	"	407091
(124.)	49062139937803	"	7001009
(125.)	156000000000	"	520000

126. Divide one hundred and twenty-seven thousand, by three thousand, seven hundred and forty-six.

127. Divide four million, six hundred and sixty-three thousand, seven hundred and seventy-eight, by three thousand, seven hundred and forty-three.

128. Divide ten million, two hundred and five thousand, seven hundred and twenty-one, by three thousand two hundred and forty-three.

129. Divide one hundred and forty-one thousand, by two thousand, three hundred and fifty.

130. Divide eight million, eight hundred and sixty thousand, and sixty, by one thousand and thirty.

131. Divide ninety-two million and eighty thousand, by one hundred and two.

132. Divide twenty-three million and forty thousand, by ninety-six hundred.

133. Divide two million, seven hundred and thirty-six thousand, three hundred and seventy, by three thousand and seven.

PRACTICAL EXAMPLES.

134. If it would take 1 man 3540 days to build a house, how long will it take 12 men to build it ?

135. At $65 each, how many cattle can be bought for $38740.

136. If $38805 will buy 597 cattle, what is the price per head ?

137. If 137 acres of land cost $17125, what is the price per acre ?

138. At $125 an acre, how many acres of land can be bought for $17250.

139. At $37 each, how many cows can be bought for $14689.

140. If 396 cows cost $14652, what is the price per head.

141. At $23 each, how many coats can be bought for $5451.

142. If 235 coats cost $5405, what is the cost of each ?

Many more such examples will be found among the Promiscuous Examples.

Art. 20.—General Principles in Division.

1. Multiplying the dividend, or dividing the divisor by any number, multiplies the quotient by that number; thus $24 \div 4 = 6$. $(24 \times 2) \div 4 = 12$, or (6×2). $24 \div (4 \div 2) = 12$.

2. Dividing the dividend, or multiplying the divisor, by any number divides the quotient by that number; thus, $24 \div 4 = 6$; $(24 \div 2) \div 4 = 3$, or $(6 \div 2)$; $24 \div (4 \times 2) = 3$, or $(6 \div 2.)$

3. Multiplying or dividing both the dividend and divisor by the same number, does not alter the quotient; thus, $24 \div 4 = 6$; $(24 \times 2) \div (4 \times 2) = 6$; $(24 \div 2) \div (4 \div 2) = 6$.

Art. 21.—Promiscuous Examples in Addition. Subtraction, Multiplication and Division.

[Pupils are now supposed to know *how* to add, subtract, multiply and divide. The following examples are designed to teach them *when* to apply the different rules. They should be fully explained by the pupils.]

MENTAL EXERCISES.

EXAMPLE 1.—A boy had 25 cents, and his father gave him 25 more; how many did he then have?

Answer.—He had as many as the sum of 25 cts. added to 25 cts., which is 50 cts. $25+25=50$.

Ex. 2.—A girl had 50 cents, and paid 25 of them for ribbon; how many had she left?

Ans.—She had as many as the difference between 25 and 50 cents; or as the remainder after subtracting 25 from 50 cts., which is 25 cts. $50-25=25$.

Ex. 3.—John has 25 cents, and his brother has 3 times as many; how many has his brother?

Ans.—He has as many as 3 times 25 cts., which are 75 cents. $25 \times 3 = 75$.

Ex. 4.—Three boys have 75 cents to be divided equally between them; how many will each boy have?

Ans.—Each one will have as many cents as there are times 3 in 75, or $\frac{1}{3}$ of 75 cts., which is 25 cts. $75 \div 3 = 25$.

Ex. 5.—If 75 cents be divided equally among some boys, and each one receive 25 cents, how many boys will there be ?

Ans.—There will be as many boys as there are times 25 cts. in 75 cts., which are 3 times. Therefore there will be 3 boys. $75 \div 25 = 3$.

6. At 4 cts. each, how many oranges can be bought for 32 cents.

7. At 4 cents each, how much will 9 oranges cost ?

8. If 12 oranges cost 48 cents, what is the price of each ?

9. A boy picked 20 quarts of chestnuts one day, and 14 quarts the next ; how many did he pick in the two days ?

10. A boy had 20 quarts of chestnuts, and sold 14 quarts of them ; how many had he left ?

11. At 9 cents a quart, how many quarts of plums can be bought for 72 cents ?

12. At 9 cents a quart, how much will 12 quarts of plums cost ?

13. If 10 quarts of plums cost 100 cents, what is the price per quart ?

14. James has agreed to pick 100 quarts of strawberries in 5 days ; how many must he pick each day ?

15. James has agreed to pick 144 quarts of strawberries ; how long will it take him if he pick 12 quarts each day ?

16. James picked 12 quarts of strawberries a day ; how many quarts did he pick in 8 days ?

EXERCISE I.—FOR THE SLATE OR BLACKBOARD.

1. A farmer has 327 sheep in one flock, and 258 in another ; how many has he in both ?

2. A farmer had 640 lambs, and has sold 325 ; how many has he left ?

3. A farmer has 1950 bushels of oats, and can carry to market 75 bushels at a load ; how many loads will there be ?

4. A farmer carried to market 25 loads of oats, and 75 bushels at a load ; how many did he carry ?

5. A farmer has 1800 bushels of oats, and wishes to carry

them all to market in 24 loads; how many bushels must he carry each time?

6. If a horse eat 12 quarts of oats a day, how long will it take him to eat 1728 quarts?

7. If a horse eat 12 quarts of oats a day, how many will he eat in 132 days?

8. If a horse eat 1740 quarts of oats in 145 days, how many will he average each day?

9. If each horse eat 12 quarts of oats a day, how many horses will eat 1800 quarts in the same time?

10. If 100 horses eat 1000 quarts of oats in a day, how many quarts on an average will each horse eat?

MENTAL EXERCISES.

1. Henry picked 36 quarts of cherries and his brother 27 quarts; how many more did Henry pick than his brother?

2. At 8 cents a quart, how many quarts of cherries can be bought for 80 cents?

3. At 8 cents a quart, how much will 12 quarts of cherries cost?

4. If 11 quarts of cherries cost 99 cents, what is the price per quart?

5. Henry picked 29 quarts of cherries and his brother 37, how many did they both pick?

6. At 18 cents each, how many knives can be bought for 72 cents?

7. At 18 cents each, how much will 5 knives cost?

8. If 6 knives cost 90 cents, what is the price of each?

EXERCISE II.—FOR THE SLATE OR BLACKBOARD.

11. A barrel of flour contains 196 pounds, how many pounds are there in 679 barrels?

12. How many barrels of the same will contain 97412 pounds?

13. If there are 97216 pounds of flour in 496 barrels, how many pounds are there in each?

14. A flour dealer bought 5624 barrels of flour, and has since sold 3768 of them, how many has he left?

15. At $14 a barrel, how many barrels of flour can be bought for $11578 ?

16. At $15 a barrel, how much will 3246 barrels of flour cost ?

17. If 45 barrels of flour cost $540, what is the price per barrel ?

18. If a ship sail 96 miles each day, how long will it take her to sail 2688 miles ?

19. If a ship sail 96 miles each day, how far will she sail in 27 days ?

20. If a ship sail 2784 miles in 29 days, how far will she sail on an average each day ?

MENTAL EXERCISES.

1. If 56 yards of calico will make 7 dresses, how many yards will make 1 dress ?

2. If 9 yards of calico will make a dress, how many yards will make 8 dresses ?

3. If 9 yards of calico will make a dress, how many dresses will 45 yards make ?

4. Jane has 26 yards of ribbon for trimming her dress, and Anna 17, how many more has Jane than Anna ?

5. How many yards have both together ?

6. At $4 a week, how many weeks can a person board for $416 ?

7. At $6 a week, how much will 26 weeks' board cost ?

8. If 23 weeks' board cost $115, what is the price per week?

EXERCISE III.—FOR THE SLATE OR BLACKBOARD.

21. In a large hotel, 857 pounds of beef are consumed daily ; how many pounds will be consumed in 365 days.

22. If 1000 men consume 137970 pounds of beef in 365 days, how much will they consume in a day ?

23. If 378 pounds of beef be consumed daily, how long will it take to consume 137592 pounds ?

24. At $18 a barrel, how many barrels of sugar can be bought for $32166 ?

25. At $18 a barrel, how much will 1700 barrels of sugar cost?

26. If 1700 barrels of sugar cost $32300, what is the price per barrel?

27. A merchant bought 320 barrels of molasses for $4800; 2000 barrels for $28000; 1900 barrels for $29730; how much did they all cost?

28. At $15 a barrel, how many barrels of molasses can be bought for $29730?

29. At $14 a barrel, how much will 2000 barrels of molasses cost?

30. If 320 barrels of molasses cost $4800, what is the price per barrel?

EXERCISE IV.

31. At $67 an acre, how many acres of land can be bought for $122878?

32. At $53 an acre, how much will 234 acres of land cost?

33. If 872 acres of land cost $47088, what is the price per acre?

34. If a person travel 26 miles a day, how far will he travel in 14 days?

35. If a person travel 52 miles a day, how long will it take him to travel 728 miles?

36. If a person travel 390 miles in 15 days, at what rate per day does he travel?

37. If a man travel 1020 miles the first week, and 965 the next, how far will he travel in the two weeks?

38. If a man travel away from home 3400 miles, and 765 miles on his return, how far from home will he be?

39. A manufacturer paid 19 journeymen $57 apiece, what was the amount paid?

40. A manufacturer paid his journeymen $1140, and each one received $57, how many were there?

EXERCISE V.

41. If 235 barrels of mackerel cost $3055, what is the price per barrel?

42. At $14 a barrel, how much will 235 barrels of mackerel cost ?

43. At $13 a barrel how many barrels of mackerel can be bought for $3042 ?

44. A speculator having $15000 lost $7000, and afterwards gained $9653, how much did he then have ?

45. At $19 each, how much will 346 overcoats cost ?

46. If 345 overcoats cost $6555, what is the price of each ?

47. At $18 each, how many overcoats can be bought for $6228.

48. At 13 cents a pound, how many pounds of cheese can be bought for 8775 cents.

49. At 24 cents a bushel, how much will 496 bushels of apples cost ?

50. If a man travel 28 miles a day, how long will it take him to travel 4256 miles ?

EXERCISE VI.

51. A huckster carried 6867 melons to market in 27 loads; how many in each load ?

52. If a huckster carries 325 melons at a load, how many will he carry in 21 loads ?

53. A huckster carries 337 melons at a load; in how many loads will he carry 8175 melons ?

54. A huckster took to market at one time 179 cabbages; at another 268; at another 947; and at another 144. He finally sold 1000 and brought back the rest for his cattle; how many did he bring back ?

55. At 63 cents a basket, how much will 325 baskets of peaches cost ?

56. At 65 cents a basket, how many baskets of peaches can be bought for $208 ?

57. If 325 baskets of peaches cost $195, what is the price per basket ?

58. What cost 45 cows, at $40 each ?

59. A carriage-maker sold 77 carriages, for $212 each; how much did he receive for all of them ?

60. A speculator bought 1400 acres of land, at $56 an acre, and selling it he gained $6600; for what did he sell it per acre?

EXERCISE VII.

61. John's arithmetic contains 296 pages, and he wishes to review it for examination in 18 days; how many pages must he review each day?

62. After reviewing it 14 days, how many pages would be left?

63. James reviewed 18 pages a day in the same arithmetic; in how many days could he finish it?

64. A country merchant went to New York to buy goods, and paid for them in cash 1215 dollars; in notes 1238 dollars, in barter 2512 dollars; all his expenses were 65 dollars; he sold them for 6000 dollars; what did he gain?

65. A horse dealer having 2549 dollars, bought 21 horses, and after paying for them had 113 dollars left; what was the average price of the horses?

66. At 200 dollars each, how many horses can be bought for 3000 dollars?

67. At 150 each, how much will 16 horses cost?

68. If a family's expenses are 18312 dollars in 24 years; how much do they average a year?

UNITED STATES MONEY.

Art. 22.—United States or Federal Money is the legal money of the United States.

It consists of **Eagles, Dollars, Dimes, Cents,** and **Mills.**

Its **Coins** are in—

Gold.—*Double Eagle, Eagle, Half Eagle, Quarter Eagle, Three Dollars,* and *Dollar.*

Silver.—*Dollar, Half Dollar, Quarter Dollar, Dime,* (Ten Cents,) *Half Dime, Three Cents.*

United States Money is a species of compound numbers; but may also be treated much like simple numbers, since it increases in the same ratio.

TABLE.

10 mills (m.) make	1 cent. (ct.)
10 cents make	1 dime.
10 dimes, or 100 cents make	1 dollar. ($.)
10 dollars make	1 eagle,

Art. 23.—Aliquot Parts of a number are such as will divide it without a remainder.

ALIQUOT PARTS OF U. S. MONEY.

5	mills	= ½	cent.	33⅓	cents	= ⅓	dollar.
10	cents	= $\frac{1}{10}$	dollar.	37½	"	= ⅜	"
12½	"	= ⅛	"	50	"	= ½	"
16⅔	"	= ⅙	"	62½	"	= ⅝	"
20	"	= ⅕	"	75	"	= ¾	"
25	"	= ¼	"	87½	"	= ⅞	"

Art. 24.—NOTATION OF U. S. MONEY.

RULE.—*Write the dollars as in simple numbers, with a point* (.) *on the right; next to this, if there are cents and mills, write two figures or ciphers for cents, and then one figure or cipher for mills.*

EXAMPLES TO BE WRITTEN.

1. Ten dollars fifteen cents and seven mills. $10.157
2. Seven dollars seven cents and seven mills.
3. Sixty dollars and six mills.
4. Fifty dollars fifty cents and five mills.
5. Nine dollars six cents and eight mills.
6. Sixty-three dollars four cents and two mills.
7. One hundred dollars and twenty-five cents.
8. Two hundred and ten dollars and five mills.
9. Seventy-five dollars two cents and one mill.
10. Five hundred dollars and fifty cents.
11. Twelve and a quarter dollars.
12. Sixty-one dollars thirty-seven and a half cents.
13. Twenty and a half dollars.
14. Seventy-five and three quarter dollars.
15. One thousand dollars twelve and a half cents.

Art. 25.—Numeration of U. S. Money.

Rule.—*Read the figures before the separating point as dollars; the next two (if there are any) as cents, and the third as mills.*

EXAMPLES TO BE READ.

1. $12.375. Twelve dollars, thirty-seven cents and five mills or half a cent.

2. $10.25	7. $250.043	12. $ 21.25
3. $ 9.375	8. $125.000	13. $202.458
4. $12.	9. $87.00	14. $405.50
5. $12.00	10. $121.	15. $700.
6. $12.000	11. $63.405	16. $700.00

Art. 26.—Reduction of U. S. Money.

Reduction of U. S. Money is changing dollars to cents, and cents to mills, or mills to cents, and cents to dollars, &c.

Since there are 100 cents in 1 dollar, and 10 mills in 1 cent, any number of dollars is equal to as many hundred cents or thousand mills; thus,

$1.=100 cts.=1000 m. $12.=1200 cts.=12000 m. $3.87=387 cts.=3870 m. $4.375=437 cts. 5 m.=4375 m.; hence,

Rules.—*To reduce dollars to cents, multiply by* 100 *or annex two ciphers.*

To reduce cents to mills, multiply by 10 *or annex one cipher.*

To reduce dollars to mills, annex three ciphers.

To reduce dollars and cents to cents, or dollars, cents, and mills to mills, remove the separating point.

This is the same as reducing the dollars and adding the cents or mills.

Again, since 10 mills make 1 cent, and 100 cents make 1 dollar, every 10 mills in any number make 1 cent, and every thousand mills or hundred cents make 1 dollar; thus,

1000 m.=100 cts.=$1. 15000 m.=1500 cts.=$15. 250 cts.=$2.50. 6375 m.=637 cts. 5 m.=$6.375; hence, Rules.—*To reduce mills to cents, divide by* 10, *or point off the right hand figure.*

To reduce cents to dollars, divide by 100, *or point off two figures.*

To reduce mills to dollars, point off three figures.

MENTAL EXERCISES.

How many mills in 2 cents? 3? 5? 8? 9? 10? 13? 16? 20? 23? 28? 31? 40? 56? 75?

How many cents in $2? $4? $5? $7? $10? $15? $20? $24? $36? $42? $50? $75? $87? $90? $100?

How many cents in 10 mills? 20? 40? 50? 65? 24? 30? 36? 45? 50? 100? 210? 750?

How many dollars in 200 cents? 400? 500? 800? 1000?

How many cents in 20 dimes? 30? 50? 80?

How many dimes in 20 cents? 30? 50? 80?

How many cents in $3? 20 dimes? 30 mills? $5? 50 mills? 50 dimes? $60? 60 dimes? 60 mills?

How many dollars, &c., in 125 cents? 125 dimes? 2000 mills? 3250 mills? 375 cents?

How many mills in 25 cents and 3 mills? 20 cts. 5 mills? 7 cts. 5 mills? 80 cts. 9 mills?

How many cents in $2.37? $6.25? 75 dimes? 75 mills? $75?

EXAMPLES FOR THE SLATE.

Reduce or change—

1. $25 to cents.
2. 10250 cents to dollars, &c.
3. 250 mills to cents.
4. 1100 cents to mills.
5. 1100 cents to dollars.
6. $3.75 to mills.
7. $10.25 to cents.
8. 170 cents to mills.
9. 170 mills to cents.
10. 170 cents to dollars.
11. $70 to mills.
12. $60 to cents.
13. $24.25 to mills.
14. $30.375 to mills.
15. $35.50 to cents.
16. 75375 mills to dollars.
17. 75375 cents to dollars.
18. 34000 mills to cents.
19. $45 to mills.
20. $250 to mills.

21. 1275 mills to dollars, &c.
22. 1000 mills to cents.
23. 1000 cents to dollars.
24. 1375 mills to dollars, &c.
25. 1000 dimes to dollars.
26. $927.25 to cents.
27. 3760 cents to dollars.
28. 1275 cents to mills.
29. 1325 mills to dollars.
30. 675 mills to cents.
31. $87 to cents.
32. $16.37 to cents.
33. $21.04 to mills.
34. 13405 cents to dollars.
35. 759 cents to mills.
36. 287 cents to dollars.
37. $300 to mills.
38. 1200 mills to dollars.

Art. 27.—Application of the Fundamental Rules to U. S. Money.

Since numbers in U. S. money increase from right to left in a ten-fold ratio, the same as simple numbers, they may be added, subtracted, multiplied, and divided by *nearly* the same rules.

Art. 28.—Addition of U. S. Money.

Rule.—*Write the numbers so that the separating points will be under one another, and proceed as in simple Addition.*

Example 1.—Add $5.125, $17.062, $10.43, $8.055, $15.706.

Process.

	$ c.m.
	5.12 5
	17.06 2
	10.43
	8.05 5
	15.70 6
Ans.	$56.37 8

2. Add $18.15, $24.45, $7.21, $9.38, $11.33.

3. Add $19.041, $17.315, $112.18, $75.873, $60.50.

4. Add $44.76, $28.19, $18.657, $270.508, $87.60, $67.005.

5. Add $10.625, $112.35, $1.75, $11.875, $100, $17.37.

6. What is the sum of $21 10cts., $17 4cts. 6m., $23 17cts. 3m., $19 18cts. 6m., $25, $16 8 cts., $15 5m.?

7. Add $200, $43.875, $56.937, $18.50. $12.315.

8. What is the amount of $304 50cts., $304 4m., $820 35cts. 4m.

9. What is the sum of $25 8cts., $40 21cts. 3m., $108 5m., $63 4cts., $312 1ct. 7m., $1000 15cts., $50 8m.?

10. Add $6 6cts. 3m., $14 17cts., $21 8cts. 6m., $25 50cts., $17 8m., $100 10cts. 3m., $1 1ct. 1m., $10 10cts. 7m.

11. Add $5 4cts. 3m., $1 14cts., $98, $2 2m., 3cts. 3m., $15 16cts. 4m.

12. What is the amount of $300, $4 4cts, $50 5m., $70 7cts., $45 5m.?

13. What is the sum of 35 dollars 6 cents 7 mills, $11 4cts. 6m., $17 18cts. 9m., $400 83cts., $12 20cts. 2m.?

14. Add 3 dollars 12 cents 5 mills, $50 50cts., $300 6m., 75cts., $75 7cts. 5m., $201 3cts.

15. What is the sum of $18¾, $12½, $6½, $¾, $5½? (See Table of Aliquot Parts.)

16. What is the amount of 9 dollars 62½ cents, 87½cts, $15¼, $108 62½ cts., $1 8cts, $27¾, $63 12¼cts.?

17. Add 10½ dollars, 87½ cents, $105 62½cts., $16¼ 37½ cents, $21¾.

18. Add 9 dollars 12½ cents, $6 3m., $28 87½cts, $56 5cts. 5m.

19. What is the sum of $39, $109 12½cts., 5m, $5 5m., $1 12½cts.

20. What is the amount of $67 12½ cts., $60¾, $62½, $¾, $1½, $¼?

Add the following numbers in U. S. money—

21. Three hundred dollars and three cents,
Three dollars and three mills,
Five hundred dollars,
Five hundred cents,
Five hundred mills.

22. Eighty-five dollars,
Sixty dollars sixty-two and a half cents,
Thirty-seven and a half cents,
Forty dollars four cents and five mills,
Forty cents and four mills,
Forty-four mills.

23. Seventy dollars,
Five dollars, eighty-seven and a half cents,
Fifty dollars fifty cents and five mills,
Six and three-quarter dollars,
Five and a half dollars and a half cent,
Ten and one quarter dollars.

24. Two dollars and two mills,
Seven dollars eighty-seven and a half cents,
Nine dollars thirteen cents and three mills,
Sixty-seven dollars and eight mills,
Four dollars and seventy-five cents,
One and three-quarter dollars.

25. Seven dollars and eighty cents,
Twelve dollars and twenty-five cents,
Ten dollars and two mills,
Sixty-five dollars,
Six cents and five mills,
One dollar one cent and one mill.

26. Ten eagles ten dollars ten dimes ten cents and ten mills.

27. One half eagle one half dollar and one half cent.

28. Thirty-seven and a half dollars,
Thirty-seven and a half cents,
Twenty-four and three-quarter dollars,
Six and a quarter dollars and a half cent,
Twelve and a half cents.

29. A family has paid for beef \$19.15, flour \$17.375, butter \$10.125, and sugar, \$4.65, what is the amount?

30. A farmer bought a horse for \$135, a pair of oxen for \$97.375, a cow for \$35, and 20 sheep for \$50, how much did he pay for them all?

31. A young man bought a suit of clothes for \$56, a watch for \$87½, a watch chain for \$12¾, and a pair of gloves for 87½ cents, what did they all cost?

32. A young lady bought a silk dress for \$26, a shawl for \$18¾, a bonnet for \$7¼, and a pair of gaiters for \$3.37½, what did they all cost?

Art. 29.—Subtraction of U. S. Money.

Rule.—*Write the numbers so that the separating points will be under each other, and proceed as in simple Subtraction.*

Example 1. From \$125 take \$37.053.

Process.

	$ c. m.
	125.00 0
	37.05 3
Ans.	\$87.94 7

2. From \$39.25 take \$16.246.
3. From \$127.384 take \$15.60.
4. From \$95.28 take \$45.183.
5. From \$118.05 take \$67.45.
6. From \$95 take \$33.60.
7. From \$25 take 25 cents.
8. From \$100 take 100 cents.
9. From \$10 take 10 cents.
10. From \$1 take 1 cent.
11. From 1 cent take 1 mill.
12. From \$5 take 5 mills.
13. A man paid \$175 for a carriage, and \$162½ for a horse, how much more did he pay for the carriage than the horse?
14. A merchant bought a hogshead of molasses for \$26¾ and sold it for \$35, how much did he gain?
15. A young man sold his watch for \$37½, and it cost \$45. how much did he lose?
16. A lady having \$50, spent \$27.62½ in shopping, how much was left?
17. A laborer has earned \$100, and been paid \$53.87½, how much is still due to him?
18. A man owed \$500, and has paid \$263.62½, how much does he still owe?
19. A person having bought a bill of goods amounting to \$7.12½, gave in payment a ten dollar bill, how much change did he receive?
20. One man earns \$1.62½, and another \$1¾ a day, how much more does one earn than the other?

Art. 30.—Multiplication of U. S. Money.

Rule.—*Proceed as in simple Multiplication, and place the separating point as far from the right as it is in the multiplicand, sometimes used as the multiplier.*

Example 1.—Multiply $8.625 by 15.

Process.

$$\begin{array}{r} \$8.625 \\ 15 \\ \hline 43125 \\ 8625 \\ \hline \textit{Ans. } \$129.375 \end{array}$$

Multiply—

(2.)	$112.08	by	7	(3.)	$1.25	by	10
(4.)	$94.375	"	9	(5.)	$12.50	"	100
(6.)	$65.	"	12	(7.)	$48.375	"	35
(8.)	$100.	"	100	(9.)	$10.	"	100
(10.)	75 cts.	"	14	(11.)	5 mls	"	25
(12.)	12½ cts	"	10	(13.)	$12½	"	20
(14.)	$8¼	"	5	(15.)	$18¾	"	1000

In multiplying U. S. money by 10, 100, &c., it is sufficient to remove the separating point as many places to the right as there are ciphers in the multiplier; as $4.50×100=$450.

16. At $1.25 a bushel, how much will 20 bushels of corn cost?

17. At 62½ cents a bushel, what will 15 bushels of apples oost?

18. If an acre of land cost $87½, how much will 100 acres cost?

19. If a cord of wood cost $6¾, how much will 12 cords cost?

20. At $5.62½ a yard, how much will 3 yards of cloth cost?

21. At 12½ cents a quart, how much will 7 qts. of cherries cost?

22. If one doz. eggs are worth $¼, how much are 100 doz. worth?

23. If a pound of butter is worth 22½ cts., what are 14 pounds worth?

24. At $10¾ a ton, what are 10 tons of hay worth?

Art. 31.—Division of U. S. Money.

RULE.—*Proceed as in simple Division, observing that either the divisor or quotient must be of the same name as the dividend, reduced if necessary to cents or mills, and have the separating point in the same place; while the other has no separating point except in decimal fractions.*

The dividend is the price of the whole quantity.

The price of the whole divided by the quantity gives the price of each part; or

The price of the whole divided by the price of each part gives the quantity.

\$5.00÷100 lbs., etc.=5 cts. \$5.00÷.05 cts.=100 lbs., etc.

EXAMPLE 1.—Divide \$316.753 by 5.

Process.—Short Division—

```
    5)$316.753
Ans.  $63.350+3
```

Ex. 2.—Divide \$225.50 by 18.

Process.—Long Division—

If there is a remainder after dividing any given number of dollars or cents, reduce the dollars to cents and the cents to mills.

```
   $  c.   $ c.m.
18)225.50(12.527+
   18
   --
    45
    36
    --
     95
     90
     --
      50
      36
      ---
      140 mills.
      126
      ---
       14
```

Ex. 3.—Divide \$460 by 62½ cents = 625 mills.

Process.—

```
c.m.  $   c.m Ans.
625)460 00 0(736
    437 5
    -----
      2250
      1875
      ----
       3750
       3750
```

Divide—

Ex. 4.	\$124.64	by	\$3.28,	or into	38	equal parts.
5.	\$62.64	"	\$7.83,	"	8	"
6.	\$108.837	"	\$12.093,	"	9	"
7.	\$1862.42	"	\$35.14,	"	53	"
8.	\$368.288	"	\$23.018,	"	16	"
9.	\$2.125	"	\$0.125,	"	17	"
10.	\$2.50	"	\$0.25,	"	10	"

To divide U. S. Money by 10, 100, &c., it is sufficient to remove the separating point as many places to the left as there are ciphers in the divisor ; thus, \$45÷100=\$0.45.

Ex. 11. At 12½ cents a pound, how many pounds of sugar can be bought for \$2.00.

12. If 24 pounds of sugar cost \$3.00, what is the price per pound ?

13. At 28 cents a pound, how many pounds of butter can be bought for \$2.24 ?

14. If 9 pounds of butter cost \$2.52, what is the price per pound ?

15. At 65 cents a bushel, how many bushels of corn can be bought for \$130 ?

16. If 200 bushels of corn cost \$120, what is the price per bushel ?

17. At 37½ cents a bushel, how many bushels of oats can be bought for \$21 ?

18. If 60 bushels of oats cost \$24, what is the price per bushel ?

19. If a bushel of wheat cost \$1.37½, how many bushels can be bought for \$49.50 ?

20. If 40 bushels of wheat cost \$55, what is the price of one bushel ?

21. At \$1¼ a day, in how many days can a man earn \$25 ?

22. If a man earn \$20 in 16 days, how much is it a day ?

Art. 32.—Aliquot Parts of a Dollar.

(*See Table of U. S. Money.*)

When the price of anything is an aliquot part of $1, it shortens the operation,

To divide by the number of parts, instead of multiplying by the number of cents, and to multiply instead of dividing.

EXAMPLE 1. What will 49 yards of calico cost, at 25 cts. a yard?

Process.—Since 1 yard costs 25 cts.=(\$$\frac{1}{4}$,) 1 quarter of a dollar, 49 yards will cost 1 quarter of 49 dollars.

$$\begin{array}{r} 4)49 \\ \hline \$12\frac{1}{4} = 12.25 \end{array}$$

Ex. 2. How many yards of calico can be bought for $10, at 25 cts. a yard?

Process.—Since 25 cents, or (\$$\frac{1}{4}$) will buy one yard, $1 will buy 4 yards, and $10 will buy (4 times 10) 40 yards.

$$\begin{array}{r} 10 \\ 4 \\ \hline 40 \end{array}$$

EXAMPLES.

3. At 20 cts. each, what will 400 writing books cost?

4. At 25 cents each, how many writing books can be bought for $5?

5. At $33\frac{1}{3}$ cts. a gallon, what cost 30 gallons of vinegar?

6. At $33\frac{1}{3}$ cts. a gallon, how many gallons of vinegar can be bought for $12?

7. At 50 cents a bushel, how many bushels of apples can be bought for $25?

8. At 50 cts. a bushel, how much will 100 bushels of apples cost?

9. At $12\frac{1}{2}$ cts. a pound, how many pounds of rice can be bought for $6?

10. At $12\frac{1}{2}$ cents a pound. how much will 16 pounds of rice cost?

11. If a boy earn $33\frac{1}{3}$ cts. a day, how much will he earn in 6 days?

12. If a boy earn $33\frac{1}{3}$ cts. a day, in how many days will he earn $1?

Art. 33.—Price per Hundred or Thousand.

When the given price is per hundred, call the dollars cents; when per thousand, call them mills, which will be the price of *one* of the things specified.

Example 1.—What will a bale of hay weighing 156 pounds cost at $1 per hundred?

Process.—$1=100 cts., and 100 cts. per hundred pounds is 1 cent per pound, and 156×.01=$1.56.

$$\begin{array}{r} 156 \\ .01 \\ \hline \textit{Ans. } \$1.56 \end{array}$$

Ex. 2. What cost 12500 shingles at $18 per thousand?

3. What cost 7500 bricks at $9 per thousand?

4. At $9 per thousand, how many bricks can be bought for $63?

5. What cost 56 pounds of flour at $4 per hundred?

6. What cost 615 feet of pine boards, at $21 per thousand?

Examples of this kind, in which the price is cents, involve decimal fractions, but the process is the same.

Art. 34.—Promiscuous Examples in the Fundamental Rules, including U. S. Money.

EXERCISE I.

1. A young man bought a horse for $150; a watch for $53.875; a suit of clothes for $46.937; a hat for $4.50; a pair of boots for $4.00, and some other things for $2.313; what was the amount?

2. From $100 subtract $1, 1 cent and 1 mill.

3. At 25 cents a yard, how many yards of ribbon can be bought for $4.

4. At 25 cts. a yard, how much will 12 yards of ribbon cost?

5. If 20 yards of ribbon cost $5, what is the price per yard?

6. At $8.05 a ton, how much will 20 tons of hay cost?

7. At 34 cents a yard, how many yards of muslin can be bought for $30.26?

8. In a case of broadcloth there are 19 pieces, containing in all 437 yards; how many yards in a piece on an average?

9. A farmer carried to market 20 loads of oats, and each load contained 75 bushels; how many bushels in all?

10. A farmer had 1200 bushels of wheat, and could carry 50 bushels at a load; how many loads were there?

11. A lady went shopping with \$5 in her purse; she paid 75 cents for a collar; \$1,50 for kid gloves; 50 cts. for ribbon and 25 cts. for needles and pins; how much had she left?

12. At \$1.60 a day, how much will a man earn in 40 days?

13. At \$1.12½ a bushel, how many bushels of wheat can be bought for \$208.

14. At 43½ cts. a bushel, what will 750 bushels of buckwheat cost?

15. A farmer owed a merchant \$500, and paid him 435 bushels of oats at 45 cts. a bushel; how much does he still owe?

EXERCISE II.

16. At 7½ cts. a quart, how many quarts of cherries can be bought for \$1.35?

17. If a clerk's salary is \$800 a year, how much is it for each day he is employed in business (313 in the year)?

18. At \$2.25 a day, how much will a laborer earn in 313 days working time in a year?

19. At \$17.565 an acre, how many acres of land can be bought for \$2722.575?

20. If 3 men gain \$1000, what is each one's equal share?

21. A lady having \$26, bought a silk dress for \$13.10; a shawl for \$6, and gloves for 75 cts; how much had she left?

22. What cost 8 pieces of calico, each containing 19 yards, at 23 cts. a yard?

23. At \$5.67 a yard, how many yards of cloth can be bought for \$136.08?

24. If 168 lambs cost \$451.92, what is the price of each?

25. What cost 17 firkins of butter, each containing 51 pounds, at 14 cents, 7 mills per pound?

EXERCISE III.

26. At $12 a barrel, how many barrels of flour can be bought for $1512 ?

27. If 670 pounds of cheese cost $87.10, what is the price per pound ?

28. What number multiplied by 9 will produce 315 ?

29. At $8 a ton, how many tons of coal can be bought for $1728 ?

30. A farmer sold his pork for $21.75, and received sugar, $3.75 ; molasses, $2.50 ; tea, $1.35 ; cheese, $1 ; pepper, 25 cts. ; ginger, 18 cts. ; the rest in cash ; how much cash did he receive ?

31. How much coffee at 13 cents a pound can be bought for $18.59 ?

32. At $1.43 a day, how much will a man earn in 312 days ?

33. If 54 bushels of wheat cost $67.50, what is the price per bushel ?

34. At $10 and 5 mills an acre, what will 150 acres of land cost ?

35. If 17 bags of coffee, each weighing 51 pounds, cost $127.449, what is the price per pound ?

EXERCISE IV.

36. If 137 shares of bank stock are worth $17125, what is a share worth ?

37. If 1000 men consume 856 pounds of beef in a day, how many pounds will last them 365 days ?

38. At $34 a barrel, how many barrels of sugar can be bought for $128.690 ?

39. A lady bought a cloak for $25.125, a muff for $12.375, a bonnet for $9.15, and gave the merchant a $50 bill ; how much change was due her ?

40. What cost 24 arithmetics, at $0.37½ each ?

41. At $0.37½ each, how many arithmetics can be bought for $13.50 ?

42. If 18 arithmetics cost $6.75, what is the price of each ?

43. If a horse travel 34 miles a day, how far will he travel in 75 days ?

44. If a horse travel 34 miles a day, how long will it take him to travel 1700 miles ?

45. What cost 8 barrels of sugar containing 225 lbs. each, at $6\frac{1}{2}$ cents a pound ?

EXERCISE V.

46. A drover bought 397 cattle for $14689, what is the average price of each ?

47. In a pile there are 237 boards, each containing 23 square feet, how many square feet in all ?

48. A manufacturer has a contract for 273249 yards of calico to be made in 313 days ; how many yards must he average daily ?

49. A man bought a lot for $375, paid for building a house on it $750, and for improvements 160.87\frac{1}{2}$; he then sold the place for $1500 ; what did he gain ?

50. A laborer was paid $23.75 for 19 days' work ; how much did he have a day ?

51. At 1.12\frac{1}{2}$ a day, how much will a man earn in 16 days ?

52. If 25 men earn $35.50 in a day, how much will they earn in 50 days ?

53. If 25 men earn $1775 in 50 days, how much will one man earn in the same time ?

54. At 37$\frac{1}{2}$ cents a bushel, how much corn can be bought for $58.50 ?

55. Bought 20 pieces of muslin, each measuring 19 yards, for $87.40 ; what was the price per yard ?

EXERCISE VI.

56. A farmer paid his hired man $165 for 12 months, how much was it a month ?

57. A laborer dug 29 bushels of potatoes a day, how many did he dig in 42 days ?

58. A laborer agreed to dig 1212 bushels of potatoes in 42 days, how many must he dig a day on an average ?

59. Reduce 37500 mills to dollars.

60. A merchant bought some cloth for 26 dollars and 3 cents, a bale of sheeting for 50 dollars and 90 cts., 20 pieces of calico for 49 dollars, 1 cent, 12 pieces of merino for $108.14, 10

pieces of silk for $77.25, 15 pieces of linen for $83.68, and 3 dozen kid gloves for $40 and 8 cents; what was the amount of the bill?

61. A young man having $40, paid $20.20 for a coat; how much had he left?

62. At $8.245 a yard, how much will 10 yards of cloth cost?

63. At $6.25 a barrel, how many barrels of eggs can be bought for $256.25?

64. If 40 barrels of eggs cost $250, what is the price per barrel?

65. Paid 16 men $516 for work at 75 cts. a day; how many days did they work?

EXERCISE VII.

66. A steamship, after consuming 17,500 pounds of coal, has 30,000 pounds left; how many had she at first?

67. A steamship had 50000 pounds of coal, and has consumed 27035 pounds; how much remains?

68. If a merchant gain $65 a day, how much will he gain in (313 business days) a year?

69. If a merchant gain 18780 dollars a year, how much will be the average gain a day for 313 business days?

70. If a merchant gain $50 a day, how long will it take him to gain $10000?

71. If a ship sail 75 miles a day, how far will she sail in 60 days?

72. If a ship sail 4500 miles in 50 days, how many miles will she average in a day?

73. If a ship sail 60 miles a day, how long will it take her to sail 4500 miles?

74. If a 150 quarts of cherries cost $9375, what is the price per quart?

75. At $3.95 a bushel, how many bushels of timothy seed can be bought for $146.15?

EXERCISE VIII.

76. In a case of broadcloths there are 12 pieces, and in each piece 49 yards; how many yards in all?

77. In another case there are 12 pieces containing 564 yards, how many yards on an average in each piece?

78. In another case each piece contains 45 yards, and there are in all 540 yards, how many pieces?

79. A merchant has in cash $576.32, notes $135.375, flour $97.10, butter $57.19, for which he wishes to purchase goods amounting to $1000, the balance to remain on credit, how much will the balance be?

80. If 360 laborers receive $405 a day, what will each one of them receive?

81. At $7.625 a bushel, how many bushels of flaxseed can be bought for $1807.125?

82. At $6.625 a bushel, how much will 237 bushels of flaxseed cost?

83. If 250 bushels of flaxseed cost $1312.50, what is the price per bushel?

84. At $5.72 a cord, what will 23 cords of wood cost?

85. At $5.75 a cord, how many cords of wood can be bought for $138.

EXERCISE IX.

86. A farmer sold 56 loads of hay, each weighing 1400 pounds; how many pounds did they all weigh?

87. A farmer received $700 for 56 loads of hay; what was the price of a load?

88. Bought 18 barrels of sugar, each containing 235 pounds; how many pounds in all?

89. In 20 barrels of sugar there are 4115 pounds; how many pounds will they average?

90. If it would take 1 man 567 days to build a house, in how many days could 28 men build it?

91. A merchant bought dry goods amounting to $5862.97, and groceries amounting to $1279.50; he paid in cash $4000 and gave notes for the balance; what was the amount of the notes?

92. Bought 75 books at $1.25; how much did they all cost?

93. Bought 25 pairs of shoes for $23.75; what was the price of a pair?

94. Paid $51.75 for 9 hats; what was the average price of each?

95. What cost 23 cases of boots, at $37.52 a case?

96. At $0.375 each, how many books can be bought for $4.50?

97. What cost 128 barrels of sugar, at $18.96 a barrel?

98. If 254 barrels of sugar cost $2407.92, what is the price of a barrel?

99. At $63.75 an acre, how much will 200 acres of land cost?

100. If 100 acres of land cost $6375, what is the price per acre?

Art. 35.—Bills in U. S. Money.

A bill is a written account of what is to be paid for, as goods, labor, &c.

EXAMPLE I.

NEW YORK, *June* 1st.

J. KING, *Dr.*

To W. BROWN.

5 lbs. of Tea,		at 62½ cts.	$3.125
8 "	Coffee,	at 15 cts.	1.20
3 "	Starch,	at 12½ cts.	.375
14 "	Sugar.	at 11 cts.	1.54
6 gals. Molasses,		37½ cts.	2.25

What is the amount? *Ans.* $8.49

EXAMPLE II.

6 yds. of	Cloth,	at $4.37½	$
18 "	Calico,	at .21	
10 "	Muslin,	at .19	
3 spools	Cotton,	at .09	
5 sheets	Wadding,	at .12½	

What is the amount?

EXAMPLE III.

10 lbs. of	Sugar,	at $0.16	
5 "	Tea,	at 1.12½	
17 "	Butter,	at .22½	
9 "	Coffee,	at .14	
2 bbls.	Flour,	at 9.50	

What is the amount?

EXAMPLE IV.

9	yds. Silk,	at	$1.25	....
15	" Calico,	at	.18½	....
20	" Muslin,	at	.21	....
7	" Gingham,	at	.62½	....
6	skeins Silk,	at	.05	....

What is the amount ?

EXAMPLE V.

175	bushels of Wheat,	at	$1.62½	....
300	" Corn,	at	.81	....
625	" Oats,	at	.37½	....
92	" Buckwheat,	at	.56	....
112	" Rye,	at	.75	....

What is the amount ?

EXAMPLE VI.

1250	bushels of Potatoes,	at	.87½	....
625	" Turnips,	at	.37½	....
172	" Carrots,	at	.35	....
85	" Beets,	at	.68	....
126	barrels of Apples,	at	$3¼	....

What is the amount ?

EXAMPLE VII.

8	yds. Merino,	at	$1.37½	....
13	" Mus. de Laine,	at	.44	....
11	" Alpaca,	at	.75	....
1	" Figured Satin,	at	3.00	....
9	" Col'd Cambric,	at	.12½	....
14	" Drab Fringe,	"	.62½	....

What is the amount ?

EXAMPLE VIII.

J. Adams, Dr.

To T. Bailey.

3 pieces of Ribbon,	*at*	*\$1.37½*	....	
6 doz. spools of Thread,	*at*	*.44*	....	
9 skeins of Silk,	*at*	*.07*	....	
5 linen Handkerchiefs,	*at*	*.62½*	....	
6 pair of Hose,	*at*	*.38*	....	———

What is the amount?

In like manner write out and find the amounts of the following bills:

9. T. White bought of L. Camp, 3 yards of cloth at \$6.50; 2 yds. of cassimere, at \$2.75; 5 yds. cambric, at 37½cts.; 2 doz. buttons, at 12½ cts.; 3 skeins of silk, at 6¼ cts.

10. W. Savage bought of L. Stearns, 2 bbls. of flour, at \$9.50; 25 lbs. of sugar, at 18¾ cts.; 10 lbs. coffee, at 37½ cts.; 3 lbs. tea, at \$1.12½; 1 bar of soap, at 15 cts.

11. Mrs. Nelson bought of A. Halsted, 12 yards of silk, at \$2¼; 3 yds. satin, at \$12½; 6 yds. of cambric, at 16 cts.; 3 pairs of hose, at 56¼ cts.; 8 skeins of silk, at 4 cts.; 1 doz. spools of cotton, at 62½ cts.

12. S. Hoes bought of I. Ball & Co., 4 yards of cloth, at \$6; 12 yds. of satinet, at 87½ cts.; 3 woolen shirts, at \$1¾; 6 collars, at 20 cts.; 1 pair gloves, at \$1.50.

COMPOUND NUMBERS.

Art. 36.—Compound Numbers are numbers having different denominations, or names, under a general name, to express one quantity ; as, pounds, shillings, pence and farthings under English or Sterling Money, and pounds, ounces, etc., under Weights.

The general names include Money Weights and Measures of different kinds ; the different denominations of which are exhibited in the following tables.

Money.

United States Money is a species of compound numbers, but, being much like simple numbers, has been already introduced.

Art. 37.—English or **Sterling Money** is the money used in England. It was formerly used in this country, and is still used to some extent, though its value has been changed, and varies in different States.

TABLE.

4 farthings (far.) make	1	penny.	(d.)
12 pence	1	shilling.	(s.)
20 shillings	1	pound or sovereign.	(£.
21 shillings	1	guinea.	

£1 is valued at $4.84.

Weights.

Art. 38.—Troy Weight is used in weighing gold, silver, and gems.

TABLE.

24 grains (gr.) make	1	pennyweight.	(pwt.)
20 pennyweights	1	ounce.	(oz.)
12 ounces	1	pound.	(lb.)

Art. 39.—Avoirdupois Weight is used in weighing heavy and common articles.

TABLE.

16 drams (dr.) make	.........1 ounce.	(oz.)
16 ounces	.........1 pound.	(lb.)
25 pounds	.........1 quarter.	(qr.)
4 quarters, or 100 lbs	.........1 hundred-weight.	(cwt.)
20 hundred-weight	.........1 ton.	(T.)

Formerly 28 lbs. made 1 qr., and 112 lbs. made 1 cwt. 1 lb. (av.)=7000 grains (troy.)

Art. 40.—Apothecaries' Weight is used in weighing medicines at retail. The pound and ounce are the same as in Troy Weight.

TABLE.

20 grains (gr.) make	........1 scruple.	(sc. or ℈)
3 scruples	........1 dram.	(dr. or ʒ)
8 drams	........1 ounce.	(oz. or ℥)
12 ounces	........1 pound.	(℔.)

Art. 41.—MISCELLANEOUS TABLE OF WEIGHTS.

196 lbs. make	.........1 barrel of flour.
200	.........1 barrel of beef, pork or fish.
56	.........1 firkin of butter.
60	.........1 bushel of wheat.
56	.........1 bushel of rye or corn.
48	.........1 bushel of barley.
32	.........1 bushel of oats.

Measures.

Art. 42.—Cloth Measure is used in measuring cloths, and other goods sold by the yard.

TABLE.

2¼ inches (in.) make	...............1 nail.	(na.)
4 nails	...............1 quarter.	(qr.)
4 quarters	...............1 yard.	(yd.)

1 qr.=¼ yd, 2 qr.=½ yd, 3 qr.=¾ yd.

Also, 5 quarters make	....1 Ell English.	(E. E.)
3 quarters	....1 Ell Flemish	(E. Fl.)
6 quarters	....1 Ell French.	(E. F. not used.)

Art. 43.—Long Measure is used in measuring distances, or lines extended in length, breadth, heighth and depth.

TABLE.

12 inches (in.) make	..........1 foot.	(ft.)
3 feet	..........1 yard.	(yd.)
$5\frac{1}{2}$ yds., or $16\frac{1}{2}$ ft.	..........1 rod or pole.	(rd.)
40 rods	..........1 furlong.	(fur.)
8 furlongs	..........1 mile.	(m.)
3 miles	..........1 league.	(lea.)
60 geographic, or $69\frac{1}{2}$ common miles	..........1 degree.	(deg. or °.)
360 degrees	..........1 circle or circumference.	

Also 6 feet make 1 fathom, (used in measuring deep water.) 160 rods make $\frac{1}{2}$ mile. 80 rods $\frac{1}{4}$ mile.

Art. 44.—Surveyors' Measure is used in measuring roads and boundaries of land, &c., with chains.

TABLE.

$7\frac{92}{100}$ inches make	.............1 link.	(li.)
25 links	.............1 rod or pole.	(P.)
4 poles	.............1 chain.	(ch.)
10 chains	.............1 furlong.	(fur.)
8 furlongs, or 80 chains	.............1 mile.	(m.)

Art. 45.—Square Measure is used in measuring surfaces or areas, as land, floors, &c., in which both length and breadth are considered.

TABLE.

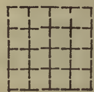
144 square inches (sq. in.) make	....1 sq. foot.	(sq. ft.)
9 square feet	....1 sq. yard.	(sq. yd.)
$30\frac{1}{4}$ sq. yards, or $272\frac{1}{4}$ sq. ft.	....1 sq. rod or pole.	(sq. rd.)
40 sq. rods	....1 rood.	(R.)
4 roods, or 160 rods	....1 acre.	(A.)
640 acres	....1 sq. mile.	(sq. m.)

This measure is directly applicable only to surfaces whose contents are known, as any number of acres, square rods, &c.

Such surfaces, depending on the length of certain lines, are found by first using long measure. Squares and rectangles are included among them.

A **Square** is a surface having four equal sides, and four equal angles, which, also, are right angles.

If the sides of a square are each one inch in length (long measure) it is a square inch; if the sides are each one foot, yard, rod or mile, it is a square foot, yard, rod or mile.



If any square is divided into square feet, as in the annexed diagram, it contains as many rows of square feet as there are linear feet in one side, and the same number of square feet in each row. Therefore if the number of feet in each row be multiplied by the number of rows, or, (which is the same,) if the number of feet in one side be multiplied by itself, the product will be the number of square feet in the whole square. The same is true of yards, rods, &c. Hence the

Rule.—*To find the contents or area of a square, multiply the length of any one of its sides by itself.*

There is no difference between a square foot and a foot square, but there is a difference between such expressions as 3 square feet, and 3 feet square. A square yard is 3 feet square, and contains 9 square feet, (see last diagram,) hence the difference between 3 feet square and 3 square feet is 6 square feet. The same is true of rods square and square rods, &c.

A **Rectangle** is like a square, only it is longer than it is wide, and its contents are found by multiplying its length by its breadth, both being reduced if necessary to the same name.

This diagram represents a rectangle 6 ft. long and 3 ft. wide—contents $6\times3=18$, the number of square feet.

Since, also, the product of two numbers divided by one of them gives the other, either side of a rectangle may be found by dividing the contents by the other side; as, 18 sq. ft.$\div$3 ft. (wide)$=$6 ft. long.

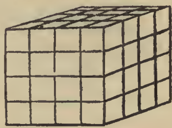
Art. 46.—Cubic Measure is used in measuring solids, in which length, breadth and thickness are considered. It is sometimes called Solid Measure.

TABLE.

1728 cubic inches (cu. in.)	make	1 cubic foot	(cu. ft.)
27 cubic feet	....	1 cubic yard	(cu. yd.)
40 feet of round or 50 feet of hewn	timber	1 ton	(T.)
16 cubic feet	....	1 cord foot	(C. ft.)
8 cord feet or 128 cubic feet	....	1 cord of wood.	

A **Cube** is a solid having six equal and square sides. If the sides are each a square inch, the solid is a cubic or solid inch ; if the sides are each a square foot or yard, the solid is a cubic or solid foot or yard, &c.

If the base of a cube be 4 feet square, it will contain 16 square feet, and a solid on it, 1 foot high, would contain 16 solid feet, 2 feet high $16\times2=32$ solid feet ; four feet high $16\times4=64$ solid feet ; hence



RULE.—*The contents of a cube, or any regular solid, may be found by multiplying together the length, breadth and thickness.*

The contents and any two of the dimensions being given, the other dimension may be found by dividing the contents by the product of the given dimensions.

Art. 47.—Wine Measure is used in measuring liquids, except beer and ale.

TABLE.

	4 gills (gi.) make	......	1 pint	(pt.)
	2 pints	......	1 quart	(qt.)
	4 quarts	......	1 gallon	(gal.)
	$31\frac{1}{2}$ gallons	......	1 barrel	(bbl.)
	63 gallons (or 2 barrels)	......	1 hogshead	(hhd.)
	2 hogsheads	......	1 pipe	(p.)
	2 pipes (or 4 hhds.)	......	1 tun	(T.)
Also	42 gallons	......	1 tierce	(tr.)
	48 gallons	......	1 puncheon	(pun.)

A gallon contains 231 cubic inches

Art. 48.—Beer Measure is used for measuring beer, and formerly milk.

TABLE.

2 pints	make	1 quart.
4 quarts	....	1 gallon.
36 gallons	....	1 barrel.
54 gallons	....	1 hogshead.

A beer gallon contains 282 cubic inches.

Art. 49.—Dry Measure is used for measuring grain, fruit, vegetables, &c.

TABLE.

	2 pints (pt.)	make	1 quart	(qt.)
	8 quarts	....	1 peck	(pk.)
	4 pecks	....	1 bushel	(bu.)
Also,	8 bushels	make	1 quarter	(q.)
	36 bushels	....	1 chaldron.	
	32 bushels	....	1 chaldron (in U. S.)	

A bushel in form of a cylinder is $18\frac{1}{2}$ in. in diameter and 8 inches deep. It contains $2150\frac{2}{5}\frac{1}{0}$ cubic inches.

Art. 50.—Time Measure is used in making divisions of time.

TABLE.

60 seconds (sec.)	make	1 minute	(m.)
60 minutes	....	1 hour	(h.)
24 hours	....	1 day (and night)	(d.)
7 days	....	1 week	(w.)
4 weeks or 30 days	....	1 month	(mo.)
12 months (calendar) or 13 " (lunar) or (365d. nearly)		1 year.	(y.)

The Solar year consists of 365 days, 6 hours (very nearly) in which the earth makes one revolution around the sun.

The Civil year is 365 days. This makes a difference of one day in four years, which is added to the month of February, making it consist of 29 instead of 28 days. This occurs whenever the number of years is divisible by 4, which is Leap Year

The addition of one day every Leap Year, makes about one day too much in 100 years, hence it is not added during the year which completes a century, as A.D. 1800, A.D. 1900.

The names of the months and the number of days in each, are as follows:

January	(2d Winter month,)	....	31 days.
February	3d " "	....	28 or 29 days.
March	1st Spring "	....	31 days.
April	2d " "	....	30 "
May	3d " "	....	31 "
June	1st Summer "	....	30 "
July	2d " "	....	31 "
August	3d " "	....	31 "
September	1st Fall "	....	30 "
October	2d " "	....	31 "
November	3d " "	....	30 "
December	1st Winter "	....	31 "

To assist in remembering the number of days in each month the following lines have been much used:

Thirty days hath September,
April, June, and November,
February twenty-eight alone,
All the rest have thirty-one;
Except in Leap Year, then is the time
When February has twenty-nine.

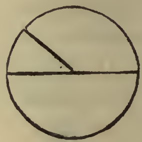
TABLE,

EXHIBITING THE NUMBER OF DAYS FROM ANY DAY OF ONE MONTH TO THE SAME DAY OF ANY OTHER MONTH IN THE SAME YEAR.

FROM ANY DAY OF	TO THE SAME DAY OF											
	JAN.	FEB.	MAR.	APL.	MAY.	J'NE.	J'LY.	AUG.	SEP.	OCT.	NOV.	DEC.
JANUA'Y	367	31	59	90	120	151	181	212	243	273	304	334
FEBR'Y.	334	365	28	59	89	120	150	181	212	242	273	303
MARCH.	306	337	365	31	61	92	122	153	184	214	245	275
APRIL..	275	306	334	365	30	61	91	122	153	183	214	244
MAY...	245	276	304	335	365	31	61	92	123	153	184	214
JUNE ..	214	245	273	304	344	365	30	61	92	122	153	183
JULY ..	184	215	243	274	304	335	365	31	62	92	123	153
AUGUST	153	184	212	243	273	304	334	365	31	62	92	122
SEPTM'R	122	153	181	212	242	273	303	334	365	30	61	91
OCTOB'R	92	123	151	182	212	243	273	304	335	365	31	61
NOVEM.	61	92	120	151	181	212	242	273	304	334	365	30
DECEM.	31	62	90	121	151	182	212	243	274	304	335	365

The number of days is opposite the first given month and under the other.

Art. 51.—Circular Measure is used in measuring circles, as in finding the latitude and longitude of places or of heavenly bodies.



A **Circle** is a figure bounded by a curved line, in which every point is equally distant from the centre.

The **Circumference** is the curved line. The **Diameter** is the straight line passing through the centre to opposite points in the circumference. A **Radius** is any straight line from the centre to the circumference.

Circles are divided into 360 parts called degrees, varying according to the size of the circles.

TABLE.

60 seconds (″) make	....	1 minute	(′)
60 minutes	....	1 degree	(°)
30 degrees	....	1 sign	(S.)
12 signs or 360°	....	1 circle	(C.)

Art. 52.—MISCELLANEOUS TABLE.

12 units or things make	....	1 dozen.
12 dozen	....	1 gross.
12 gross	....	1 great gross.
20 units or things	....	1 score.

PAPER.

24 sheets make	....	1 quire.
20 quires	....	1 ream.

BOOKS.

A sheet folded in two			leaves makes a folio.		
"	"	four	"	"	a quarto or 4to.
"	"	eight	"	"	an octavo or 8vo.
"	"	twelve	"	"	a duodecimo or 12mo.
"	"	eighteen	"	"	an 18mo.
"	"	twenty-four	"	"	a 24mo.

Reduction.

Art. 53.—Reduction of Compound Numbers is changing one denomination or name to another, without altering its value, and is either **descending** or **ascending.**

Art. 54.—Reduction descending is changing a greater denomination to a less, as pounds to shillings, &c.

Reduction ascending is changing a less denomination to a greater, as farthings to pence, &c.

Examples illustrating reduction descending :

EXAMPLE 1.—Reduce £24 15s. 10½d. to farthings.

£	s.	d.	f.
24	15	10	2

20
495 shill'gs
12
5950 pence.
4
Ans. 23802 fart'gs.

Process.—Reduction descending because it is changing greater denominations (pounds, &c.) to a less (farthings.)

1. Change the pounds to shillings. Since there are 20s. in £1, there are in £24, 20 times as many shillings, which are 480s., to which add the 15s. and the sum will be 495s.

2. Change the shillings to pence. Since there are 12d. in 1s., there are in 495s. 12 times as many pence, which are 5940d., to which add the 10½d. and the sum will be 5950½ or 5950d. 2far.

3. Change the pence to farthings. Since there are 4 far. in 1d., there are in 5950d. 4 times as many farthings, which are 23800far., to which add the 2far. and the sum will be 23802far., the answer.

Ex. 2. In 15 rods, 4 yds., 2 ft., 9 in., how many inches ?

rds.	yds.	ft.	in.
15	4	2	9

5½
7½
75
86½ yds.
3
261½ ft.
12
Ans. 3147 in.

Process.—Reduction descending, as in the preceding example.

1. Change the rods to yards. Since there are 5½ yards in one rod, in 15 rods there are 5½* times as many yards, which are 82½ yds., to which add the 4 yds., and the sum will be 86½ yds.

2. Change the yards thus found into feet. Since there are 3 feet in 1 yard, in 86½ yds. there are 3 times as many feet, which are 259½ ft., to which add the 2 ft., and the sum will be 261½ ft.

3. Change the feet to inches. Since there are 12 inches in 1 foot, in 261½ ft., there are 12 times as many inches, which are 3138 in., to which add the 9 in., and the sum will be 3147 inches, the answer.

---

* Since this is by a rule in Fractions, the teacher should explain the process in such cases, as it is explained in Fractions.

RULE.—*In reduction descending, multiply the greatest denomination by that number which it takes of the next less to make one of this greater, and add to the product any given number of the same name.*

Proceed thus till the required denomination is found.

EXAMPLES ILLUSTRATING REDUCTION ASCENDING.

Ex. 3. Reduce 23802 farthings to pounds, &c.

4)23802 far.
12)5950 d. +2far.
20)495s.+10.
24+15s.+10½d.

Process.—Reduction ascending, changing a less denomination (farthings) to greater (pounds, &c.)

1. Change the farthings to pence. Since 4 farthings make 1 penny, there are as many pence in 23802 far. as there are times 4 farthings, which are 5950 times, and 2 farthings remaining. Therefore 23802far. are 5950d. 2far.

2. Change the pence thus found to shillings. Since 12 pence make 1 shilling, there are as many shillings in 5950d. as there are times 12d. which are 495 times, and 10d. rem. Therefore 5950d. are 495s. 10d.

3. Change the shillings to pounds. Since there are 20 shillings in 1 pound, there are as many pounds in 495s. as there are times 20s. which are 24 times, and 15s. rem. Therefore 495s. are £24 15s.

Also 23802far. are £24 15s. 10½d.

Ex. 4. In 3147 inches, how many rods, &c.

12)3147 in.
3)262 ft.+3 in.
5½)87 yds.+1 ft.
2
11)174
15 rds.+9 half yds.
(4 yds. 1 ft. 6 in.)
Ans. 15 rds, 4 yds. 2 ft. 9 in.

Process.—Reduction ascending as in the preceding example.

1. Change the inches to feet. Since 12 in. make 1 ft., there are as many feet in 3147 in. as there are times 12 in., which are 262 times, and 3 in. rem. Therefore 3147 in. are 262 ft. 3 in.

2. Change the feet thus found to yards. Since 3 ft. make 1 yd., there are as many yds. in 262 ft. as there are times 3 ft., which are 87 times. Therefore 262 ft. are 87 yds. 1 ft.

3. Change the yards to rods. Since 5½ yds. make 1 rod, there are as many rods in 87 yds. as there are times 5½ yds. which are 15 times,* and 4½ yds. (=4 yds. 1 ft. 6 in.) rem. Therefore 87 yards are 15 rods, 4 yds. 1 ft. 6 in.

And 3147 in. are 15 rods, 4 yds. 2 ft. 9 in.

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* By a rule in Fractions.

RULE.—*In reduction ascending, divide the given denomination by that number which it takes of this denomination to make one of the next less.*

Proceed thus until the greatest denomination required is found, which, with the remainders, in regular order, will be the answer.

Art. 55.—English or Sterling Money.

MENTAL EXERCISES.

How many farthings in 3 pence?

Process.—Since there are 4far. in 1 penny, in 3d. there are 4 times as many farthings as pence, which are 12 farthings; or there are 4 times 3 far.

How many farthings in 5d? 7? 9? 11? 4? 6? 8? 10? 12? in 1 shilling? 1 pound?

In 50 farthings how many pence?

Process.—Since 4 farthings make 1 penny, in 50 farthings there are as many pence as there are times 4 farthings, which are 12 times, and 2 farthings remaining. Therefore 50 far. are 12d. 2far.

How many pence in 8 farthings? 16? 24? 32? 40? 48? 10? 20? 25? 30? 42? 50?

How many pence in 2 shillings? 4? 6? 8? 10? 5? 7? 9? 12? in 1 pound?

How many shillings in 24 pence? 48? 72? 36? 60? 96? 30? 40? 56? 63? 81? 100? 144?

How many shillings in 2 pounds? 4? 6? 9? 3? 5? 7? 10?

How many pounds in 40 shillings? 60? 80? 50? 65? 72? 84? 100? 120?

How many shillings in 2 guineas? 4? 6? 3? 5? 7? 10?

How many guineas in 42 shillings? 63? 84? 50? 65? 76?

How many farthings in 1 shilling? 2? 3?

How many pence in 1 pound? 2? 3?

How many shillings in 48 far.? 96?

EXAMPLES FOR THE SLATE, ETC.

Reduce—

(5.) £17 2s. 6d. to pence.
(6.) £11 15s. 4d. to farthings.
(7.) £15 18s. 10d. to pence.
(8.) £24 12s. 8d. to far.
(9.) £45 8s. $7\frac{1}{2}$d. to far.
(10.) £8 4s. to pence.
(11.) 10s. 9d. 3far to far.
(12.) £18 13s. $8\frac{1}{2}$d. to far.
(13.) £12 14s. to shillings.
(14.) 11d. 2far. to farthings.
(15.) £4 6d. to farthings.
(16.) £5 15s. to pence.
(17.) 10s. 3far. to far.
(18.) £10 $0\frac{1}{2}$d. to far.
(19.) 23475far. to pounds, &c.
(20.) 3756d. to pounds, &c.
(21.) 7620s. to pounds, &c.
(22.) 4325far. to shillings, &c.
(23.) 2754d. to shillings, &c.
(24.) 3240far. to pence, &c.
(25.) 4360s. to pounds, &c.
(26.) 1345d to pounds, &c.
(27.) 7563far. to shillings, &c.
(28.) $2560\frac{1}{2}$d. to pounds.
(29.) 1781s. to pounds, &c.
(30.) 1564d. to shillings, &c.
(31.) 1682far. to pence, &c.
(32.) $5146\frac{1}{2}$d. to pounds, &c.

Reduce—

(33.) £15 10s. 9d. to pence.
(34.) 23470far. to pence.
(35.) 18s. 7d. 3far. to far.
(36.) 1340d. to pounds.
(37.) 12560far. to pounds, &c.
(38.) £16 4s. to pence.
(39.) 1260s. to pounds.
(40.) 17s. $6\frac{1}{4}$d. to far. &c.

Art. 56.—Troy Weight.

MENTAL EXERCISES.

How many grains in 2 pennyweights? 4? 6? 3? 5? 7? 9? 10?

How many pennyweights in 48 grains? 72? 96? 30? 56? 81?

How many pennyweights in 2 ounces? 4? 6? 3? 5? 8? 10? 7? 12?

How many ounces in 40 pennyweights? 60? 100? 50? 75? 110?

How many ounces in 2 pounds? 4? 6? 8? 3? 7? 10? 5? 9? 12?

How many pounds in 24 ounces? 36? 48? 72? 96? 144? 30? 56? 68? 112?

EXAMPLES FOR THE SLATE.

Reduce—

(41.) 59125 gr. to oz.
(42.) 10 lbs. 10 oz. 10 pwts. to pwts.
(43.) 6743 pwts. to lbs.
(44.) 1000 lbs. to pwts.
(45.) 1000 pwts. to oz.
(46.) 62 lbs. 5 oz. to pwts.
(47.) 2840 grs. to oz.
(48.) 2840 lbs. to oz.
(49.) 12 lbs. 15 pwts. to grains.
(50.) 2895 pwts. to lbs.

Art. 57.—Avoirdupois Weight.

MENTAL EXERCISES.

How many drams in 2 ounces? 4? 6? 8? 3? 5? 7? 9? 10?

How many ounces in 32 drams? 48? 64? 36? 50? 75?

How many ounces in 2 pounds? 4? 6? 8? 3? 5? 7? 10? 9? 11?

How many pounds in 32 ounces? 48? 64? 30? 54? 75?

How many pounds in 2 quarters? 4? 3? 8? 5? 10?

How many quarters in 50 pounds? 75? 100? 40? 80? 108?

How many quarters in 2 hundred weight? 3? 5? 7? 4? 6? 9? 12?

How many hundred weight in 8 quarters? 16? 24? 12? 20? 10? 15? 25?

How many hundred weight in 2 tons? 4? 6? 3? 5? 7? 10?

How many tons in 40 hundred weight? 60? 80? 100? 50? 75?

EXAMPLES FOR THE SLATE.

Reduce—

(51.) 3 cwt. to pounds.
(52.) 4815 lbs. to cwts.
(53.) 2748000 dr. to tons.
(54.) 4 T. 15 lbs. to pounds.
(55.) 67200 oz. to cwt.
(56.) 15 cwt. 1 qr. 12 oz. to oz.
(57.) 22500 lbs. to tons.
(58.) 3 T. 15 cwt. 16 lbs. to lbs.

(59.) 4 cwt. 3 qrs. to pounds.
(60.) 1120 oz. to pounds.
(61.) 1225 lbs. to drams.
(62.) 1750 lbs. to cwts.
(63.) 2000 lbs. to ounces.
(64.) 4000 lbs. to cwts.
(65.) 28000 lbs. to tons.

Art. 58.—Apothecaries' Weight.

MENTAL EXERCISES.

How many grains in 2 scruples ? 4 ? 6 ? 3 ? 5 ? 8 ? 7 ? 9 ? 10 ?

How many scruples in 40 grains ? 60 ? 100 ? 50 ? 65 ? 87 ? 112 ?

How many scruples in 3 drams ? 5 ? 7 ? 2 ? 4 ? 6 ? 9 ? 11 ?

How many drams in 2 ounces ? 4 ? 6 ? 3 ? 7 ? 9 ? 8 ? 10 ? 12 ?

How many ounces in 3 pounds ? 2 ? 4 ? 6 ? 5 ? 7 ? 10 ? 9 ? 12 ?

EXAMPLES FOR THE SLATE.

Reduce—

(66.) 47230 gr. to ounces.
(67.) 5375 lbs. to ounces.
(68) 74376 dr. to pounds.
(69.) 10752 gr. to ounces.
(70.) 1728 lbs. to ounces.
(71.) 1730 dr. to ounces.
(72.) 6 lb. 4 oz. to drams.
(73.) 5000 gr. to ounces, &c.

Art. 59.—Cloth Measure.

MENTAL EXERCISES.

How many inches in 2 nails ? 4 ? 6 ? 3 ? 5 ? 7 ? 10 ? 12 ?

How many nails in 5 inches ? 10 ? 15 ? 20 ? 6 ? 12 ?

How many nails in 2 quarters ? 4 ? 3 ? 5 ? 7 ? 6 ? 8 ? 11 ?

How many quarters in 3 yards ? 5 ? 8 ? 10 ? 12 ?

How many yards in 8 quarters ? 16 ? 20 ? 12 ? 9 ? 15 ? 18 ? 21 ? 24 ?

EXAMPLES FOR THE SLATE.

Reduce—

(74.) 128 yds. to E. E.
(75.) 75 yds. to E. Fl.
(76.) 764 qrs. to yds.
(77.) 10 yds. 3 qrs. to nails.
(78.) 11¼ yds. to E. E.
(79.) 16½ yds. to quarters.
(80.) 20 yds. 3 qrs. to inches.
(81.) 6 E. E. to E. F.

Art. 60.—Long Measure.

MENTAL EXERCISES.

How many inches in 3 ft.? 5? 8? 7? 9? 11? 12?

How many feet in 24 inches? 36? 48? 60? 30? 56? 63? 72? 108? 144?

How many feet in 3 yards? 2? 4? 6? 5? 7? 9? 10? 12?

How many yards in 6 ft.? 9? 15? 18? 24?

How many yards in 3 rods? 6? 5? 7? 10?

How many feet in 2 rods? 4? 3? 5?

How many rods in 3 furlongs? 4? 6? 5?

How many miles in 16 furlongs? 24? 40? 32? 56? 72? 64? 10? 20? 25?

EXAMPLES FOR THE SLATE.

Reduce—

(82.) 103 in. to yards.
(83.) 7 yds. 10 in. to inches.
(84.) 3840 rods to miles.
(85.) 7 M. 6 fur. 30 rods to rods.
(86.) 2910 rds. to miles.
(87.) 5 yds. 1 in. to inches.

Art. 61.—Square Measure.

MENTAL EXERCISES.

How many square feet in 2 square yards? 4? 6? 8? 3? 5? 7? 10? 12?

How many square yards in 18 square feet? 27? 54? 36? 63? 99? 20? 44? 56?

How many square feet on a board 1 ft. wide and 12 ft. long (12 ft. by 1 ft.?) in a door 6 ft. high and 3 ft. wide (6 ft. by 3 ft.?) in a room 12 ft. by 11 ft.?

What is the length of a board 2 ft. wide and containing 24 square feet? a pane of glass 9 inches wide and containing 108 square inches? of a room 12 feet wide and containing 180 square feet? of a field 10 rods wide and containing 1 acre?

What is the difference between 4 square feet and 4 feet square? 5 square rods and 5 rods square?

EXAMPLES FOR THE SLATE.

Reduce—

(88.) 243 sq. rods. to acres, &c.
(89.) 8 sq. yds to sq. in.
(90.) 4176 sq. in. to sq. yds.
(91.) 24000 sq. rds. to acres.
(92.) 1 sq. M. to sq. rods.
(93.) 71680 sq. rds. to acres.
(94.) 16 A. 18 sq. rds. to sq. rds.
(95.) 78436 ft. to acres.

96. How many square inches in a window pane 15 in. by 12 in.?

97. How many sq. ft. in a looking glass 36 in. long and 24 in. wide (36 in. by 24 in.?)

98. How many sq. ft. in a floor 12 ft. by 10 ft.?

99. How many sq. yds. in a floor 18 ft. by 15 ft.?

100. How many sq. rods in a field 40 rds. by 20 rds.?

101. How many acres in a field 45 rds. by 30 rds.?

102. How many acres in a farm ¼ M. by 60 rds.?

103. What is the length of a window pane 15 in. wide and containing 300 square inches?

104. What is the width of a looking glass 60 in. long and containing 15 square feet?

105. What is the length of a room 12 ft. wide and containing 192 square feet?

106. What is the width of a room which requires 30 sq. yds. of carpeting, and is 18 feet long?

107. How many sq. yds. in 6 M. 7 sq. rods?

108. How many square miles in 6400 acres?

109. How many sq. yds. of paper will cover the walls of a room 24 feet by 20, and 12 feet high; and how many square yards of ceiling are there?

110. How many square miles in 92160 acres?

111. How many yards of muslin 3 qrs. wide, will line a quilt 8 feet square?

Art. 62.—Cubic Measure.

Reduce—

112. 4 cord, 16 cub. feet to cub. feet.

113. 1 cub. yd., 10 cub. feet to cub. inches.

114. 34 cords, 64 cub. feet to feet.

115. 31104 cub. inches to cub. feet.
116. 3584 cub. feet to cords.
117. 442368 cub. inches to cords.

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118. How many cubic feet of wood in a load of wood 8 ft. long, 4 ft. wide, and 5 ft. high ?

119. How many cubic yards in a cellar 24 ft. long, 15 ft. wide, and 6 ft. deep ?

120. How many cords of wood in a pile 40 ft. long, 8 ft. wide, and 6 ft. high ?

121. How many bricks will it take to lay the foundation of a house 32 ft. by 30, the height of the foundation being 8 ft. and the thickness 1 ft.; the bricks being 8 in. long, 4 in. wide, and 2 in. thick ?

Art. 63.—Wine Measure.

MENTAL EXERCISES.

How many gills in 3 pints ? 5 ? 7 ? 10 ?
How many pints in 8 gills ? 16 ? 20 ? 32 ?
How many pints in 2 quarts ? 4 ? 6 ? 8 ? 12 ?
How many quarts in 3 gallons ? 5 ? 7 ? 9 ? 12 ?
How many quarts in 4 pints ? 8 ? 10 ? 16 ? 22 ?
How many gallons in 8 quarts ? 12 ? 20 ? 25 ?
How many gallons in 2 barrels ? 2 hogsheads ?

EXAMPLES FOR THE SLATE.

Reduce—

(122.) 2 hhd. 2 qt. 1 pt. to gills.
(123.) 17 gal. 2 qts. to pints.
(124.) 140 pts. to gallons, &c.
(125.) 40736 gills to tuns, &c.

Reduce by Beer Measure—

(126.) 3 bbls. 16 gals. to pts.
(127.) 1730 pts. to barrels, &c.

128. How many gills in 1 hogshead of wine ?
129. How many pints in 1 barrel of beer ?
130. How many quarts in 5 hhds., 31½ gals. of vinegar ?

131. How many quarts in 3 hhds. of ale ?
132. How many hhds., &c., in 1200 gals. ?
133. How many barrels in 850 gals. of beer ?

Art. 64.—Dry Measure.

MENTAL EXERCISES.

How many pints in 4 quarts ? 6 ? 9 ? 12 ? 15 ?
How many quarts in 4 pints ? 6 ? 12 ? 9 ? 15 ?
How many pecks in 16 quarts ? 24 ? 40 ? 56 ?
How many pecks in 2 bushels ? 5 ? 3 ? 7 ?
How many bushels in 16 pecks ? 32 ? 48 ? 72 ?

EXAMPLES FOR THE SLATE.

Reduce—

(134.) 12 bus. 3 pks. to qts.
(135.) 3 pks. 1 pt. to pints.
(136.) 15 bu. 6 qts. to qts.
(137.) 4½ pks. to pints.
(138.) 1000 quarts to bushels.
(139,) 560 pecks to bushels.
(140.) 32140 pints to pecks.
(141.) 2764 qts. to bushels.

142. In 100 bushels how many quarts ?
143. In 1200 pints how many pecks ?

Art. 65.—Time.

MENTAL EXERCISES.

How many seconds in 2 minutes ? 4 ? 6 ?
How many minutes in 120 seconds ? 180 ?
How many minutes in 3 hours ? 5 ? 7 ? 10 ?
How many days in 2 weeks ? 4 ? 7 ? 12 ?
How many weeks in 3 months ? 5 ? 7 ? 10 ?
How many years in 24 months ? 36 ? 48 ? 65 ?

EXAMPLES FOR THE SLATE, ETC.

Reduce—

144. 2 hours, 3 minutes, 50 seconds, to seconds.
145. 3 days, 30 minutes, to minutes.
146. 1 year, 6 months, 27 days to hours.
147. 3 years, 2 weeks, 10 hours to seconds.

148. 9 months, 15 days, 10 minutes to minutes.
149. 1 solar year to seconds.
150. 12000 minutes to days, &c.
151. 99840 seconds to days, &c.
152. 7200 seconds to hours.
153. 56000 minutes to months.
154. 450000 seconds to days.
155. 12500 days to years.
156. How many hours in 5 years, 6 months, 3 weeks, 4 days ?
157. How many days in 365000 seconds ?
158. How many seconds in 1 day ?

66.—Circular Measure.

Reduce—
159. 10 S. 15°, 45′, 30″ to seconds.
160. 3000000 seconds to signs.
161. 18°, 36′, 12″ to seconds.
162. 25000″ to degrees.

Art. 67.—Promiscuous Examples in Reduction of Compound Numbers.

EXERCISE I.

1. How many farthings in £5 10s. 6d. ?
2. How many pounds in 3540 pwt. ?
3. How many pounds in 3 T. 7 cwt. 3 qr. ?
4. How many pounds in 7580 scruples. ?
5. How many nails in 25 yd. 3 qr. ?
6. How many miles in 500000 ft. ?
7. How many sq. yards in 5 A. 1 R. 12 rd. ?
8. How many sq. yards in 350 000 sq. in. ?
9. How many sq. inches in a looking glass 30 in. long and 18 in. wide ?

10. How many cubic feet in 6 C. 72 ft. ?

11. How many cubic feet in a load of wood 8 feet long, 4 feet wide, 5 feet high ?

12. How many gallons in 12548 gills ?

13. How many gallons in 3 T. 2 hhd. 21 gal. ?

14. How many quarts in 9 hhd. 15 gal. 3 qt. of beer ?

15. How many pints in 1 bu. 4 qts. ?

16. How many days in 325760 seconds ?

17. How many seconds in 14° 16′ 15″ ?

EXERCISE II.

18. How many pounds in 2540 farthings.

19. How many pennyweights in 12 pounds, 9 ounces ?

20. How many hundredweight in 36456 drams ?

21. How many grains in 7 pounds, 8 ounces, 1 scruple ?

22. How many yards in 1650 nails ?

23. How many yards in 5 miles, 3 furlongs, 4 yards ?

24. How many acres in a field 69 rods long, 45 rods wide ?

25. How many cords of wood in a pile 5 rods long, 8 feet wide and 6 feet high ?

26. How many pints in 5 hhd. 36 gallons, 1 quart, 1 pint ?

27. How many barrels in 5000 quarts of beer ?

28. How many bushels in 3540 quarts ?

29. How many minutes in 3 years, 4 months, 1 week, 2 days ?

30. How many degrees in 9863 seconds of a circle ?

EXERCISE III.

31. How many pence in £6 12s. 4d.

32. How many pounds in 75603 grains of silver ?

33. How many pounds in 5 T. 10 cwt. 20 pounds ?

34. How many pounds in 7856 grains of calomel ?

35. How many yards in 6 ells English ?

36. How many yards in 432 inches ?

37. How many yards in 7 miles 12 rods ?

38. How many acres in 5 square miles ?

39. How many cubic feet in the walls of a brick house 42 feet long, 30 feet wide, 25 feet high, the walls being 1 foot thick ?

40. How many barrels in 6481 pints ?

41. How many pints in 25 barrels of beer?
42. How many quarts in 15 bushels, 3 pecks?
43. How many days in 72000 minutes?
44. How many minutes in 27° 12′?

EXERCISE IV.

45. How many shillings in 260 farthings?
46. How many shillings in £12 10s.?
47. How many ounces in 12 lbs. 10 oz. of gold?
48. How many ounces in 12 lbs. 10 oz. of tea?
49. How many ounces in 856 scruples?
50. How many ells English in 5 yards?
51. How many inches in 1 M. 32 rods?
52. How many square inches in 5 rods square? in 5 square rods?
53. How many cubic feet in the walls of a stone house, 30 feet long, 26 feet wide, and 24 feet high, the walls being 1 foot thick?
54. How many gallons in 1 pipe, 1 hhd. 1 barrel?
55. How many barrels in 648 gallons of beer?
56. How many bushels in 3840 quarts?
57. How many days in 5 years?
58. How many signs in 1800′?

EXERCISE V.

59. How many pence in £10 10s.?
60. How many pence in 500 farthings?
61. How many pounds in 5672 pwt.?
62. How many pounds in 4 T. 16 cwt. 21 lbs,?
63. How many pounds in 6742 gr. of camphor?
64. How many yards in 672 nails?
65. How many rods in 3½ miles?
66. How many rods in 10,672 inch?
67. How many acres in a field containing 320 sq. rods?
68. How many acres in a field 160 rods sq.?
69. How many cubic feet in a ditch around a garden 6 rods long and 4 rods wide, the ditch being 2 feet wide and 3 feet deep?

70. How many gallons in 4728 gills ?
71. How many gallons in 2 hhd. 1 barrel of beer ?
72. How many pints in 15 bushels 6 quarts ?
73. How many months in 273456 minutes ?
74. How many seconds in 27° 30′ ?

EXERCISE VI.

75. How many farthings in £18 7s. 10½d. ?
76. How many grains in 11 ounces of gold ?
77. How many hundredweight in 12760 ounces of flour ?
78. How many grains in 56 ounces of morphine ?
79. How many ells English in 10 ells Flemish ?
80. How many feet in 312 inches ?
81. How many feet in 275 rods ?
82. How many square inches of gilding will cover the frame of a looking glass 36 inches long, 20 in. wide, the width of the frame being 4 inches ?
83. How many cords of wood in pile 64 feet long, 4 feet wide and 5 feet high ?
84. How many quarts in 9 hhds. 3 qts. ?
85. How many barrels in 3472 qts. of beer ?
86. How many pecks 712 pints ?
87. How many seconds in 1 month ?
88. How many degrees in 56312 seconds of a circle ?

EXERCISE VII.

89. How many shillings in 187½ pence ?
90. How many ounces in 6512 grains of silver ?
91. How many ounces in 11 cwt. 12 lbs. ?
92. How many ounces in 678 grains of laudanum ?
93. How many nails in 7¼ yards ?
94. How many rods in 12¼ miles ?
95. How many rods in 1428 feet ?
96. How many acres in a town 5 miles square ?
97. How many acres in 6 square miles ?
98. How many solid feet in 72 tons of hewn timber ?
99. How many hogheads in 6854 gallons ?
100. How many half-pints in 2 barrels of beer ?
101. How many pints in 36 bushels ?

102. How many hours in 36000 seconds?
103. How many minutes in ½ a circle?

Promiscuous Examples in Reduction of Compound Numbers occurring in business, &c.

EXERCISE VIII.

104. A silversmith made a gold cup weighing 8 oz. 10 pwt.; what did it cost at 5 cts. a grain?

105. A manufacturer bought 7 cwt. 16 lbs. of wool; what did it cost at 37½ cts. a pound?

106. A druggist sold 5 dr. 2 sc. morphine; what was the amount at 8 mills a grain?

107. If a tailor uses 4 yds. 2 qrs. of cloth in making an overcoat, how many can he make from 29 yards?

108. A contractor agreed to make a road for $325.50 a mile; what was the price per rod?

109. A farmer sold 3 acres 1 R. of land for 45 cts. a square rod; what did he receive for it?

110. At $5 a cord, what costs 1 cord foot of wood?

111. A merchant bought 15 gallons 2 quarts of oil at 1 shilling a pint; what did it all cost?

112. If 3 bushels 4 quarts of salt cost $4, what is the price per pint?

113. If a man's income is $1200 a year, how much is it per day?

114. If a star move from W. to E. at the rate of 10′ 30″ daily, how long will it be in completing a circle?

115. A stationer sells paper for 20 cents a quire; what will 3 reams cost?

EXERCISE IX.

116. A silversmith paid $300 for 1 lb. of gold; what was the price per grain?

117. A blacksmith used 8 ounces of iron in making a horse shoe; how many did he make from 60 pounds?

118. Paid a druggist $1.25 for 4 ounces of jalap; what was the price per grain?

119. A merchant tailor made 12 vests, each containing 3 qrs. of silk velvet; how many yards did he use?

120. A telegraphic company paid $1250 for wire at 3 cents a yard; how many miles did it extend?

121. A man bought a field 150 rods long and 75 rods wide, at $64 an acre; what did it cost?

122. A ditcher agreed to dig a ditch 60 rods long, 2 ft. wide, and 3 ft. deep, for $44; what did he receive for each solid yard?

123. A brewer sold 5 hogsheads, 15 gallons of ale at 4 pence a quart; what did he receive for it?

124. A woman sold a quantity of blackberries for $6.25, at 6 cents a quart; how many bushels did she sell?

125. A laborer earned $56 in chopping wood at 75 cents a day; how many months did he work?

126. The apparent motion of the sun being one circle in the heavens a year, how many seconds does it move daily?

127. What will 6 gross of buttons cost at 8 pence a dozen?

EXERCISE X.

128. If a silversmith uses 8 ounces of silver in making a cup, how many cups can he make from 30 pounds?

129. If a family consumes 2 pounds 12 ounces of flour daily, how long will 1 barrel last them?

130. How many powders, each composed of 4 grains, can be made from a mixture of medicine weighing 1 lb. 6 oz. 2 dr. 1 sc.?

131. How many vests, each containing 3 quarters, can be made from a piece of valentia measuring 16½ yards?

132. At 12½ cents a foot, how much will a lead pipe cost extending from a house to a spring ¼ mile distant?

133. How many panes of glass, 8 inches by 10 inches, are in a box containing 100 square feet?

134. What is a pile of wood worth which is 112 feet long, 4 feet wide, and 6 feet high, at $4½ a cord?

135. How many bottles, containing 3 pints, each can be filled from a hogshead of wine?

136. At 6 cts. a quart, how much will 5½ bushels of salt cost?

137. If a carriage wheel turn round once in passing over 8 feet 9 inches, how many times will it turn in going 3¼ miles?

138. If a clock tick 4 times in a second, how many times does it tick in a day?

139. A stationer sells paper for $4 a ream; how much is it a quire?

EXERCISE XI.

140. At 4d. a grain, how many pounds sterling will a gold cup cost weighing 7 oz. 10 pwt.?

141. If a pound of wool cost 4s. 6d., how many pounds sterling will 8 cwt. 3 qrs. cost?

142. At ½d. a grain, how much will 2 oz. 5 dr. 2 sc. of quinine cost?

143. At 10d. a foot, how much will a gas pipe cost measuring 3 rds. 4 yds.?

144. At 10s. 6¼d. a square yard of oil cloth, how much will it cost to cover a hall floor 30 feet by 6 feet?

145. At 8d. a solid foot, how much will a ton of hewn timber cost?

146. If a pint of Port wine cost 7s. 6d., how much will 10½ gallons cost?

147. If a laborer earn 6d. an hour, how much will he earn in a month of working time (6 days a week and 10 hours a day?)

148. If 10 gross of buttons cost $6, what is the price per doz.?

EXERCISE XII.

149. If a silversmith buy 1 pound of silver for $40, and sell it for 1 cent a grain, how much will he gain?

150. If a merchant buy 10 cwt. 2 qrs. of sugar for $115.50, for what must he sell it to gain 1 cent. a pound.

151. If a druggist pay 8 mills a grain for calomel and sell it for 9 mills, how much will he gain on a pound?

152. A contractor agreed to make a road 7½ miles long for $2500 and paid his laborers $1 a rod; how much did he gain?

153. A speculator bought city lots containing 1½ acres for

$50,000, and sold them for $1 a square foot; how much did he gain?

154. A job mason agreed to build the foundation of a house 32 feet long, 24 feet wide, the foundation to be 9 feet high, 18 inches thick, for $12, but was obliged to pay his journeymen 1 cent a solid foot; how much did he lose?

155. A merchant bought 3 hhds. of molasses for £28 7s. and sold it for 1 shilling a quart, how much did he gain?

156. A merchant in selling peas at 20 cents a quart gains 5 cents; what did they cost a bushel?

157. If a watch lose 2 seconds an hour, how much will it lose in a week?

EXERCISE XIII.

158. A silversmith paid $175.68 for gold ore at three cents a grain; how much did he buy?

159. A merchant bought 1 T. 1 cwt. of rice at 9d. a pound; what did it all cost?

160. A druggist sells opium for 4d. a scruple; how many ounces can be bought for £2 10s.?

161. How many suits of clothes, each containing 5 yards 3 quarters; can be cut from 69 yards?

162. How much will it cost to make a wire fence 4 wires high, around a field 40 rods long and 32 rods wide at 3 cents a yard?

163. What will an acre of ground cost at 75 cents a square yard?

164. How many cubic inches in a block of marble 1 yard long, 2 feet wide, 18 inches thick?

165. What will a hogshead of vinegar cost at 5 mills a gill?

166. What will 2 bushels 3 pecks of plums cost at 4 cents a pint?

167. If a ship sail 12 miles an hour, how far will she sail in 2 months, 1 week, 3 days.

168. If a planet move 1° 30′ a day; how long will it be in moving through each sign of the Zodiac?

169. If one gross of steel pens cost 72d., what does 1 pen cost?

EXERCISE XIV.

170. If 24 English Bibles cost 22 guineas, 6s., what is the price of each ?

171. A silver dollar contains $412\frac{1}{2}$ grains ; how many dollars can be made from 25 lbs. 15 pwt. 15 gr. of silver ?

172. In 6 tons 14 cwt. 3 qrs. 9 lbs. of butter, how many firkins ?

173. How many acres in a field 430 rods long and 132 feet wide ?

174. How many yards of muslin 3 quarters wide will line 10 yards of merino $1\frac{1}{4}$ yards wide ?

175. How many sheets of tin 15 inches by 12 inches, will cover the roof of a house 40 feet long and 25 feet wide, the rafters on each side being 16 feet long ?

176. How many blocks of granite 3 feet long, 21 inches wide, and 18 inches thick, will it take to build the walls of a church 80 feet long, 62 feet wide, 36 feet high, the walls being 18 inches thick ?

177. If a barrel of ale cost $7.25, what will be gained by retailing it a 4 cents a pint ?

178. How long would it take a locomotive to travel 3000 miles, at the rate of 3 rods a second ?

179. If a ream of paper cost $4 and be sold for 25 cents a quire, how much would be gained ?

180. At 5 cents a dozen, how many gross of buttons can be bought for $3 ?

EXERCISE XV.

181. How many yards of carpeting 2 feet wide, will cover a room 24 feet by 18 feet ?

182. How many pieces of paper 9 yards long, 15 inches wide, will cover the sides of a room 18 feet long 15 feet wide, and 11 feet high, there being one door 6 feet 3 inches by 2 feet 9 inches, and two windows 5 feet by 4 feet ?

183. How many square yards of plastering in the same room ?

184. How much will it cost to pave a sidewalk 10 rods long, 10 feet wide, at $10 a square rod ?

185. If a piece of land containing 6 acres be divided into

building lots 8 rods long and 2 rods wide, and each be sold for $300, for how much will it all be sold?

186. How many shingles will cover the roof of a house 30 feet long, the rafters on each side being 15 feet long, allowing one shingle for every 20 square inches?

187. How many bricks will it require to build a chimney, averaging 4 feet by 3 feet and 30 feet high, the walls being 8 inches thick, and the bricks 8 inches long, 4 inches wide and 2 inches thick?

EXERCISE XVI.

188. How many spoons, each weighing 2 ounce 10 pwt., can be made from 5 lbs. of silver?

189. What will 6 hogsheads of beer cost at 3 cents a quart?

190. What will 6 hogsheads of wine cost at 1 shilling a pint?

191. How many bushels of nuts can be bought for $20 at 4 cents a quart?

192. What will it cost to plaster a room 30 feet long, 20 feet wide and 8 feet high, at 25 cents a square yard, there being two doors 7 feet by 4 feet, and two windows 5 feet by 3 feet, and a mop board 6 inches wide?

193. What is the value of a pile of wood 108 feet long, 6 feet high, and 4 feet wide, at $7½ a cord?

194. How many rods of fence will inclose a tract of land 1 mile long and 180 rods wide?

195. How many acres in a town 5 miles square?

196. How many acres in a town 6 miles long and 3 miles wide?

197. How much will it cost to carpet a room 24 feet long, 18 feet wide, the carpet being 2 feet wide and the price $1¼ a yard?

198. What will it cost to dig a cellar 40 feet long 32 feet wide, and 8 feet deep at 25 cents a cubic yard?

199. What will it cost to dig a ditch around a garden (outside) which is 6 rods long and 4 rods wide, the ditch to be 3 feet deep and 2 feet wide, at ½ cent a cubic foot?

EXERCISE XVII.

200. How many yards of muslin 3 quarters wide, will it take to line 21 yards of silk 2 quarters 2 nails wide?

201. How much will it cost to paper a room 27 feet long, 18 feet wide and 12 feet high, each piece of paper containing 9 yards, 1 foot 6 inches wide and costing 30 cents ?

202. How many bricks will pave a side-walk 120 rods long, 12 feet wide, each brick being 8 inches long and 4 inches wide ?

203. How high must a load of wood be to contain a cord, if it is 9 feet long and 4 feet wide ?

204. How wide must a field be to contain six acres, if it is 80 rods long ?

205. How many bricks in a pile 20 feet long, 12 feet wide, and 10 feet high, each brick being 8 inches long 4 inches wide and 2 inches thick ?

206. How many bricks of the same size in a furnace chimney 60 feet high, having four equal sides, averaging 5 feet, and the walls being 8 inches thick ?

The Fundamental Rules Applied to Compound Numbers.

Art. 68.—Addition of Compound Numbers.

Example 1. What is the sum of £9 10s. 7d. ; £7 11s. 4d. 1 far. ; £2 18s. 7½d. ; 4s. 9d. 3 far.

Process.—First, add the farthings, the sum of which is 6 farthings (by reduction ascending) 1d. 2 farthings. Write the 2 farthings under the column of farthings, and add or carry the 1d. to the pence in the next column. Add and reduce (if possible) the other denominations in the same manner.

	£	s.	d.	far.
	9	10	7	0
	7	11	4	1
	2	18	7	2
		14	9	3
Ans.	20	15	4	2

Rule.—*Write the numbers of the same denomination, or name under one another in order. Add each denomination, beginning with the least, as in simple numbers, and reduce the sum to the next greater denomination. Write the remainder under the column added, and carry the quotient to the next column.*

PROOF—*As in simple Addition.*

In writing compound numbers, if any denomination is wanting, supply its place with a cipher.

EXAMPLES.

2. Add 5 lbs. 10 oz. 12 pwt. 9 gr; 1 lb. 11 oz. 13 pwt. 7 gr; 6 lbs. 9 oz. 7 pwt. 14 gr.; 3 lbs. 6 oz. 12 pwt. 12 gr.

3. Add 9 lbs. 8 oz. 17 gr.; 8 lbs. 10 pwt. 12 gr.; 7 oz. 12 pwt. 15 gr.; 4 lbs. 5 oz. 21 pwt.

4. What is the sum of 7 T. 19 cwt. 1 qr. 15 lbs.; 10 T. 8 cwt. 2 qrs. 18 lb.; 12 T. 10 cwt. 3 qrs. 14 lbs.; 5 T. 11 cwt. 1 qr. 9 lbs.?

5. Add 5 cwt. 3 qrs. 16 lbs. 12 oz. 10 dr.; 7 cwt. 1 qr. 18 lbs. 14 oz.; 9 cwt. 8 lbs. 10 oz. 10 d.; 3 qrs. 18 lbs. 9 oz. 8 dr.

6. Add 3 lbs. 2 oz. 4 dr. 2 sc. 10 gr.; 4 lbs. 8 oz. 5 dr. 15 gr.; 6 lbs. 11 oz. 7 dr. 2 sc.; 2 oz. 4 dr. 1 sc. 15 gr.

7. What is the sum of 12 yd. 2 qr. 2 na.; 8 yd. 3 qr. 1 na. 4 yd. 1 qr. 2 na.; 6 yd. 2 qr. 2 na.?

8. Add 12 m. 6 fur. 18 rod. 2 ft.; 10 m. 4 fur. 16 rod. 10 ft.; 9 m. 7 fur. 15 rod. 9 ft.; 7 m. 5 fur. 10 rod. 12 ft.

9. How many acres in four fields, the first containing 6 A. 3 R. 25 rod.; the second 5 A. 1 R. 30 rds.; the third 4 A. 2 R. 35 rds.; the fourth 7 A. 3 R. 28 rd.?

10. How many cords in three piles of wood measuring as follows. 8 C. 24 ft.; 7 C. 64 ft.; 9 C. 84 ft.?

11. What is the sum of 10 hhd. 42 gal. 1 qt. 1 pt.; 9 hhd. 30 gal. 1 pt. 2 gills; 54 gal. 3 qt. 1 pt. 2 gills; 6 hhd. 25 gal. 2 qt. 1 pt.?

12. Add 5 y. 4 mo. 12 d. 16 m. 20 sec.; 4 y. 3 mo. 15 d. 45 m.; 6 y. 6 mo. 20 d. 14 m.; 6 m. 6d. 6 h. 40 sec.; 7 y. 28 d. 20 m. 10 sec.

13. What is the difference between 45° 16′ 28″ north latitude and 30° 43′ 32″ south latitude?

Art. 69.—SUBTRACTION OF COMPOUND NUMBERS.

EXAMPLE 1. From 3 hhds. 18 gals. 1 qt. 1 pt. 1 gi., take 1 hhd. 31 gals. 2 qts. 1 pt. 3 gi.

Process.—Since 3 gills cannot be taken from 1 gill, add 1 pint reduced to 4 gills, to the 1 gill, then subtract the 3 gills, and write the remainder, 2 gills, underneath, also carry the 1 pint to the next denomination. Thus proceed.

	hhd.	gal.	qt.	pt.	gi.
	3	18	1	1	1
	1	31	2	1	3
Ans.	1	49	2	1	2

RULE.—*Write the less compound number under the greater, so that numbers of the same name will be under one another. Subtract each denomination as in simple numbers. If any number is greater than the one above it, add to the upper number* 1 *of the next denomination reduced to the same name, then subtract and carry* 1 *to the next denomination.*

PROOF—*As in simple Subtraction.*

EXAMPLES.

2. From £11 12s. 8d. 2 far., subtract £4 15s. 6d. 3far.

3. From 16 lbs. 10 oz. 14 pwts. 20 grs., subtract 8 lbs. 8 oz. 20 pwts. 10 grs.

4. What is the difference between 17 T. 11 cwt. 1 qr. 10 lbs. and 10 T. 15 cwt. 3 qr. 6 lbs.?

5. What is the difference between 16 lbs. 8 oz. 6 dr. 2 sc. and 8 lbs. 4 oz. 6 dr. 12 grs.?

6. From 14 yds. 1 qr. 2 na. subtract 8 yds. 2 qrs. 3 na.

7. From 127 A. 1 R. 30 rds. subtract 56 A. 2 R. 24 rds.

8. What is the difference between 4 y. 5 mo. 10 d. 6 h. and 1 y. 9 mo. 21 d. 45 m.?

9. What is the difference between 20° 12′ 24″ and 5° 15′ 45″?

Art. 70.—*To find the time between two dates.*

Ex. 10. How old was a person, born May 15, 1820, when he died, Oct. 21, 1856?

Process.—(October is the 10th and May the 5th month.)

Died	1856	10	21	
Born	1820	5	15	
Age	36	5	6	*Ans.*

RULE.—*Subtract the first date from the last, numbering the months in their order.*

30 days is considered a month, and 12 months a year.

EXAMPLES.

11. A note dated Jan. 1, 1860, was due July 15, 1861; for what time was it given?

12. A father was born May 16, 1815, and his son June 23, 1855; how much older is the father than his son?

13. A note dated Aug. 15, 1865 was due May 1, 1866; for what time was it given?

14. A man was born April 12, 1828, and his wife Sept. 21, 1833; how much older is the man than his wife?

15. On the 19th of June, 1860, a man gave his note to be paid July 1, 1861; how long was the time?

Art. 71.—MULTIPLICATION OF COMPOUND NUMBERS.

EXAMPLE 1.—Bought 5 barrels of molasses, each containing 34 gal. 3 qt. 1 pt. 2 gills; how much did they all contain?

Process.—The same as in addition, except we multiply instead of adding each denomination.

	hhd.	gal.	qt.	pt.	gi.
	0	34	3	1	2
					5
Ans.	2	46	2	1	2

RULE.—*Multiply each denomination separately, as in simple multiplication, beginning with the least, and reduce the product to the next greater denomination. Write the remainder under the number multiplied, and carry the quotient to the next denomination.*

PROOF.—*The same as in simple Multiplication.*

The multiplier is properly an abstract number, though it may be of a particular denomination or name.

EXAMPLES.

2. Multiply 3 oz. 12 pwts. 18 grs. by 5.
3. Multiply 14 cwt. 1 qr. 15 lbs. by 6.
4. Multiply 4 oz. 3 drs. 2 sc. 10 grs. by 7.
5. Multiply 3 yds. 1 qr. 2 na. by 8.
6. Multiply 2 m. 3 fur. 9 rods, 8 ft. by 9.
7. Multiply 26 A. 2 R. 20 rods by 10.
8. Multiply 5 C. 112 ft. by 11.
9. Multiply 7 hhds. 45 gals. 2 qts. by 12.
10. 8 y. 6 mo. 7 d. 30 min. by 15.
11. 12° 12′ 12″ by 8.

Art. 72.—DIVISION OF COMPOUND NUMBERS.

CASE 1. *To divide a compound number into any number of equal parts.*

EXAMPLE 1.—Divide 12 cwt. 2 qr. 18 lbs. equally among 5 persons.

Process.—12 cwt.÷5=2 cwt. and 2 cwt. remainder; write the quotient under cwt. and reduce the remainder to quarters; 2 cwt.×4=8 qrs., to which add the 2 qrs. and the sum will be 10 qrs., which divide and thus proceed.

$$\begin{array}{r} 5)\underline{12 \text{ cwt. } 2 \text{ qrs. } 18 \text{ lbs.}} \\ 2 \text{ `` } \; 2 \text{ `` } \; 3\tfrac{3}{5} \text{ ``} \end{array}$$

RULE.—*Divide each denomination separately, beginning with the greatest, and write the quotient under it. Reduce the remainder to the next less denomination, to which add it and divide again.*

PROOF.—*The same as in Simple Division.*

Ex. Divide 48 T. 10 cwt. by 25.

Process.—The same except by long division.

```
      T.   cwts.
   25)48   10(1 T.
      25
      --
      23
       20
   25)470(18 cwt.
      25
      ---
      220
      200
      ---
       20
        4
    25)80(3 qrs.
       75
       --
        5
       25
   25)125(5 lbs.
      125
```

Ans. 1 T. 18 cwt. 3 qr. 5 lb.

EXAMPLES.

3. Divide £100 15s. 10d. by 4.
4. Divide 10 lbs. 8 oz. 10 pwt. by 6.
5. Divide 12 cwt. 1 qr. 14 lbs. by 9.

6. Divide 4 lbs. 6 oz. 4 dr. 2 scr. by 11.
7. Divide 15 yds. 2 qrs. 3 na. by 15,
8. Divide 16 m. 3 fur. 20 rd. 12 ft. by 21.
9. Divide 124 A. 3 R. 150 rd. by 100.
10. Divide 50 C. 72 ft. by 16.
11. Divide 1200 bu. 3 pk. by 175.
12. Divide 9 hhd. by 11.

CASE 2. *To divide one compound number by another.*

Ex. 13. Divide £7 10s. among as many persons as possible, giving to each 12s. 6d.

Process.—Reduce both the dividend and divisor to pence, and then divide.

Since this is performed by reduction and simple division it is not usually included in Conpound Division.

Art. 73.—Special Application of Compound Numbers to Longitude and Time.

Since the apparent motion of the sun around the earth is in a circle of 360° in 24 hours—

360°÷24 or 15°=1 hour of time.

1°=$\frac{1}{15}$ hour or 4 minutes of time.

15°÷60 or 15′=1 minute of time.

1′=$\frac{1}{15}$ minute or 4 seconds of time.

15′÷60 or 15″=1 second of time.

Art. 74.—To Find the Difference of Time when the Difference of Longitude is Known.

EXAMPLE 1. The difference of longitude between two places is 18° 35′; what is the difference of time?

Process.—1st, since 15°=1 hour, 18°=1 hour and 3° remainder; 3°=180′, to which add the 35′ and the sum will be 215′; and since 15′=1 min. 215′=14 min. and 5′ remainder, which reduced to (″) and divided, is equal to 20 sec.

$$\begin{array}{r} 15)\underline{18°\ 35'\quad 0''} \\ 1\text{h. } 14\text{ m. } 20\text{ sec.} \end{array}$$

Process.—2d, since 1′=4 sec. 35′=140 sec. or 2 min. 20 sec. And since 1°=4 min. 18°=72 min. to which add the 2 min. and the sum will be 74 min.= 1 h. 14 min. Therefore 18° 35′=1 h. 14 min. 20 sec.

$$\begin{array}{r} 18°\quad 35' \\ \underline{4} \\ 1\text{ h. } 14\text{ m. } 20\text{ sec.} \end{array}$$

RULE.—*Divide the longitude by* 15 *as in Compound Numbers and the quotient of degrees will be hours, of minutes (′) minutes of time, and of seconds (″) seconds of time.*

Or Multiply the longitude by 4, *and the product of minutes (′) will be seconds of time, and of degrees minutes of time, which may be reduced to hours.*

EXAMPLES.

2. The difference of longitude between Boston and London is 71° 4′; what is the difference of time?

3. The difference of longitude between Washington and London is 76° 53′; what is the difference of time?

4. The difference of longitude between New York and San Francisco is 51° 26′; what is difference of time?

When the longitudes of two places are given, their difference may be found by subtraction when they are both East or West; by addition when one is East and the other West.

Art. 75.—TO FIND THE DIFFERENCE OF LONGITUDE WHEN THE DIFFERENCE OF TIME IS GIVEN.

EXAMPLE 5. The difference of time between Philadelphia and Cincinnati is 37 min. 20″; what is the difference of longitude?

Process.—1st, since 1 sec.=15″ of long. 20 sec.=300″=5′ and since 1 min.=15′ of long. 37 min.=555 min., to which add the 5′ and the sum will be 560′=9° 20′

	m.	sec.
	37	20
		15
Ans.	9° 20′	″

Process.—2d, since 4 min.=1° of long. 37 min. =9° and 1 min. rem., 1 min.=60 sec. and 20 sec. more are 80 sec., and since 4 sec.=1′ of long. 80 sec.=20′ of long. Therefore 37 min. 20 sec.=9° 20′ of long.

	m.	sec.
	4)37	20
Ans.	9°	20′

RULE.—*Multiply the time by* 15, *as in Compound Numbers, and the product of seconds of time will be (″) seconds of longitude, of minutes (′) minutes of longitude, and of hours, of degrees of longitude.*

Or Multiply the hours by 15 *to find degrees, and divide the minutes by* 4 *to find more degrees, and the seconds by* 4 *to find minutes of longitude.*

EXAMPLES.

6. The difference of time between Washington and Cincinnati is 29 minutes 36 seconds ; what is the difference of longtude ?

7. The difference of time between Washington and London is 5 hours, 8 minutes, 4 seconds; what is the difference of longitude ?

8. The difference of time between New York and San Francisco is 3 hours, 25 minutes, 44 seconds ; what is the difference of longitude ?

Art. 76.—Promiscuous Examples in Addition, Subtraction, Multiplication, and Division of Compound Numbers.

EXERCISE I.

1. A man when married was 20 years, 9 months, 12 days old ; he and his wife lived together 21 years, 4 months, 15 days, and he lived after the death of his wife 12 years, 8 months, 3 days ; what was his age when he died ?

2. A railroad when completed is to be 224 miles long. The part already finished is 109 miles, 3 furlongs 18 rods; how much remains to be made ?

3. A druggist sold 9 boxes of medicine, each weighing 3 ounces, 4 drams, 2 scruples ; what was the whole weight ?

4. A wholesale grocer shipped 1 ton of sugar in 9 barrels of equal size ; how much was there in each barrel ?

5. A note dated July 10, 1860, was due January 1, 1862 ; how long was the time ?

6. The longitude of Jerusalem is 35° 32 East, and that of Baltimore 76° 37′ West, what is the difference of time ?

7. The difference of time between Boston and London is 4 hours 44 minutes 32 seconds, what is the difference of longitude ?

EXERCISE II.

8. Bought 4 pieces of cloth measuring as follows 23 yards,

3 quarters, 2 nails ; 27 yards, 2 quarters, 1 nail ; 25 yards, 20 quarters, 3 nails; how many yards in all of them ?

9. A party started on a journey of 315 miles. After travelling 156 miles, 4 furlongs, 20 rods, how much farther had they to travel ?

10. A farm is divided into 16 fields, and each field contains 9 acres, 130 square rods ; how large is the farm ?

11. A man wishes to draw 12 cords of wood in 14 loads; how much must he draw in each load ?

12. A note was dated June 15, 1861, and was paid June 1st, 1862 ; how long was the time ?

13. The difference of longitude between Washington and London is 77°, what is the difference of time ?

14. The difference of time between New York and San Francisco is 3 hours, 25 minutes, 44 seconds ; what is the difference of longitude ?

EXERCISE III.

15. A farmer carried to market four loads of corn; in the first 36 bushels, 2 pecks, 4 quarts ; in the second 42 bushels, 3 pecks, 1 quart ; in the third 38 bushels, 1 peck, 3 quarts ; in the fourth 40 bushels, 6 quarts ; how many bushels in all ?

16. A merchant has sold from a hogshead of molasses 38 gallons, 1 quart, 1 pint ; how much is unsold ?

17. What is the weight of a dozen silver spoons, each weighing 6 ounces, 8 pennyweights, 10 grains ?

18. If a pound of rhubarb be divided into 100 doses ; how much will a dose contain ?

19. John Q. Adams, was born July 11, 1767 ; what was his age when he died, February 23, 1848 ?

20. What is the difference in the time of two places, one in longitude 56° 30′, and the other in longitude 125° 20′, both East ?

21. When it is noon at Greenwich England, a captain of a ship finds that is 4 o'clock P.M. where he is ; in what longitude is he ?

EXERCISE IV.

22. Two men building a stone fence, built the first day 5

rods, 8 feet, 9 inches; the second 6 rods, 4 feet, 6 inches; the third 6 rods; and the fourth they finished it, building 4 rods 8 inches; how long was the fence?

23. A merchant tailor bought a piece of cloth supposed to contain 27 yards, 2 quarters, but on measuring it he found that it contained only 25 yards, 3 quarters; how much did it fall short?

24. A lumber merchant sold 10 loads of hewn timber, each containing 1 ton, 32 feet, 1600 inches; how much did he sell?

25. A farmer having 256 acres of land, divided it equally among his seven children; how much did he give each?

26. A note was dated May 16, 1858, and was due October 1st, 1860; how long was the time?

27. What is the difference of time in two places, one 36° 30′ East longitude, and the other 40° 15′ West longitude?

28. A sea captain found that it was 20 minutes past 9 o'clock where he was, when it was noon at Greenwich, England. In what direction from Greenwich was he, and in what longitude?

EXERCISE V.

29. Bought four loads of coal, weighing as follows: 18 cwt. 3 qrs. 20 lbs.; 16 cwt. 1 qr. 12 lbs.; 19 cwt. 21 lbs.; 17 cwt. 2 qrs. 9 lbs.; 18 cwt. 3 qrs.; how much in all?

30. A merchant bought 12 hogsheads, 36 gallons of oil, and sold 7 hogsheads, 42 gallons, 3 quarts, how many had he left?

31. At £1 12s. 8d. a yard, how much will 20 yards of cloth cost?

32. If 18 yards of cloth cost £28, what is the price per yard?

33. Washington was born February 22, 1732; what was his age when he died, December 14, 1799?

34. The longitude of Boston is 71° 4′, and that of St. Louis is 90° 15, what is the difference of time?

35. A sea captain having sailed from Philadelphia, longitude 75° 10, finds his watch 1 hour 20 minutes slower than the time where he is; supposing his watch has kept good time, in what longitude is he?

CANCELLATION.

Art. 77.—Cancellation is the omission of equal factors in the dividend and divisor, or any corresponding terms, for the purpose of shortening the operation. It is on the principle that dividing both the dividend and divisor by the same number, does not alter the quotient. (Art. 20, 3.)

EXAMPLE 1.—Multiply 12 by 4, and divide the product by 4.

Process.—$12\times4=48$ and $48\div4=12$. We therefore omit both the multiplication and division, or cancel the 4, by drawing a line across it, in both the dividend and divisor.

$$\frac{12\times\cancel{4}}{\cancel{4}}=12$$

Ex. 2.—Bought 12 oranges at 4 cents each, and gave in exchange for them 8 quarts of nuts; what was the price of the nuts per quart?

Process.—$12\times4=48$, the price of all the oranges, and $48\div8=6$ the price of the nuts per quart, but we cancel the factor 4 by drawing a line across it in the dividend and dividing the divisor 8 by it, which gives the same result; we also cancel the 2.

$$\frac{\overset{6}{\cancel{12}}\times\cancel{4}}{\underset{\cancel{2}}{\cancel{8}}}=6$$

RULE.—*Write the factors of the divisor under those of the dividend, and omit equal factors in both the dividend and divisor, or corresponding terms, drawing a line across them, then proceed as in other similar operations.*

Since cancelling is the same as dividing, the place of a cancelled factor must be supplied by 1 if there is no other factor left.

EXAMPLES.

Cancel equal factors in—

(3.) $\dfrac{2\times3\times4\times5\times7}{3\times4\times5\times2}$

(4.) $\dfrac{4\times6\times8\times10}{2\times3\times4\times2}$

(5.) $\dfrac{4\times9\times11\times15}{3\times22\times5}$

(6.) $\dfrac{2\times13\times28}{26\times14}$

7. Multiply together the factors 8, 12, 15 and 18, and divide the product by the factors 4, 5 and 9.

8. Multiply the factors 6, 9, 11 and 12, and divide the product by the factors 3, 4, 3, 4.

9. Multiply the factors 9, 14, 15, 16, and divide the product by the factors 2, 3, 7, 5, 8, 4.

10. Multiply the factors 11, 24, 22, 30, and divide the product by the factors 4, 5, 44, 33.

11. Bought 25 firkins of butter, each weighing 54 lbs., at 30 cts. a pound, and paid for them in exchange, potatoes in barrels, 3 bushels in each, at 75 cts. a bushel; how many barrels?

12. How many boxes of starch, each containing 20 lbs., at 12 cts. a pound, will pay for 10 sacks of corn, each containing 3 bushels, at 72 cts. a bushel?

13. How many bags of coffee, each weighing 60 lbs., at 30 cts. a pound, can be bought for 20 firkins of butter, each containing 90 lbs., at 25 cts. a pound?

14. How many barrels of sugar, each weighing 225 pounds, at 16 cts. a pound, can be bought for 25 barrels of flour at 8 dollars a barrel?

Art. 78.—Properties of Numbers.

A number is either *odd* or *even*; odd when it cannot be exactly divided by 2, and even when it can be, as 1, 3, 5, 13, &c., are *odd* numbers; 2, 4, 8, 12, &c., are even numbers.

Numbers are, also, either *prime* or *composite.*

A *prime* number is one which is not the product of any numbers greater than 1, or of any number into itself; as 3, 13, 127.

A *composite* number is the product of other numbers greater than 1; as $4=2\times2$; $12=4\times3$; $150=10\times5\times3$.

Art. 79.—To resolve Composite Numbers into Prime Numbers or Factors.

Small composite numbers may be thus resolved by inspection ; thus, the prime factors of 6 are 3 and 2 ; of 15 5 and 3.

The prime factors of larger numbers are found by trial.

Example 1.—What are the prime factors in 42 ?

Process.—Beginning with the least prime number, 2, we find that it is a factor in 42, and 21 another factor ; and by further trial we find that 3 is a factor in 21, and 7 another factor. Hence 2, 3, and 7, being all prime numbers, are the prime factors in 42.

$$\begin{array}{r} 2\overline{)42} \\ 3\overline{)21} \\ \hline 7 \end{array}$$

Rule.—*Divide the given number by any prime number, and the quotient in the same manner, till it is found to be also a prime number. The several divisors, and the last quotient will be the prime factors of the given number.*

[Any one familiar with the multiplication or division table can easily distinguish prime numbers less than 144 from composite numbers. To find other prime numbers less than 1000, reference may be made to the following]

Table of Prime Numbers.

149	193	241	293	353	409	461	509	571	617	661	727	773	829	883	947
151	197	251	307	359	419	463	521	577	619	673	733	787	839	887	953
157	199	257	311	367	421	467	523	587	631	677	739	797	853	907	967
163	211	263	313	373	431	479	541	593	641	683	743	809	857	911	971
167	223	269	317	379	433	487	547	599	643	691	751	811	859	919	977
173	227	271	331	383	439	491	557	601	647	701	757	821	863	929	983
179	229	277	337	389	443	499	563	607	653	709	761	823	877	937	991
181	233	281	347	397	449	503	569	613	659	719	769	827	881	941	997
191	239	283	349	401	457										

EXAMPLES.

What are the prime factors in

(2.) 8	(5.) 21	(8.) 32	(11.) 288
(3.) 10	(6.) 115	(9.) 230	(12.) 720
(4.) 14	(7.) 35	(10.) 256	(13.) 1728

The Greatest Common Divisor.

Art. 80.—The **Greatest Common Divisor** (g. c. d.) of two or more numbers is the **greatest** number that will divide each of them without a remainder; as 3 is the g. c. d. of 6 and 9.

Art. 81.—To Find the Greatest Common Divisor of any Numbers.

This may be often done by inspection; thus the g. c. d. of 18 and 27 is evidently 9.

Or the greatest common divisor may be found by trial.

Ex. 1. What is the g. c. d. of 96, 144, 1728.

Process.—We find by trial that 12 is a common divisor of the given numbers; also that 4 is a common divisor of the quotients, and that there is no other common divisor. Since dividing by 12 and 4 is the same as dividing by 48, the greatest common divisor is 48.

12)	96	144	1728
4)	8	12	144
	2	3	36

Rule.—*Divide the given numbers by any number that will divide each of them without a remainder, and the quotients in the same manner, till the last common divisor is found; the product of the several common divisors will be the greatest common divisor.*

The products of the prime factors common to all the given numbers will also be the g. c. d.

The common method of finding the g. c. d., has been to divide the greater of two numbers by the less, and the last divisor by the last remainder, till nothing remains. The last divisor is the g. c. d. Proceed in the same manner with the divisor thus found and another number.

Ex. 2.—What is the greatest common divisor of 48, 114, 132?

Process.—Any divisor of 48 is a divisor of $48 \times 2 = 96$. Therefore any divisor of 48 and 114 is a divisor of 114 and 96, also of their difference, 18. For the same reason it is a divisor of 12, the next remainder, and 6, the divisor of 12, is the g. c. d. Proceed in the same way with 6 and 132.

48)114(2
96
18)48(2
36
12)18(1
12
6)12(2
12
0

6)132
22

This method depends on the principle, that the divisor of any number is the divisor of any product of it, and the divisor of any two numbers is also the divisor of their sum or difference.

EXAMPLES.

What is the greatest common divisor of

(3.)	85	105	(7.)	72	84	(11.)	12	18	24
(4.)	96	152	(8.)	84	96	(12.)	18	32	40
(5.)	100	150	(9.)	98	112	(13.)	44	66	88
(6.)	108	114	(10.)	100	120	(14.)	81	90	117

The Least Common Multiple.

Art. 82.—The **Multiple** of any number is the product of that number multiplied by another number; as 15 is the multiple of 5 by 3 or 3 by 5.

The **Common Multiple** of two or more numbers is the same or common product of each multiplied by other numbers; as, 36 is the common multiple of 12 by 3; 9 by 4; 6 by 6; 4 by 9; 3 by 12.

The **Least Common Multiple** (l. c. m.) of two or more numbers, is the **least** common product of each by other numbers; as, 24 is the l. c. m. of 12 (by 2); 8 (by 3); 6 (by 4).

Art. 83.—To Find the Least Common Multiple of Two or More Numbers.

Ex. 1. What is the l. c. m. of 4, 6, 12.

Process.—By inspection we find that 12 is a common multiple of 4 (by 3), and 6 (by 2), and there can be no less multiple of itself; therefore it is the least common multiple of the given numbers.

Rule I.—*When the largest given number is a common multiple of the others, it is the least common multiple.*

Ex. 2. What is the least common multiple of 6, 8, 12?

Process.—By inspection we find that 12×2 will be a common multiple of 6 (×4) and 8 (×3). Therefore it is the l. c. m.

Rule II.—*When the largest number is not already a common multiple of the others, multiply it by the least number* *that will evidently make it such, and the product will be the least common multiple.*

If it is not known by what number the largest given number must be multiplied to make it a common multiple of the others, the l. c. m. may be found by the following method.

The common multiple of 6, 8, 12 (Ex. 2) is

$$\left.\begin{array}{r} 6\times8\times12= \\ 8\times6\times12= \\ 12\times6\times\ 8= \end{array}\right\} 576.$$

But 6, a common factor in two of the numbers (6 and 12), and 4, a common factor in 8 and 12, are common factors in the multipliers of the given numbers, and therefore may be cancelled, leaving

$$\left.\begin{array}{r} 6\times2\times2= \\ 8\times1\times3= \\ 12\times1\times2= \end{array}\right\} 24 \text{ the l. c. m.}$$

Since each product is the same, it is not necessary to repeat the process, and it makes no difference whether the common factors be cancelled in the multipliers of the given numbers, or in the numbers themselves; hence

2*d Process.*—Cancel the 6 because it is a factor in 12, and 4 the greatest factor in 8, because it is also a factor in 12. Then multiply the given number, 12, by the factor 2, and the product, 24, is the least common multiple.

$$\left\{\begin{array}{l} \cancel{6} \quad \overset{2}{\cancel{8}} \quad 12 \\ 2\times12=24 \ \textit{Ans.} \end{array}\right.$$

Rule III.—*Cancel any number or greatest factor that is also a factor in another number, and the product of the remaining numbers and factors will be the least common multiple.*

Sometimes two factors in the same number may be omitted when they are also factors in different numbers.

If there is no common factor, the product of the given numbers will be the least common multiple.

The common method has been the following:

$$\begin{array}{r|rrr} 2 & 6 & 8 & 12 \\ \hline 2 & 3 & 4 & 6 \\ \hline 3 & 3 & 2 & 3 \\ \hline & 1 & 2 & 1 \end{array}$$

$2\times2\times3\times2=24$ *Ans.*

Rule IV.—*Divide the numbers by any prime number that will divide two or more of them without a remainder; place the quotients and undivided numbers in another line; divide these also in the same manner, and continue the process till no two* *numbers can be thus divided. Then multiply together all the divisors and undivided numbers.*

EXAMPLES.

What is the l. c. m. of

(3.) 4, 8, 9 ?	(7.) 5, 10, 12, 24 ?
(4.) 6, 9, 12 ?	(8.) 6, 12, 14, 28 ?
(5.) 9, 15, 18 ?	(9.) 8, 14, 21, 30 ?
(6.) 10, 20, 30 ?	(10.) 9, 11, 22, 27 ?

Art. 84.—*Promiscuous Examples in Properties of Numbers.*

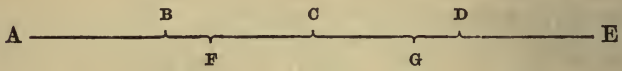
1. What are the prime factors in 24 ? 35 ? 196 ?
2. What is the g. c. d. of 9, 18, 24, 30 ?
3. What is the l. c. m. of 8, 12, 20, 32 ?
4. What are the prime factors in 14 ? 26 ? 34 ?
5. What is the g. c. d. of 21, 28, 35, 70 ?
6. What is the l. c. m. of 9, 11, 18, 22 ?
7. There are three rooms respectively, 12, 18 and 24 feet wide ; how wide may be the widest oil-cloth that will exactly fit them all without being cut ?
8. There are three horses running a circuit ; the first can complete it in 10 minutes, the second in 12, and the third in 15 ; if they start together and keep running, how long will it be before they will be together again ?
9. There is a garden the sides of which are respectively, 112, 126, 140 and 154 feet ; what must be the length of the longest boards that will fence it without being cut ?
10. Three ferry boats, starting together, are making regular trips to different points and back ; the first in 20, the second in 25, and the third in 30 minutes ; how long will it be before they will all return together ?

FRACTIONS.

Art. 85.—A **Fraction** is one or more equal parts of a number or thing, and expresses division.

If a number or thing is divided into two equal parts, one of them is called *one-half;* if divided into three equal parts one of them is *one-third,* two of them *two-thirds;* if divided into four equal parts, one of them is called *one-fourth,* or *one-quarter.*

If the number thus divided is not expressed, it is understood to be *one,* or a *single* thing; thus in the line AE, A C is one-half, A B one-quarter; A D three-quarters; A F one-third; A G two-thirds of the line AE.



Art. 86.—Fractions are of two kinds, **Common** and **Decimal.**

Common fractions are such as express any number of equal parts; as, one-half, two-thirds.

Decimal fractions are such as express only one or more of ten equal parts, or ten times ten, &c.; as, one-tenth, three-hundredths, five-thousandths, &c.

Common Fractions.

Art. 87.—Common fractions are usually written with two numbers, one the above other, with a line between them; as, $\frac{1}{2}$, one-half; $\frac{2}{3}$, two-thirds.

The number above the line is called the **Numerator,** and the number below the line the **Denominator;** both together are called the **Terms** of a fraction.

The **Denominator** shows the *number* of equal parts into which anything is divided, and corresponds with the divisor in division; also gives name to the fraction.

The **Numerator** shows *how many* of the equal parts are taken, or it may be a number divided by the denominator, and the same as the dividend in division.

ILLUSTRATION.—In the fraction $\frac{2}{3}$, the denominator (3) shows that something is considered as divided into three equal parts, and the numerator (2) shows that there are two such parts, or two things thus divided. The fraction expresses either two-thirds of one, or one-third of two; as, one-third of two dollars, or two-thirds of one dollar.

Art. 88.—Common fractions are usually divided into **Simple,** (either *proper* or *improper,*) **Compound, Complex,** and **Mixed Numbers.**

A **Simple** fraction has but *one* numerator and *one* denominator, both whole numbers. It is called a **Proper** fraction when its numerator is *less* than its denominator, and **Improper** when its numerator is *equal to or greater* than its denominator; as, $\frac{7}{8}$, (proper) $\frac{8}{7}$, or $\frac{7}{7}$, (improper.)

A **Compound** fraction is a fraction of a fraction; as $\frac{2}{3}$ of $\frac{4}{5}$.

A **Complex** fraction is one whose numerator or denominator is a fraction or mixed number; as, $\frac{\frac{1}{2}}{2\frac{1}{4}}$, $\frac{3}{5\frac{1}{2}}$.

A **Mixed Number** is a whole number and fraction written together; as $3\frac{2}{5}$.

It will be found convenient, also, to divide common fractions into two other kinds, **Like** and **Unlike.**

Like fractions are such as have the same name or common denominator; **Unlike,** such as have different denominators; as, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$ are **Like** fractions; $\frac{3}{4}$, $\frac{3}{5}$, $\frac{3}{7}$ are **Unlike.**

Art. 89.—The **Value** of a fraction is the quotient of the numerator divided by the denominator; as, the value of $\frac{8}{4}$ is 2; of $\frac{3}{3}$ is 1. The value of a proper fraction is less than 1, and therefore can only be expressed in the form of a fraction; as $\frac{2}{3}$.

The value of the equal parts of a fraction depends on the denominator, or the number of parts. The less the denominator the greater the value of each part; the greater the denominator the less the value of each part; thus $\frac{1}{4}$ is more than $\frac{1}{8}$, $\frac{2}{3}$ than $\frac{2}{9}$.

Art. 90.—Since the numerator of a fraction corresponds with the dividend in division, the denominator with the divisor, and the value with the quotient, the following propositions are of frequent application in fractions.

I. *Multiplying the numerator or dividing the denominator by any number, multiplies the value of the fraction by that number; thus*—

$$\tfrac{16}{8}=2;\ \frac{16\times4}{8}=\frac{64}{8}=8\ (2\times4).\quad \frac{16}{8\div4}=\frac{16}{2}=8\ (2\times4.)$$

II. *Dividing the numerator or multiplying the denominator by any number, divides the value of the fraction by that number; thus*—

$$\frac{16\div2}{4}=\frac{8}{4}=2\ (4\div2);\ \frac{16}{4\times2}=\frac{16}{8}=2\ (4\div2.)$$

III. *Multiplying or dividing both numerator and denominator by the same number does not alter the value of a fraction; thus*—

$$\frac{16}{8}=\frac{16\times4}{8\times4}=\frac{64}{32}=2;\ \frac{16\div4}{8\div4}=\frac{4}{2}=2.$$

EXERCISES.

Read, name the kind, and explain the terms, of the following fractions:

$\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{3}$, $\frac{1}{3}$ of $\frac{4}{5}$, $3\frac{1}{4}$, $\frac{3}{7}$, $\frac{9}{7}$, $\frac{\frac{1}{2}}{\frac{3}{4}}$, $\frac{7}{8}$, $\frac{11}{8}$, $\frac{\frac{2}{3}}{3\frac{1}{4}}$, $6\frac{2}{5}$, $\frac{10}{12}$, $\frac{2}{5}$ of $\frac{3}{7}$.

Also, mention some of the same that are like, and others that are unlike.

Write and describe the following fractions:

Three-fifths, seven-ninths, nine-fourths, eleven-quarters, two-thirds of four-ninths, nine and a half, five-sevenths, seven-thirds, four-fifths of seven-eighths, six and three-quarters, 3-sevenths, 5-ninths, 8-elevenths, 10-twelfths.

Read the following in order according to the value of their equal parts, beginning with the greatest. Repeat the same, beginning with the least :

$\frac{4}{9}$, $\frac{3}{15}$, $\frac{1}{2}$, $\frac{3}{8}$, $\frac{4}{16}$, $\frac{1}{4}$, $\frac{7}{20}$, $\frac{2}{3}$, $\frac{11}{18}$, $\frac{2}{5}$, $\frac{3}{10}$, $\frac{2}{7}$, $\frac{5}{6}$, $\frac{7}{12}$,

Write a fraction of each kind repeatedly till it can be done correctly.

MENTAL EXERCISES.

If an apple or anything is divided into two equal parts, what is each part called ? If divided into three equal parts ? 4 ? 5 ? 6 ? 7 ? 8 ? 9 ? 10 ? 11 ? 12 ? 13 ? 15 ? 18 ? 20 ? 25 ? 50 ? 65 ? 84 ? 100 ?

If an orange or anything is divided into 12 equal parts, what is one of them called ? 3 of them ? 6 ? 9 ? 4 ? 7 ? 10 ? 12 ?

Which is the greater, $\frac{1}{10}$ or $\frac{1}{2}$? Why ? $\frac{3}{8}$ or $\frac{3}{4}$? $\frac{4}{5}$ or $\frac{4}{20}$? $\frac{5}{12}$ or $\frac{5}{4}$? $\frac{7}{9}$ or $\frac{7}{18}$? $\frac{3}{25}$ or $\frac{3}{50}$? $\frac{1}{100}$ or $\frac{1}{1000}$?

Art. 91.—Reduction of Fractions.

Reduction of fractions is changing their form without altering their value ; as, $\frac{2}{4}=\frac{1}{2}$, $\frac{4}{6}=\frac{2}{3}$, $\frac{9}{4}=2\frac{1}{4}$.

Fractions are reduced to their *simplest form* when they are reduced to *simple and proper fractions in their lowest terms, whole or mixed numbers ;* thus the simplest form of $\frac{12}{24}$ is $\frac{1}{2}$; of $\frac{6}{3}$ is 2 ; of $\frac{7}{4}$ is $1\frac{3}{4}$; $\frac{2}{3}$ of $\frac{4}{5}$ is $\frac{8}{15}$.

Case I.

Art. 92.—*To reduce a fraction to its lowest terms or simplest form.*

A fraction is in its lowest terms, when its value is expressed by the least numbers possible ; thus $\frac{6}{12}$ reduced to its lowest terms is $\frac{1}{2}$.

Example 1.—Reduce $\frac{18}{24}$ to its lowest terms.

The fraction $\frac{18}{24}$ can be reduced to lower terms because both its numerator and denominator can be divided by 2 and 3, or 6, which (Prop. III. Art. 90) does not alter its value.

Process.—Dividing both terms by 3, the result is $\frac{6}{8}$, which may also be divided by 2, the result being $\frac{3}{4}$, the lowest terms. Or dividing $\frac{18}{24}$ by 6 the result is the same.

$$3)\frac{18}{24}=2)\frac{6}{8}=\frac{3}{4}$$

$$\text{or } 6)\frac{18}{24}=\frac{3}{4}$$

RULE.—*Divide the numerator and denominator by any number that will divide them both without a remainder, and continue dividing till the lowest terms are found.*

Or divide both terms by their greatest common divisor.

EXAMPLES.

(2.) $\frac{8}{12}$	(6.) $\frac{16}{32}$	(10.) $\frac{120}{144}$	(14.) $\frac{63}{108}$
(3.) $\frac{20}{25}$	(7.) $\frac{18}{90}$	(11.) $\frac{121}{143}$	(15.) $\frac{104}{120}$
(4.) $\frac{16}{20}$	(8.) $\frac{12}{48}$	(12.) $\frac{135}{180}$	(16.) $\frac{105}{112}$
(5.) $\frac{14}{42}$	(9.) $\frac{27}{45}$	(13.) $\frac{96}{112}$	(17.) $\frac{256}{258}$

CASE II.

Art. 93.—*To reduce an improper fraction to a whole or mixed number, or its simplest form.*

MENTAL EXERCISES.

In 2 half dollars or 2 halves ($\frac{2}{2}$) of one dollar, how many dollars? In $\frac{3}{2}$? $\frac{4}{2}$? $\frac{5}{2}$? $\frac{6}{2}$? $\frac{9}{2}$? $\frac{12}{2}$? $\frac{3}{3}$? $\frac{4}{3}$? $\frac{5}{3}$? $\frac{6}{3}$? $\frac{8}{3}$? $\frac{9}{3}$? $\frac{10}{3}$? $\frac{4}{4}$? $\frac{7}{4}$? $\frac{8}{4}$? $\frac{9}{4}$? $\frac{11}{4}$? $\frac{5}{5}$? $\frac{6}{5}$? $\frac{9}{5}$? $\frac{10}{5}$? $\frac{14}{5}$? $\frac{25}{5}$? $\frac{28}{5}$? $\frac{6}{6}$? $\frac{9}{6}$? $\frac{14}{6}$?

EXAMPLES FOR THE SLATE.

EXAMPLE 18.—Reduce $\frac{27}{3}$ to a whole number.

Process.—Since the denominator (3) shows the number of equal parts in 1, and the numerator (27) how many such parts there are, the fraction $\frac{27}{3}$ is as many times 1 as 3 is contained times in 27, which are 9 times. Therefore $\frac{27}{3}=9$ a whole number.

$$\frac{27}{3} \qquad 3)27 \qquad 9$$

2. Reduce $\frac{18}{5}$ to a mixed number.

Process.—$\frac{18}{5}$

$$5)18$$

$3\frac{3}{5}$ a mixed number.

RULE.—*Divide the numerator by the denominator, and the quotient will be the whole or mixed number.*

EXAMPLES.

(19.) $\frac{16}{2}$	(22.) $\frac{19}{3}$	(25.) $\frac{25}{5}$	(28.) $\frac{38}{6}$	(31.) $\frac{40}{8}$
(20.) $\frac{17}{2}$	(23.) $\frac{20}{4}$	(26.) $\frac{28}{5}$	(29.) $\frac{42}{7}$	(32.) $\frac{43}{8}$
(21.) $\frac{18}{3}$	(24.) $\frac{23}{4}$	(27.) $\frac{30}{6}$	(30.) $\frac{45}{7}$	(33.) $\frac{48}{9}$

34. In $\frac{27}{2}$ of a mile, how many miles?
35. In $\frac{25}{3}$ of a pound how many pounds?
36. In $\frac{30}{4}$ of a yard how many yards?
37. In $\frac{21}{8}$ of a bushel how many bushels?

CASE III.

Art. 94.—*To reduce whole or mixed numbers to improper fractions.*

MENTAL EXERCISES.

In 1 apple or anything how many halves? [Let the answer be first oral, then written.] How many thirds? fourths? quarters? fifths? &c.

In 3 apples? 5? 7? 8? &c.

In $1\frac{1}{2}$ apples how many halves? $2\frac{1}{2}$? $4\frac{1}{2}$? $6\frac{1}{2}$? $7\frac{1}{2}$? $10\frac{1}{2}$?

In $1\frac{1}{3}$ apples how many thirds? $2\frac{2}{3}$? $3\frac{1}{3}$? $5\frac{2}{3}$? $7\frac{1}{3}$? $8\frac{2}{3}$? $11\frac{1}{3}$? $12\frac{2}{3}$?

In $1\frac{3}{4}$ apples how many quarters? $2\frac{1}{4}$? $3\frac{3}{4}$? $5\frac{1}{4}$? $6\frac{3}{4}$?

EXAMPLES FOR THE SLATE.

38. Reduce $6\frac{3}{5}$ to an improper fraction.

Process.—Since $1=\frac{5}{5}$ $6=6$ times $\frac{5}{5}=\frac{30}{5}$ to which add $\frac{3}{5}$ and the sum will be $\frac{33}{5}$, *Ans.* Or,
Since there are $\frac{5}{5}$ in 1, in 6 there are 5 times as many fifths as there are times 1, and 5 times 6 are 30. Therefore $6=\frac{30}{5}$ to which add $\frac{3}{5}$, and the sum will be $\frac{33}{5}$, *Ans.*

$$\begin{array}{r} 6\frac{3}{5} \\ 5 \\ \hline 33 \\ \hline 5 \end{array}$$

EXAMPLE 39.—Reduce 8 to an improper fraction.

Process.—Since $1=\frac{1}{1}$, $8=8$ times $\frac{1}{1}=\frac{8}{1}$.

RULE.—*To reduce a mixed number to an improper fraction, multiply the whole number by the denominator of the fraction, add the numerator, and write the denominator under the sum.*

To reduce a whole number to an improper fraction, write 1 *under it with a line between them.*

EXAMPLES.

(40.) $6\frac{1}{2}$	(43.) $9\frac{2}{5}$	(46.) $12\frac{2}{5}$	(49.) 25
(41.) $7\frac{2}{3}$	(44.) 10	(47.) $15\frac{5}{8}$	(50.) $30\frac{5}{12}$
(42.) $8\frac{3}{4}$	(45.) $11\frac{1}{4}$	(48.) $18\frac{4}{9}$	(51.) $53\frac{8}{11}$

52. In $3\frac{4}{5}$ pounds, how many fifths of a pound?
53. In $11\frac{2}{3}$ hours, how many thirds of an hour?
54. In $12\frac{7}{9}$ rods, how many ninths of a rod?

CASE IV.

Art. 95.—*To reduce unlike fractions (having different denominators,) to like fractions (having a common denominator.)*

EXAMPLE 55. James has $\frac{1}{2}$ of an orange, John $\frac{2}{3}$, Henry $\frac{3}{4}$; how can these unequal parts be divided into equal parts?

Since the denominator of a fraction shows the number of equal parts into which anything is divided, fractions having different denominators must be reduced to a common denominator, that all the parts may be equal.

Process.—To reduce $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ to a common denominator, we multiply the numerator and denominator of each fraction by all the other denominators which (Prop. III. Art. 90) does not alter their value; thus

$$\frac{1\times3\times4}{2\times3\times4}=\frac{12}{24};\quad \frac{2\times2\times4}{3\times2\times4}=\frac{16}{24};\quad \frac{3\times2\times3}{4\times2\times3}=\frac{18}{24}$$

The fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, are thus reduced to equivalent fractions $\frac{12}{24}$, $\frac{16}{24}$, $\frac{18}{24}$, having a common denominator. Therefore if $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, of an orange be divided respectively into 12, 16, 18 equal parts all the parts will be equal, and each part will be $\frac{1}{24}$ of an orange.

RULE I.—*Multiply the numerator and denominator of each fraction by all the other denominators.*

Since the denominators thus found will be the same, after one is found it may be taken for the common denominator.

EXAMPLES.

(56.) $\frac{1}{3}$ $\frac{3}{4}$ $\frac{4}{5}$	(59.) $\frac{3}{5}$ $\frac{5}{7}$ $\frac{3}{8}$	(62.) $\frac{2}{7}$ $\frac{7}{8}$ $\frac{8}{9}$
(57.) $\frac{1}{4}$ $\frac{2}{5}$ $\frac{3}{7}$	(60.) $\frac{6}{7}$ $\frac{5}{8}$ $\frac{4}{9}$	(63.) $\frac{5}{6}$ $\frac{1}{7}$ $\frac{2}{11}$
(58.) $\frac{2}{3}$ $\frac{4}{5}$ $\frac{2}{9}$	(61.) $\frac{7}{8}$ $\frac{4}{5}$ $\frac{2}{11}$	(64.) $\frac{2}{3}$ $\frac{3}{5}$ $\frac{5}{7}$

Art. 96.—*To reduce unlike fractions to equivalent like fractions having the least common denominator.*

The fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ (Ex. 55,) reduced to a common denominator, are $\frac{12}{24}$, $\frac{16}{24}$, $\frac{18}{24}$, but these may be reduced to lower terms, so that the common denominator will be less, as $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$. 12 is the least common denominator, because the fractions cannot be reduced to lower terms and still have a common denominator.

To find the least common denominator of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, we multiply the numerator and denominator of the fraction $\frac{1}{2}$ by 6, $\frac{2}{3}$ by 4, $\frac{3}{4}$ by 3, because we observe that the fractions thus multiplied will have a common denominator, the least that can be found, and become $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, *Ans.*

RULE II.—*Multiply the numerator and denominator of each fraction by the least number that will produce a common denominator.*

If such a number cannot be conveniently found by inspection, find the least common multiple of the denominators, and divide it by the denominator of each fraction. The least common multiple will be the least common denominator, therefore the denominators need not be multiplied.

Before reducing fractions to a common denominator reduce them to their lowest terms.

EXAMPLES.

(65.)	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{8}$	(68.)	$\frac{3}{4}$	$\frac{1}{6}$	$\frac{11}{12}$	(71.)	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$
(66.)	$\frac{1}{3}$	$\frac{5}{6}$	$\frac{4}{9}$	(69.)	$\frac{1}{3}$	$\frac{3}{4}$	$\frac{5}{12}$	(72.)	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{7}{12}$
(67.)	$\frac{1}{4}$	$\frac{2}{3}$	$\frac{5}{6}$	(70.)	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{4}{9}$	(73.)	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{5}{6}$

When there are no two denominators of the fractions to be reduced, which can be divided by the same number, the first rule for finding the common denominator must be used.

EXAMPLES UNDER BOTH RULES.

(74.)	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{5}$	(77.)	$\frac{3}{4}$	$\frac{4}{6}$	$\frac{10}{12}$	(80.)	$\frac{6}{9}$	$\frac{4}{8}$	$\frac{3}{12}$
(75.)	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{8}$	(78.)	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{12}{14}$	(81.)	$\frac{2}{10}$	$\frac{3}{21}$	$\frac{2}{18}$
(76.)	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{5}$	(79.)	$\frac{1}{4}$	$\frac{4}{12}$	$\frac{2}{10}$	(82.)	$\frac{7}{8}$	$\frac{5}{10}$	$\frac{3}{4}$

The object of reducing fractions to a common denominator, is to prepare them for addition and subtraction.

Art. 97.—Reduction of Compound Fractions is properly included in Multiplication of Fractions, and Reduction of Complex Fractions in Division of Fractions.

Art. 98.—Promiscuous Examples in Reduction of Fractions.

1. Reduce $\frac{18}{84}$ to its lowest terms, or simplest form.
2. Reduce $\frac{27}{4}$ to a mixed number, or its simplest form.
3. Reduce $9\frac{2}{5}$ to an improper fraction.
4. Reduce $\frac{7}{9}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{2}{3}$ to like fractions or a common denominator.
5. Reduce $\frac{121}{132}$ to its simplest form.
6. Reduce $12\frac{2}{7}$ to an improper fraction.
7. Reduce $\frac{35}{8}$ to its simplest form.
8. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{2}$, 4 to like fractions.
9. Reduce $\frac{48}{7}$ to a mixed number.
10. Reduce $\frac{48}{64}$ to its lowest terms.
11. Reduce $13\frac{5}{8}$ to an improper fraction.
12. Reduce $\frac{3}{7}$, $\frac{6}{15}$, $\frac{4}{9}$, 3 to like fractions.
13. Reduce $\frac{63}{81}$ to its simplest form.
14. Reduce $\frac{132}{11}$ to its simplest form.
15. Reduce $14\frac{7}{11}$ to an improper fraction.
16. Reduce $\frac{4}{5}$, $\frac{9}{12}$, $\frac{5}{8}$, 6 to like fractions.
17. Reduce $\frac{42}{63}$ to its lowest terms.
18. Reduce $\frac{64}{16}$ to a whole number.
19. Reduce $15\frac{9}{41}$ to an improper fraction.
20. Reduce $\frac{4}{9}$, $\frac{2}{3}$, $\frac{5}{6}$, 5 to like fractions.
21. Reduce $\frac{3}{4}$, $\frac{7}{9}$, $\frac{11}{12}$, $\frac{6}{2}$ to like fractions.
22. Reduce $15\frac{8}{9}$, to an improper fraction.
23. Reduce $\frac{123}{11}$ to its simplest form.
24. Reduce $\frac{18}{54}$ to its simplest form.
25. Reduce $\frac{3}{5}$, $\frac{4}{7}$, $\frac{2}{9}$, 2 to like fractions.
26. Reduce $\frac{3}{4}$, $\frac{5}{6}$, $\frac{3}{8}$, 1 to like fractions.
27. Reduce $\frac{39}{7}$ to a mixed number.
28. Reduce $9\frac{7}{8}$ to an improper fraction.
29. Reduce $\frac{49}{7}$ to a whole number.
30. Reduce $\frac{3}{7}$, $\frac{4}{9}$, $\frac{3}{12}$, 7 to like fractions.
31. Reduce $\frac{81}{99}$ to its lowest terms.
32. Reduce $\frac{2}{6}$, $\frac{2}{18}$, $\frac{5}{6}$, 3 to like fractions.

33. Reduce $16\frac{4}{11}$ to an improper fraction.
34. Reduce $\frac{321}{7}$ to a mixed number.
35. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{12}$ to like fractions.
36. Reduce $\frac{3}{7}$, $\frac{4}{9}$, $\frac{6}{11}$, 4 to like fractions.
37. Reduce $\frac{1728}{1872}$ to its lowest terms.
38. Reduce $\frac{1728}{144}$ to its simplest form.
39. Reduce $\frac{3}{7}$, $\frac{4}{9}$, $\frac{51}{14}$, 3 to like fractions.
40. Reduce $\frac{5}{6}$, $\frac{7}{9}$ $\frac{2}{3}$, $\frac{7}{12}$ to like fractions.
41. Reduce $\frac{2}{8}$, $\frac{3}{12}$, $\frac{6}{9}$, $\frac{2}{3}$ to like fractions.

Addition of Fractions.

Art. 99.—Addition of Fractions is finding their sum.

Example 1.—Add $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$.

Since $\frac{1}{5}$, $\frac{2}{5}$, &c., are like fractions they can be added by adding their numerators, the same as 1 cent, 2 cents, &c.

Process.—$\frac{1}{5}+\frac{2}{5}=\frac{3}{5}$ and $+\frac{3}{5}=\frac{6}{5}$ and $+\frac{4}{5}=\frac{10}{5}=2$ *Ans.*

$$\begin{array}{rll} 1 & \text{fifth.} & \\ 2 & \text{"} & \\ 3 & \text{"} & \\ 4 & \text{"} & \\ \hline 10 & \text{"} & \frac{10}{5}=2 \end{array}$$

Ex. 2. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$.

Since $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$ are unlike fractions, they cannot be added as they are, and make $\frac{6}{2}$, $\frac{6}{3}$, or $\frac{6}{4}$ any more than 1 dollar 2 cents and 3 mills can be added in one column and make 6 dollars 6 cents or 6 mills.

Process.—Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ to like fractions, $\frac{12}{24}$, $\frac{16}{24}$, $\frac{18}{24}$, or $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, and add as above $\frac{12}{24}+\frac{16}{24}+\frac{18}{24}=\frac{46}{24}=\frac{23}{12}=1\frac{11}{12}$ or $\frac{6}{12}+\frac{8}{12}+\frac{9}{12}=\frac{23}{12}=1\frac{11}{12}$.

$$\begin{array}{cc} 12 & 6 \\ 16 & 8 \\ 18 & 9 \\ \hline \end{array}$$

$$\textit{Ans.}\ \frac{46}{24}=\frac{23}{12}=1\frac{11}{24}$$

Rule.—*Reduce the fractions to like fractions or a common denominator, add their numerators, and under the sum write the common denominator.*

1. Whole and mixed numbers may be reduced to improper fractions and then added like other fractions: but it better to add the fractional parts separately, and the whole numbers to the result.
2. After adding, reduce the sum to its simplest form.

EXAMPLES.

(3.) Add	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$		(9.) Add	$\frac{3}{8}$	$\frac{4}{8}$	$\frac{5}{8}$	
(4.) "	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$		(10.) "	$2\frac{1}{2}$	$3\frac{1}{4}$	$\frac{4}{5}$	
(5.) "	$\frac{1}{4}$	$\frac{2}{5}$	$\frac{5}{6}$		(11.) "	$\frac{7}{8}$	$\frac{5}{12}$	$\frac{3}{14}$	
(6.) "	$\frac{1}{5}$	$\frac{2}{7}$	$\frac{3}{10}$		(12.) "	$3\frac{1}{4}$	$\frac{5}{6}$	$2\frac{1}{3}$	
(7.) "	$\frac{4}{7}$	$2\frac{3}{4}$	$\frac{3}{8}$		(13.) "	$2\frac{3}{4}$	$3\frac{1}{5}$	$\frac{5}{8}$	1
(8.) "	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{7}{9}$		(14.) "	$\frac{3}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	3

15. How many dollars will pay for a coat worth $\$12\frac{1}{2}$, pants $\$9\frac{3}{4}$, vest $\$4\frac{1}{4}$, hat $\$5\frac{1}{8}$, and a pair of boots $\$6\frac{5}{8}$?

16. How many yards in five pieces of cloth measuring $27\frac{1}{4}$, $25\frac{3}{8}$, $26\frac{1}{3}$, $24\frac{1}{2}$, $22\frac{5}{6}$?

17. How many pounds of tea in six packages weighing $6\frac{2}{3}$, $5\frac{1}{6}$, 3, $4\frac{3}{4}$, $2\frac{1}{8}$, $\frac{11}{16}$? pound ?

18. How many pounds of butter in four tubs weighing $18\frac{3}{4}$, $16\frac{1}{2}$, $20\frac{3}{16}$, $19\frac{3}{8}$?

19. How many hundred weight of sugar in five barrels weighing $2\frac{3}{4}$, $1\frac{87}{100}$, $2\frac{5}{8}$, $1\frac{19}{20}$, $2\frac{3}{5}$ cwt ?

20. How many tons in four loads of coal weighing $1\frac{1}{20}$, $1\frac{2}{5}$, $\frac{195}{200}$, $1\frac{1}{25}$ tons ?

Subtraction of Fractions.

Art. 100.—Subtraction of Fractions, is finding the difference between two fractions or a fraction and a whole number.

Ex. 1. From $\frac{4}{5}$ take $\frac{3}{5}$.

Since $\frac{4}{5}$ and $\frac{3}{5}$ are like fractions, they can be subtracted by taking the less numerator from the greater, the same as taking 3 cents from 4 cents.

Process.—$\frac{4}{5}-\frac{3}{5}=\frac{1}{5}$ *Ans.*

Ex. 2. From $\frac{3}{4}$ take $\frac{1}{3}$.

Since $\frac{3}{4}$ and $\frac{1}{3}$ are unlike fractions, they cannot be subtracted as they are, any more than 1 cent can be taken from 3 dollars and leave 2 dollars or 2 cents.

Process.—Reduce $\frac{3}{4}$, $\frac{1}{3}$ to like fractions $\frac{9}{12}$, $\frac{4}{12}$, and substract $\frac{4}{12}$ from $\frac{9}{12}$. The remainder, $\frac{5}{12}$, is the answer.

RULE.—*Reduce the fractions to like fractions or a common denominator, subtract the numerators, and under the remainder write the common denominator.*

1. After subtraction reduce the remainder to its simplest form.
2. Whole or mixed numbers may be reduced to improper fractions and then subtracted, but it is better to subtract the fractional parts separately, thus

Ex. 3. From 4 subtract $\frac{3}{5}$.

Process.—Since there is no fraction from which to take $\frac{3}{5}$, take it from 1 (a part of the 4)$=\frac{5}{5}$. $\frac{5}{5}-\frac{3}{5}=\frac{2}{5}$ and since 1 has been used in the operation, carry as in subtraction of whole numbers.

$$\begin{array}{r} 4 \\ \frac{3}{5} \\ \hline 3\frac{2}{5} \end{array}$$

Ex. 4. From $8\frac{1}{3}$ subtract $3\frac{5}{8}$.

Process.—Reduce $\frac{1}{3}$ and $\frac{5}{8}$ to like fractions, $\frac{8}{24}$, $\frac{15}{24}$, and since $\frac{15}{24}$ cannot be subtracted from $\frac{8}{24}$, add 1 (a part of the 8)$=\frac{24}{24}$ to $\frac{8}{24}$, and from their sum $\frac{32}{24}$ take $\frac{15}{24}$, and the remainder will be $\frac{17}{24}$, carry 1, &c.

$$\begin{array}{r} 8\frac{1}{3} \\ 3\frac{5}{8} \\ \hline 4\frac{17}{24} \end{array}$$

EXAMPLES.

(5.)	From	$\frac{3}{4}$	subtract	$\frac{2}{5}$	(11.)	From	$4\frac{3}{4}$	subtract	$2\frac{1}{4}$
(6.)	"	$\frac{7}{9}$	"	$\frac{3}{9}$	(12.)	"	$5\frac{7}{8}$	"	$3\frac{1}{2}$
(7.)	"	$\frac{8}{11}$	"	$\frac{2}{3}$	(13.)	"	$10\frac{3}{4}$	"	$3\frac{5}{12}$
(8.)	"	$\frac{12}{24}$	"	$\frac{3}{7}$	(14.)	"	$9\frac{5}{12}$	"	$4\frac{5}{6}$
(9.)	"	$\frac{16}{64}$	"	$\frac{1}{16}$	(15.)	"	14	"	$6\frac{3}{7}$
(10.)	"	$\frac{81}{108}$	"	$\frac{3}{4}$	(16.)	"	$18\frac{2}{9}$	"	$9\frac{7}{8}$

17. If a man have \$6, and spend \$$3\frac{3}{8}$, how many dollars will he have left?

18. If a piece of cloth contain $27\frac{1}{4}$ yds., and $9\frac{5}{8}$ yds. be cut off, how many yards will be left?

19. If I buy 25 lbs. of sugar and use $12\frac{5}{16}$ lbs., how much will be left?

20. If a man on a journey of $65\frac{1}{2}$ miles, has traveled $48\frac{3}{4}$ miles, how many more miles has he to travel?

21. If there are $37\frac{1}{8}$ gals. of wine in a cask, how many will be left after $15\frac{2}{3}$ gals. shall have been drawn from it?

22. If a boy study $5\frac{1}{4}$ hours and play $2\frac{2}{3}$ hours, how many more will he study than play?

Multiplication of Fractions.

Case I.

Art. 101.—*Multiplying a fraction by a fraction.*

Multiplying any number by $\frac{1}{2}$ is finding $\frac{1}{2}$ of it, which is the same as dividing it by 2; multiplying by $\frac{1}{3}$ is finding $\frac{1}{3}$ of it, the same as dividing it by 3, and $\frac{1}{3}$ of a number multiplied by 2, gives $\frac{2}{3}$ of it, &c.

Example 1.—Multiply $\frac{5}{8}$ by $\frac{3}{4}$.

Process.—$\frac{5}{8}\times\frac{1}{4}=\frac{5}{32}$. Therefore $\frac{5}{8}\times\frac{3}{4}=\frac{5\times3}{32}=\frac{15}{32}$ (Prop. I. II. Art. 90.) $\frac{5}{8}\times\frac{3}{4}=\frac{15}{32}$.

Ex. 2. Multiply $\frac{4}{9}$ by $\frac{9}{16}$.

Process.—Cancel 4 and 9, factors common to the numerators and denominators, before multiplying. $\frac{\cancel{4}}{\cancel{9}}\times\frac{\cancel{9}}{\underset{4}{\cancel{16}}}=\frac{1}{4}$

Case II.

Art. 102.—*Multiplying a fraction by a whole number, or a whole number by a fraction.*

Ex. 3. Multiply $\frac{7}{9}$ by 3.

Process.—3 times $\frac{7}{9}=\frac{21}{9}$ or $\frac{7}{3}$ (P. I. Art. 90)$=2\frac{1}{3}$.

Ex. 4. Multiply 3 by $\frac{7}{9}$.

Process.—$3\times\frac{1}{9}=\frac{3}{9}$ or $\frac{1}{3}$; therefore $3\times\frac{7}{9}=\frac{21}{9}$ or $\frac{7}{3}=2\frac{1}{3}$. The product of $\frac{7}{9}\times3$ is the same as that of $3\times\frac{7}{9}$, and since $3=\frac{3}{1}$ this case may be included in Case I.

Case III.

Art. 103.—*Multiplying mixed numbers.*

Ex. 5. Multiply $4\frac{2}{3}$ by $3\frac{4}{5}$.

1*st Process.*—$4\frac{2}{3}\times3\frac{4}{5}=\frac{14}{3}\times\frac{19}{5}=\frac{266}{15}=17\frac{11}{15}$. *Ans.*

2*d Process.*—
$$\begin{aligned}\tfrac{2}{3}\times\tfrac{4}{5}&=\tfrac{8}{15}\\ 4\times\tfrac{4}{5}&=\tfrac{16}{5}=3\tfrac{1}{5}\\ \tfrac{2}{3}\times3&=\tfrac{6}{3}=2\\ 4\times3&=12\end{aligned}$$

The products added are $17\frac{11}{15}$. *Ans.*

By the first process the mixed numbers are reduced to improper fractions and then multiplied as in Case I.; by the second process, the whole numbers and fractions are multiplied as in Case II.

RULE.—*Reduce whole or mixed numbers to improper fractions, cancel all factors common to the numerators and the denominators, then multiply the remaining numerators together for a new numerator and the remaining denominators for a new denominator.*

A whole number may be multiplied as the numerator of an improper fraction without being reduced.

Art. 104.—Reduction of Compound Fractions is properly included in multiplication of fractions:

$\frac{3}{4}$ of $\frac{5}{7}=\frac{3}{4}\times\frac{5}{7}=\frac{15}{28}$; $\frac{2}{3}$ of $\frac{3}{4}$ of $2\frac{3}{5}=\frac{2}{3}\times\frac{3}{4}\times\frac{13}{5}=\frac{13}{10}=1\frac{3}{10}$.

MENTAL EXERCISES.

If an apple cost $\frac{1}{2}$ cent, what will $\frac{1}{2}$ of an apple cost? $\frac{2}{3}$? $\frac{1}{6}$? $\frac{5}{8}$? $\frac{3}{4}$?

Process.—If an apple cost $\frac{1}{2}$ cent, $\frac{1}{2}$ of an apple will $\frac{1}{2}$ of $\frac{1}{2}$ cent, which is $\frac{1}{4}$ cent.

At $\frac{1}{2}$ cent each, what will 2 apples cost? 4? 5? 8? 11? 12?

If an orange cost 4 cents, what will $\frac{1}{2}$ of an orange cost? $\frac{2}{3}$? $\frac{4}{5}$? $\frac{5}{6}$? $\frac{7}{8}$?

What will 3 plums cost at $\frac{1}{3}$ cent each? 4? 5? 6? 8? 11? 12?

If a melon cost 6 cents, what will $\frac{1}{2}$ a melon cost? $\frac{3}{4}$? $\frac{2}{3}$? $\frac{1}{6}$? $\frac{5}{9}$? $\frac{7}{12}$?

If a peach cost $\frac{3}{4}$ of a cent, what will $\frac{1}{2}$ of it cost? $\frac{1}{4}$? $\frac{2}{3}$? $\frac{4}{5}$? $\frac{5}{6}$? $\frac{7}{8}$?

EXAMPLES FOR THE SLATE, ETC.

Multiply—

(6.) 4 by $\frac{3}{4}$	(13.) 6 by $\frac{5}{6}$	(20.) 8 by $\frac{6}{7}$
(7.) $\frac{4}{5}$ by 3	(14.) $\frac{6}{7}$ by 14	(21.) $\frac{7}{8}$ by 16
(8.) $\frac{2}{3}$ by $\frac{5}{6}$	(15.) $\frac{3}{11}$ by $\frac{4}{7}$	(22.) $\frac{5}{12}$ by $\frac{4}{15}$
(9.) $3\frac{1}{4}$ by 4.	(16.) $4\frac{2}{3}$ by 5	(23.) $6\frac{3}{4}$ by 3
(10.) 5 by $2\frac{2}{3}$	(17.) 6 by $7\frac{2}{3}$	(24.) 7 by $4\frac{3}{5}$
(11.) $6\frac{3}{5}$ by $3\frac{4}{7}$	(18.) $8\frac{4}{7}$ by $5\frac{3}{4}$	(25.) $9\frac{5}{8}$ by $6\frac{2}{3}$
(12.) $\frac{1}{2}$ of 3 by $2\frac{1}{4}$	(19.) $7\frac{1}{2}$ by $\frac{2}{3}$ of 6	(26.) $\frac{1}{3}$ of $\frac{3}{4}$ by $\frac{1}{2}$ of $3\frac{1}{2}$

27. At $\$\frac{1}{2}$ a bushel, what will 6 bushels of potatoes cost? 13? 27?

28. At \$5 a bushel, what will $\frac{1}{2}$ bushel of clover seed cost? $\frac{2}{3}$? $\frac{3}{4}$?

29. At $\$\frac{3}{4}$ a bushel, what $\frac{3}{4}$ of a bushel of corn cost? $\frac{2}{3}$? $\frac{7}{8}$?

30. At $\$4\frac{1}{2}$ a bushel, what will 2 bushels of timothy seed cost? 3? 5?

31. At \$4 a bushel, what will $3\frac{3}{4}$ bushels of timothy seed cost? $5\frac{2}{3}$? $6\frac{3}{8}$?

32. At $\$4\frac{1}{4}$ a bushel, what will $2\frac{3}{4}$ bushels of timothy seed cost? $4\frac{1}{2}$? $6\frac{2}{3}$?

33. At $\$6\frac{1}{4}$ a bushel, what will $\frac{2}{3}$ of $3\frac{3}{4}$ bushels of clover seed cost?

Division of Fractions.

Case I.

Art. 105.—*Dividing a fraction by a whole number.*

Ex. 1. Divide $\frac{3}{5}$ by 3.

Process.—$\frac{3}{5}\div3=\frac{1}{5}$ or $\frac{3}{15}$ (P. II. Art. 90.)

Rule.—*Divide the numerator by the whole number when it can be done without a remainder, otherwise multiply the denominator by the whole number.*

A mixed number may be reduced to an improper fraction and then divided; or the integral part may be divided first, and the remainder afterwards; thus $9\frac{1}{2}\div4=\frac{19}{8}=2\frac{3}{8}$, or 2 and a remainder $1\frac{1}{2}$ which divided by 4 equals $\frac{3}{2}\div4=\frac{3}{8}$.

EXAMPLES.

(2.) Divide $\frac{4}{7}$ by 2
(3.) Divide $\frac{5}{8}$ by 3
(4.) Divide $\frac{8}{12}$ by 4
(5.) $\frac{7}{12}$ by 5
(6.) $\frac{18}{19}$ by 6
(7.) $\frac{25}{4}$ by 100
(8.) $\frac{14}{17}$ by 7
(9.) $\frac{13}{5}$ by 8
(10.) $\frac{32}{40}$ by 8

CASE II.

Art. 106.—*Dividing by a fraction.*

Ex. 11.—Divide $\frac{5}{6}$ by $\frac{2}{3}$.

$$\frac{5}{6}\div\frac{2}{3}=\frac{5}{\cancel{6}_{2}}\times\frac{\cancel{3}}{2}=\frac{5}{4}=1\tfrac{1}{4}$$

Process.—$\frac{5}{6}\div1=\frac{5}{6}$, and $\frac{1}{3}$ is contained in any number 3 times more than 1 is, therefore $\frac{5}{6}\div\frac{1}{3}=3$ times $\frac{5}{6}=\frac{5\times3}{6}$, but $\frac{2}{3}$ is contained in a number $\frac{1}{2}$ as many times as $\frac{1}{3}$, therefore, $\frac{5}{6}\div\frac{2}{3}=\frac{5}{6}\times\frac{3}{2}=\frac{15}{12}$ the same as multiplying $\frac{5}{6}$ by the divisor inverted.

Ex. 12.—Divide 6 by $\frac{2}{4}$.

Process.—$6=\frac{6}{1}$, therefore, $6\div\frac{2}{3}=\frac{6}{1}\div\frac{2}{3}=\frac{6}{1}\times\frac{3}{2}$ or $6\times\frac{3}{2}$, the divisor inverted.

Mixed numbers may be reduced to improper fractions and then divided.

RULE.—*Multiply the dividend by the divisor inverted.*

Another method is to reduce the fractions to a common denominator and divide their numerators.

EXAMPLES.

Divide—

(13.) $\frac{5}{7}$ by $\frac{2}{3}$	(17.) 7 by $2\frac{1}{4}$	(22.) $\frac{8}{9}$ by $\frac{4}{6}$
(14.) $\frac{3}{8}$ by $\frac{2}{5}$	(18.) $\frac{11}{14}$ by $3\frac{2}{3}$	(23.) 100 by $\frac{25}{3}$
(15.) 6 by $\frac{3}{4}$	(19.) $5\frac{3}{4}$ by $2\frac{2}{5}$	(24.) $\frac{23}{24}$ by $\frac{3}{8}$
(16.) $4\frac{3}{8}$ by $\frac{4}{9}$	(20.) $\frac{6}{7}$ by $\frac{3}{14}$	(25.) $\frac{19}{20}$ by $\frac{19}{5}$
	(21.) 16 by $\frac{5}{8}$	

(26.) 12 by $\frac{5}{7}$	(29.) $7\frac{4}{9}$ by $\frac{7}{9}$	(32.) $3\frac{1}{2}$ by $\frac{3}{5}$ of $\frac{10}{12}$
(27.) $\frac{7}{9}$ by 5	(30.) 16 by $5\frac{1}{3}$	(33.) $\frac{2}{5}$ of $5\frac{1}{2}$ by $1\frac{2}{3}$ of 6
(28.) $\frac{8}{9}$ by $\frac{2}{3}$	(31.) $\frac{2}{3}$ of $\frac{4}{5}$ by $\frac{4}{5}$	(34.) $6\frac{2}{5}$ by $2\frac{5}{7}$

MENTAL EXERCISES.

If $\frac{3}{4}$ of a yard of silk cost $\$\frac{7}{8}$, what is the price per yard?

Process.—If $\frac{3}{4}$ yd. cost $\$\frac{7}{8}$, $\frac{1}{4}$ yd. will cost $\frac{1}{3}$ of $\$\frac{7}{8}=\$\frac{7}{24}$ and 1 yd. will cost 4 times $\frac{7}{24}=\frac{28}{24}=\$1\frac{1}{6}$, *Ans.*

If 3 yards of silk cost $\$2\frac{1}{8}$, what is the price per yard?

If $4\frac{1}{2}$ yards of silk cost $\$3\frac{3}{8}$ what is the price per yard?

If $\frac{5}{6}$ of a yard of silk cost \$2, what is the price per yard ?

At $\frac{3}{5}$ of a cent each, how many pears can be bought for 12 cents ?

At \$$\frac{3}{8}$ a bushel, how many bushels of oats can be bought for \$$\frac{3}{4}$? \$$1\frac{1}{2}$? \$$3\frac{3}{4}$?

What part of \$1 will 1 quart of nuts cost if 5 qts. cost \$$\frac{4}{5}$?

What will a pound of candy cost, if $\frac{3}{5}$ of a pound cost $8\frac{2}{3}$ cents ?

What will a pound of candy cost if $4\frac{1}{2}$ lbs. cost \$$\frac{5}{8}$?

EXAMPLES FOR THE SLATE.

35. If $3\frac{1}{4}$ yds. of silk cost \$$6\frac{1}{2}$, what is the price per yard ?
36. If $3\frac{5}{6}$ lbs. of tea cost \$$2\frac{3}{10}$, what is the price per pound ?
37. What will a yard of muslin cost if $26\frac{2}{3}$ yards cost \$10 ?
38. What costs a yard of cloth if $\frac{2}{3}$ of a yard cost \$$2\frac{3}{4}$?
39. If 3 yards of cloth cost \$$15\frac{3}{8}$, what is it a yard ?
40. If $3\frac{7}{8}$ yards of cloth cost \$$15\frac{1}{2}$, what is it a yard ?
41. If 9 quarts of cherries cost \$$\frac{1}{2}$, what part of \$1 will 1 quart cost ?
42. What costs 1 bushel of corn if $\frac{7}{12}$ of a bushel costs \$$\frac{7}{8}$?
43. If $12\frac{1}{2}$ bushels of wheat cost \$$31\frac{1}{4}$, what is the price per bushel ?
44. At \$$1\frac{5}{8}$ a bushel, how many bushels of wheat can be bought for \$26 ?
45. If $11\frac{1}{4}$ lbs. of rice cost \$$1\frac{1}{4}$, what is the price per pound ?
46. At $10\frac{1}{4}$ cts. a pound how many pounds of rice can be bought for $87\frac{1}{2}$ cents ?
47. At \$$\frac{3}{8}$ a pound, how much butter will \$6 purchase ?
48. If 22 lbs. of butter cost \$$7\frac{1}{3}$, what is the price per pound ?
49. At $16\frac{2}{3}$ cts. a pound, how many pounds of sugar can be bought for \$$2.33\frac{1}{3}$?

Art. 107.—Reduction of Complex Fractions to simple fractions is properly included in division of fractions, the numerators being the same as dividends and the denominators the same as divisors.

EXAMPLES.

50. Reduce $\dfrac{\frac{3}{4}}{2\frac{1}{2}}$ to a simple fraction.

Process.—$\dfrac{\frac{3}{4}}{2\frac{1}{2}}=\dfrac{3}{4}\div\dfrac{5}{2}=\dfrac{3}{\underset{2}{\cancel{4}}}\times\dfrac{\cancel{2}}{5}=\dfrac{3}{10}.$

51. Reduce $\dfrac{\frac{4}{5}}{7}$; $\dfrac{8}{1\frac{1}{2}}$; $\dfrac{6\frac{1}{4}}{\frac{4}{7}}$; $\dfrac{9\frac{5}{6}}{\frac{2}{3}}$; $\dfrac{\frac{3}{4}}{\frac{5}{8}}$; $\dfrac{\frac{5}{8}}{3\frac{1}{4}}$.

52. Add $\dfrac{3\frac{1}{2}}{6}$ to $\dfrac{8}{5\frac{1}{3}}$; $\dfrac{2\frac{3}{4}}{6\frac{2}{7}}$ to $\dfrac{2\frac{1}{2}}{8}$; $\dfrac{5}{2\frac{1}{4}}$ to $\dfrac{\frac{2}{3}}{\frac{4}{5}}$.

53. From $\dfrac{6\frac{3}{4}}{4\frac{1}{2}}$ subtract $\dfrac{\frac{1}{2}}{\frac{1}{3}}$. From $\dfrac{\frac{3}{4}}{\frac{2}{3}}$ subtract $\dfrac{\frac{1}{3}}{1\frac{1}{2}}$.

54. Multiply $\dfrac{8}{2\frac{1}{2}}$ by $\dfrac{\frac{2}{3}}{\frac{1}{5}}$; $\dfrac{6\frac{7}{8}}{2\frac{1}{4}}$ by $\dfrac{2}{\frac{5}{6}}$; $\dfrac{4\frac{2}{3}}{\frac{1}{5}}$ by $\dfrac{\frac{1}{3}}{\frac{5}{6}}$.

55. Divide $\dfrac{9\frac{1}{4}}{\frac{3}{8}}$ by $\dfrac{\frac{2}{3}}{4\frac{1}{6}}$; $\dfrac{8\frac{1}{5}}{\frac{3}{10}}$ by $\dfrac{\frac{1}{2}}{\frac{1}{4}}$; $\dfrac{3}{4\frac{1}{3}}$ by $\dfrac{\frac{1}{2}}{5\frac{1}{4}}$.

Promiscuous Examples in Common Fractions.

EXERCISE I.

1. Add $\frac{5}{8}$, $\frac{3}{10}$, $\frac{1}{4}$, $\frac{3}{5}$; $2\frac{1}{3}$, $\frac{5}{6}$, $\frac{3}{4}$ of $\frac{4}{9}$, $5\frac{1}{2}$; $\frac{1}{2}$ of $2\frac{1}{4}$, $\frac{5}{8}$, 3, $4\frac{1}{2}$.
2. From $\frac{7}{8}$ subtract $\frac{2}{3}$; $9\frac{5}{6}-4\frac{7}{12}$; $60-45\frac{3}{8}$; $16\frac{2}{3}-\frac{2}{5}$ of $7\frac{1}{4}$.
3. Multiply $\frac{11}{12}$ by 4; $3\frac{8}{9}\times\frac{2}{5}$; $5\times\frac{4}{15}$; $\frac{2}{3}$ of $6\frac{3}{4}\times5\frac{5}{9}$; $11\frac{1}{2}\times6\frac{3}{5}$.
4. Divide $\frac{14}{25}$ by 7; $8\div\frac{4}{5}$; $10\frac{1}{2}\div5\frac{1}{4}$; $\frac{3}{4}$ of $\frac{6}{9}\div\frac{2}{5}$ of $3\frac{1}{3}$.

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5. Add $\frac{3}{8}$, $\frac{7}{10}$, $\frac{5}{6}$, $\frac{4}{9}$; $\frac{2}{3}$ of $9\frac{3}{4}$, $\frac{5}{6}$, 3, $4\frac{2}{3}$; $5\frac{4}{7}$, $\frac{11}{21}$, $\frac{4}{5}$ of $\frac{7}{8}$, 6.
6. From $\frac{8}{15}$ subtract $\frac{2}{5}$; $7\frac{1}{4}-\frac{7}{12}$; $\frac{3}{4}$ of $16-2\frac{5}{12}$; $120-96\frac{7}{8}$.
7. Mul. $\frac{11}{16}$ by $\frac{3}{44}$; $7\times8\frac{3}{7}$; $\frac{4}{5}$ of $\frac{7}{12}\times\frac{2}{7}$ of 3.
8. Div. 12 by $\frac{4}{9}$; $\frac{7}{11}\div4$; $16\frac{4}{9}\div\frac{1}{3}$ of $\frac{6}{7}$; $22\frac{5}{6}\div7\frac{1}{3}$.

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9. Add $\frac{5}{8}$, $\frac{9}{24}$, $\frac{11}{16}$, $\frac{1}{12}$; $4\frac{1}{9}$, $2\frac{1}{2}$, $3\frac{2}{3}$, $\frac{1}{2}$ of $\frac{5}{6}$; $\frac{4}{9}$ of $18\frac{9}{11}$, $7\frac{1}{2}$, 8.
10. From $\frac{9}{16}$ subtract $\frac{3}{8}$; $12-6\frac{1}{4}$; $\frac{5}{6}$ of $\frac{12}{20}-\frac{2}{5}$ of $\frac{5}{8}$.
11. Multiply $\frac{10}{11}$ by $\frac{4}{5}$; $6\frac{2}{3}\times4$; $14\times\frac{3}{7}$; $\frac{4}{9}$ of $2\times\frac{2}{3}$ of $6\frac{3}{5}$.
12. Divide $\frac{6}{9}$ by $\frac{2}{3}$; $24\div3\frac{1}{5}$; $9\frac{3}{5}\div3$; $\frac{2}{7}$ of $49\div6\frac{1}{2}$.

13. Add $\frac{11}{12}$, $\frac{5}{16}$, $\frac{7}{8}$, $\frac{17}{24}$; $6\frac{3}{4}$, 4, $5\frac{3}{5}$, $\frac{2}{7}$ of $5\frac{1}{4}$; $\frac{3}{7}$ of $\frac{5}{6}$, $\frac{4}{5}$ of 12, $\frac{1}{4}$ of $6\frac{1}{3}$.
14. From $\frac{11}{18}$ subtract $\frac{3}{9}$; $14\frac{3}{4}-4\frac{7}{8}$; $\frac{3}{5}$ of $4-\frac{1}{3}$ of $\frac{9}{40}$.
15. Multiply $\frac{9}{22}$ by $\frac{11}{15}$; $8\frac{3}{4}\times6$; $12\times\frac{4}{7}$; $\frac{3}{5}$ of $11\times\frac{2}{3}$ of $\frac{6}{11}$ of $4\frac{2}{5}$.
16. Divide $\frac{7}{8}$ by $\frac{21}{32}$; $38\div9\frac{1}{2}$; $24\frac{3}{7}\div6$; $15\frac{5}{6}\div\frac{1}{2}$ of $10\frac{2}{3}$.

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17. Add $\frac{4}{5}$, $\frac{2}{3}$, $\frac{13}{15}$, $\frac{7}{9}$; $8\frac{1}{3}$, $16\frac{2}{3}$, 10, $\frac{7}{16}$; $\frac{3}{7}$ of $\frac{42}{45}$, $\frac{3}{5}$ of $6\frac{2}{3}$, $10\frac{4}{5}$.
18. From $\frac{10}{11}$ subtract $\frac{3}{11}$; $20-8\frac{3}{5}$; $11\frac{1}{5}-6\frac{5}{12}$; $\frac{14}{15}-\frac{1}{2}$ of $\frac{3}{5}$.
19. Multiply $\frac{8}{9}$ by $\frac{5}{16}$; $15\times\frac{7}{30}$; $\frac{12}{21}\times7$; $\frac{4}{5}$ of $\frac{37}{48}\times3\frac{1}{8}$.
20. Divide $\frac{9}{16}$ by $\frac{3}{4}$; $16\div\frac{5}{5}$; $\frac{8}{9}\div6$; $17\frac{1}{2}\div\frac{2}{3}$ of $13\frac{1}{8}$.

EXERCISE II.

21. How many yards $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{7}{8}$ of a yard?

22. From $8\frac{3}{4}$ subtract $2\frac{7}{8}$.

23. What will 11 apples cost, at $\frac{4}{5}$ of a cent each?

24. How many pounds of sugar at $7\frac{1}{2}$ cents a pound, can be bought for $48\frac{1}{8}$ cents?

25. At $5\frac{2}{3}$ cents a mile how much will it cost to travel 20 miles?

26. At $7\frac{2}{3}$ cts. a qt., how many quarts of cherries can be bought for $76\frac{2}{3}$ cents?

27. Five pieces of muslin contain the following numbers of yards, $18\frac{3}{4}$, $15\frac{1}{3}$, $14\frac{7}{8}$, $16\frac{5}{6}$, and $\frac{3}{4}$ of $16\frac{1}{4}$; how many yards in all the pieces?

28. A piece of cloth measured $22\frac{1}{2}$ yds., of which $\frac{3}{4}$ remain; how many yards would be left if $5\frac{2}{3}$ yards more should be cut off?

29. At $\$1\frac{1}{5}$ a head, how many sheep can be bought for $\$56\frac{2}{5}$?

30. What will $12\frac{1}{2}$ yds. of ribbon cost at $6\frac{7}{8}$ cts. a yard?

EXERCISE III.

31. How many are $6\frac{1}{4}$, $\frac{2}{3}$ of $\frac{6}{7}$, $7\frac{1}{2}$, and 3?

32. From $7\frac{1}{3}$ subtract $\frac{2}{5}$ of $\frac{10}{11}$.

33. At $\$1\frac{1}{4}$ a bushel how much wheat can be bought for \$50?

34. At $\$1\frac{1}{4}$ a bushel, how much cost $72\frac{1}{2}$ bushels of wheat?

35. At $18\frac{3}{4}$ cts. a pound, how much cost $18\frac{3}{4}$ lbs. of butter?

36. At $22\frac{1}{2}$ cts. a pound, how many pounds of butter can be bought for $\$5.37\frac{1}{2}$?

37. Four cheese weigh as follows: $25\frac{1}{4}$, $26\frac{1}{3}$, $24\frac{3}{8}$, and $\frac{4}{5}$ of $25\frac{1}{5}$ pounds; how much do all of them weigh?

38. From a piece of cloth which contained $27\frac{5}{8}$ yards have been cut off at different times, $3\frac{1}{2}$, $4\frac{1}{4}$, $5\frac{3}{5}$, and $\frac{7}{8}$ yds.; how many yards are left?

39. At $62\frac{1}{2}$ cts. a pound, how much will $7\frac{1}{4}$ lbs. of tea cost?

40. At $62\frac{1}{2}$ cts. a pound, how much tea can be bought for $\$6\frac{1}{4}$?

EXERCISE IV.

41. How many pounds are $3\frac{7}{8}$, $2\frac{1}{2}$, and $\frac{2}{3}$ pounds?

42. From $7\frac{9}{10}$ yards of ribbon $2\frac{3}{5}$ yards have been cut off, how many are left?

43. How much will $9\frac{3}{4}$ lbs. of sugar cost at $7\frac{1}{3}$ cts. a pound?

44. At $1\frac{1}{8}$ cts. a pound how much meal can be bought for $13\frac{1}{2}$ cts.?

45. At 15 cents a yard, how much cost $37\frac{2}{5}$ yards of muslin?

46. At $62\frac{1}{2}$ cts. a bushel, how many bushels of corn can be bought for $\$7.81\frac{1}{4}$?

47. Four pieces of calico measure as follows: $22\frac{3}{4}$, $20\frac{7}{8}$, $21\frac{1}{3}$, and $\frac{4}{5}$ of 25 yds., how many yards in all of them?

48. A piece of cassimere contained $16\frac{2}{3}$ yards, of which $\frac{1}{2}$ remain; if $3\frac{1}{4}$ yards more should be cut off, how many would be left?

49. At $\$6\frac{3}{4}$ a cord, how much will $8\frac{4}{9}$ cords of wood cost?

50. At $74\frac{1}{2}$ cts. a bushel, how many bushels of potatoes can be bought for $\$4.84\frac{1}{4}$?

EXERCISE V.

51. How many are $6\frac{1}{2}$, $\frac{1}{2}$ of $\frac{4}{5}$, $\frac{1}{2}$, $\frac{4}{5}$ of $\frac{5}{7}$, $2\frac{3}{4}$?

52. From $3\frac{3}{4}$ yards of linen $2\frac{1}{9}$ yards have been but off, how many remain?

53. At $\$1\frac{1}{4}$ a day what will a man receive for labor in $26\frac{1}{2}$ days?

54. At $9\frac{3}{4}$ cts. a pound, how many pounds of rice can be bought for $53\frac{5}{8}$ cts.?

55. At $\$\frac{3}{4}$ a bushel, how much will 10 bushels of corn cost?

56. At \$36 an acre, how much will $\frac{7}{9}$ of an acre of land cost?

57. How many yards $\frac{3}{5}$ of a yard wide will line 25 yards of cloth 1 yard wide?

58. From a hogshead of molasses $7\frac{5}{8}$ gal. have deen drawn, how many are left?

59. At 5 cents a pound, how much will $9\frac{3}{8}$ pounds of fish cost?

60. At $9\frac{1}{2}$ cts. a pound, how much cheese can be bought for $\$1.42\frac{1}{2}$?

EXERCISE VI.

61. How many are $5\frac{1}{3}$, $\frac{7}{8}$ of $\frac{4}{14}$, $\frac{1}{6}$ of $\frac{3}{7}$, and $3\frac{1}{4}$?

62. From $4\frac{1}{4}$ take $\frac{7}{15}$.

63. At $3\frac{4}{5}$ cts. a yard, how much will 100 yds. of tape cost?

64. At $\frac{7}{8}$ of a dollar a yard, how many yards of flannel can be bought for $\$2\frac{11}{40}$?

65. $9\frac{1}{2}$ cts. a yard, what will $15\frac{1}{4}$ yds. of muslin cost?

66. If $7\frac{3}{4}$ pounds of tea cost $\$4\frac{18}{20}$, what is the price per pound?

67. Six pieces of ribbon measure as follows: $9\frac{1}{4}$, $8\frac{1}{3}$, $10\frac{2}{5}$, $7\frac{1}{2}$, 5 yds.; how many yards in all the pieces?

68. From a piece of linen containing $11\frac{1}{3}$ yds. have been cut off at different times, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{2}$ of $\frac{1}{4}$ yds., how many are left?

69. At $12\frac{1}{2}$ cts. a dozen, what cost $7\frac{3}{4}$ dozen eggs?

70. If $13\frac{4}{5}$ bushels of apples cost $\$5.17\frac{1}{2}$, what is the price of a bushel?

EXERCISE VII.

71. How many bushels are $5\frac{1}{2}$, $3\frac{1}{4}$, $7\frac{1}{3}$ of $\frac{9}{22}$, and $\frac{2}{3}$ of $\frac{3}{5}$ bushels?

72. From $7\frac{1}{7}$ take $\frac{2}{3}$ of $\frac{4}{3}$.

73. At $16\frac{1}{2}$ cts. a yard, what cost $10\frac{3}{5}$ yards of calico?

74. Paid $\$2.53\frac{1}{8}$ for $6\frac{3}{4}$ bushels of turnips, what is the price per bushel?

75. At $7\frac{3}{4}$ cts. a pound, what cost $75\frac{1}{2}$ lbs. of sugar?

76. At $\$12\frac{3}{5}$ a ton, how many tons of coal can be bought for $\$47\frac{22}{25}$?

77. Four pieces of tape measure as follows: $23\frac{1}{8}$, $22\frac{1}{2}$, $24\frac{1}{3}$, and $\frac{3}{4}$ of $24\frac{2}{3}$ yds.; how many yards in all of them?

78. From a cheese weighing 26 pounds have been sold to different persons $5\frac{1}{4}$, $4\frac{2}{3}$, $3\frac{3}{8}$, and 6 pounds ; how much is left ?

79. At $9\frac{3}{4}$ cts. a pound, what cost $14\frac{1}{2}$ lbs. of codfish?

80. Paid \$4.41$\frac{4}{5}$ for $18\frac{4}{5}$ yds. of muslin ; what was the price per yard ?

EXERCISE VIII.

81. At \$9$\frac{1}{2}$ a barrel, how much cost 7 bbls. of flour ?

82. At \$9$\frac{1}{2}$ a barrel, how many barrels of flour can be bought for \$85$\frac{1}{2}$?

83. If 11 barrels of flour cost \$100$\frac{1}{2}$, what is the price per barrel ?

84. How many pounds of sugar are $\frac{3}{7}$, $\frac{7}{10}$, $3\frac{1}{5}$, and 4 lbs. ?

85. From 13 subtract $9\frac{3}{7}$.

86. At $5\frac{5}{8}$ cts. a pound, what cost $76\frac{2}{5}$ lbs. of iron ?

87. At $5\frac{5}{8}$ cts. a pound, how many pounds of iron can be bought for \$455$\frac{5}{8}$?

88. If $72\frac{1}{2}$ lbs. of iron cost \$4.62$\frac{1}{2}$, what is the price per pound ?

EXERCISE IX.

89. At $12\frac{1}{2}$ cts. a pound, how much lard can be bought for \$5.31$\frac{1}{4}$?

90. At $12\frac{1}{2}$ cts. a pound, how much will $16\frac{2}{3}$ lbs. of lard cost ?

91. If $18\frac{3}{4}$ lbs. of lard cost \$2.06$\frac{1}{4}$ cts., what is the price per pound ?

92. A boy paid $58\frac{1}{3}$ cts. for an arithmetic, $18\frac{3}{4}$ cts. for a slate, 9 cents for a sponge, $4\frac{7}{8}$ cts. for a lead pencil, and $12\frac{1}{2}$ cts. for paper, what did he pay for all of them ?

93. A girl bought a handkerchief for $33\frac{1}{3}$ cts. and gave the merchant $37\frac{1}{2}$ cents ; how much change should she have received ?

94. If one person consume $8\frac{3}{5}$ lbs. of beef in a week, how many persons would consume $587\frac{3}{5}$ lbs. in the same time ?

95. If one person consume $8\frac{3}{5}$ lbs. of beef in a week, how much would a family of 9 consume in the same time ?

96. If a family of 9 persons consume $73\frac{2}{5}$ lbs. of beef in a week, how many pounds will one person consume in the same time ?

EXERCISE X.

97. If $7\frac{3}{8}$ lbs. of tea cost $\$9\frac{7}{32}$, what is the price per pound?

98. At $\$1.18\frac{3}{4}$ a pound, how much will $6\frac{2}{5}$ lbs. of tea cost?

99. At $\$1\frac{3}{8}$ a pound, how many pounds of tea can be bought for $\$8\frac{1}{4}$?

100. Bought of the butcher on Monday, $6\frac{3}{16}$ lbs. of beef, on Tuesday $5\frac{1}{3}$ lbs, on Wednesday $4\frac{1}{5}$ lbs., on Thursday $7\frac{3}{4}$ lbs., on Friday $3\frac{7}{16}$ lbs., and on Saturday $12\frac{2}{3}$ lbs., how many pounds during the week?

101. A piece of beef weighed $11\frac{1}{4}$ lbs.; after being divided the smaller piece weighed $4\frac{11}{16}$ lbs.; how much did the larger weigh?

102. If a tub of lard contains $48\frac{9}{16}$ lbs., how many tubs will $582\frac{1}{2}$ lbs. fill?

103. If $11\frac{1}{2}$ tubs of lard contain $618\frac{3}{4}$ lbs. of lard, how many pounds must there be in a tub?

104. If 1 tub will contain $46\frac{15}{16}$ lbs. of lard, how many pounds will 12 tubs hold?

EXERCISE XI.

105. At $43\frac{3}{4}$ cts. a bushel, how many bushels of oats can be bought for for $25?

106. At $43\frac{3}{4}$ cts. a bushel, how much will $47\frac{3}{8}$ bushels of oats cost?

107. If $7\frac{3}{8}$ bushels of oats cost $\$3.22\frac{21}{32}$, what is the price per bushel?

108. A horse ate $4\frac{1}{8}$ bushels one week; $5\frac{1}{4}$ the second; $4\frac{11}{16}$ the third; $5\frac{1}{6}$ the fourth; how many bushels did he eat in the four weeks?

109. A man bought $27\frac{3}{7}$ bushels of oats for his horse, and fed him $\frac{2}{3}$ of $9\frac{3}{5}$ bushels; how many had he left?

110. If a horse eat $\frac{4}{7}$ of a bushel of oats in a day, in how many days will he eat $18\frac{3}{4}$ bushels?

111. If a horse eat $\frac{4}{7}$ of a bushel of oats in a day, how many will he eat in $18\frac{3}{4}$ days?

112. If a horse eat $5\frac{5}{8}$ bushels of oats in $10\frac{1}{2}$ days, how many does he eat in a day?

EXERCISE XII.

113. If a meadow yield $2\frac{3}{7}$ tons of hay on an acre, and the whole of it yield $13\frac{5}{14}$ tons, how many acres does it contain ?

114. If a meadow yields $0.2\frac{3}{7}$ tons of hay on an acre, and contains $6\frac{3}{4}$ acres, how much will the whole of it yield ?

115. If a meadow containing $7\frac{3}{5}$ acres yield $15\frac{4}{15}$ tons of hay, how much does it yield on an acre ?

116. A farmer has a load of hay in bales weighing as follow :—$147\frac{1}{4}$, $156\frac{2}{3}$, $139\frac{1}{2}$, $144\frac{9}{16}$, $153\frac{7}{8}$, $146\frac{5}{6}$ lbs. ; what is the weight of the whole load ?

117. A farmer having a load of hay weighing $24\frac{2}{9}$ cwt., and finding it too heavy for his team, took off $5\frac{4}{5}$ cwt.; what was then the weight of his load ?

118. At $\$9\frac{7}{8}$ a ton, how many tons of hay can be bought for $\$987\frac{1}{2}$?

119. At $\$9\frac{7}{8}$ a ton, how much will 50 tons of hay cost ?

120. If $86\frac{1}{4}$ tons of hay cost $\$862\frac{1}{2}$, what is the price per ton?

EXERCISE XIII.

121. If $\frac{4}{5}$ of 21 lbs. of soap cost $\frac{6}{7}$ of $\$2\frac{1}{5}$, what is the price per pound ?

122. If a pound of soap cost $8\frac{4}{9}$ cts., how much will 14 times $1\frac{5}{7}$ lbs. cost ?

123. If $\frac{5}{6}$ of $21\frac{7}{9}$ lbs. of starch cost $\frac{4}{9}$ of $\frac{2}{3}$ of $3\frac{3}{4}$ dollars, what is the price per pound ?

124. A man having a farm of $675\frac{3}{4}$ acres, gave $\frac{1}{2}$ of $\frac{3}{4}$ of it to his son, and $\frac{2}{5}$ of the remainder to his daughter ; how many acres did he give to both of them ?

125. If $\frac{3}{5}$ of 45 gals. of vinegar cost $\$3\frac{4}{7}$, what is the price per gallon ?

126. If 1 gallon of vinegar cost $31\frac{1}{4}$ cts., how much cost 11 times $\frac{4}{5}$ of a gallon ?

127. If a man's expenses are 30 times $\frac{7}{8}$ of a dollar a month, how much will they be for $\frac{3}{8}$ of 1 month ?

128. If a man's expenses are $\frac{3}{4}$ of 30 times $\frac{3}{5}$ of a dollar for $\frac{3}{4}$ of a month, how much are they for a month ?

DECIMAL FRACTIONS.

Art. 109.—Decimal Fractions are such as express one or more of 10 equal parts of anything, or of some multiple of 10, by itself; as, 100, 1000, etc.

The **Denominator,** not usually written, is always 10, 100, or 1000, etc.; when written the fraction is also common.

Since the value of figures decreases in a tenfold ratio from left to right, if, beginning at the left hand, we divide the value of any figure by 10, we find the value of the same figure in the next place, as in 111 or 222, 1(00) $\div 10 = 1(0)$, $2(00) \div 10 = 2(0)$ etc. For the same reason if we continue dividing by 10 we shall find the value of the same figure repeated any number of times after the whole number; but it will be less than 1, and therefore a fraction. To distinguish the whole number from the fraction, a decimal point (.) is placed between them, thus 111.111, &c., 222.2222, &c.

In these numbers the value of the first decimal figure next to the point is $1 \div 10 = \frac{1}{10}$; $2 \div 10 = \frac{2}{10}$, and of the next $\frac{1}{10} \div 10 = \frac{1}{100}$, $\frac{2}{10} \div 10 = \frac{2}{100}$, etc., but they are usually read as one numerator having a common denominator the same as that of the last decimal figure; thus .11 is read as $\frac{11}{100}$, .222 as $\frac{222}{1000}$, etc.

Art. 110.—Numeration Table.

Whole numbers.							Decimals.					
Millions.			Thousands.			Units or Ones.	Tenths.	Hundredths.	Thousandths.	Tens of T.	Hundreds of T.	Millionths.
1	2	3	4	5	6	7.	2	3	5	7	8	4.

Read, 1 million, 234 thousand, 567, and (decimal) 235 thousand, 784 millionths.

.2 is read.......2 tenths		=	$\frac{2}{10}$
.23 is read......23 hundredths		=	$\frac{23}{100}$
.234 is read.....234 thousandths		=	$\frac{234}{1000}$
.2345 is read....2,345 ten thousandths		=	$\frac{2345}{10000}$
.23456 is read...23,456 hundred thous.		=	$\frac{23456}{100000}$
.234567 is read..234,567 millionths		=	$\frac{234567}{1000000}$

Art. 110.—Rule for Reading Decimals.—*Read as whole numbers and add the name or denominator of the last decimal figure.*

The denominator is always 1, with as many ciphers annexed as there are decimals; $.25=\frac{25}{100}$. $.025=\frac{25}{1000}$.

Prefixing a cipher to a decimal divides its value by 10, because it multiplies its denominator by 10 without changing the numerator; as, $.5=\frac{5}{10}$, but $.05=\frac{5}{100}$; annexing a cipher does not alter the value, because it is the same as multiplying both the numerator and denominator of a fraction by 10; as $.5=\frac{5}{10}$. $.50=\frac{50}{100}=\frac{5}{10}$.

To distinguish the whole number from the decimal in reading, use the word *decimal* before the decimal expression, or when the numbers are concrete, read them as such (dollars, yards; &c.)

EXAMPLES TO BE READ.

25; .6; 2.5; .025; .3; .36; 3.6; .036; 12.5; 125.; 1.25; 125; .0125; 136.; 13.6; 1.36; .136; 00136; 147.; .147; 1.47; 14.7; .0147; .00147; 2356.; .2356; 23.56; 2.356; .235.6; .002356; 200.02; 100.; .001; 1728.; 1.728; 172.8; .1728; 17.28; .001728; 2500.25; .07; .067; 4.37; 21.21; 300.03; 40.4; 4.04; .404; .000404; .0005; 31.0031; .310031; 3.10031.

Art. 111.—Rule for Writing Decimals.—*Write what would be the numerator of a common fraction as a whole number, and prefix, if necessary, ciphers till the right hand figure is in its proper place, and then the decimal point.*

EXAMPLES TO BE WRITTEN.

Fifteen; fifteen hundredths; fifteen thousandths; 15 millionths; (15.; .15; 015; .000015;) 5 tenths; 5 thousandths; 5 millionths; 2 and 5 tenths; 25 thousandths; 25 hundred thousandths; 333 thousandths; 33 thousandths; 3 thousandths; 27 and 27 thousandths; 3 and 45 hundred thousandths; 36 hundredths; 3 tenths and 6 hundredths; 356 thousandths; 3 tenths, 5 hundredths and 6 thousandths; 28 hundredths; 128 ten thousandths; 8 millionths; 7 hundred thousandths; 4 and 4 tenths; 200 and 2 hundredths; 3000 and 3 thousandths; 45 millionths; 175 ten thousandths.

(See examples in Addition, &c.)

Art. 112.—Application of the Fundamental Rules to Decimals.

Since decimals increase from right to left in a tenfold ratio, the same as whole numbers, to which they are commonly annexed, they may be added, subtracted, multiplied, and divided by the same rules except in a few particulars.

Art. 113.—Addition of Decimals.

Example 1.—Add 6 and 5 tenths; 10 and 1 hundredth; 250 and 25 thousands; 144 and 265 thousandths; 14 and 4 tenths; 144 thousandths.

Process.—

$$\begin{array}{r@{.}l} 6 & 5 \\ 10 & 01 \\ 250 & 025 \\ 144 & 265 \\ 14 & 4 \\ & 144 \\ \hline \textit{Ans.}\ 425 & 344 \end{array}$$

Rule.—*Write the numbers so that all the decimal points will be under one another; add as in whole numbers and place the decimal point in the sum under the others.*

Pupils are supposed to be familiar with addition, and the following examples are chiefly designed to exercise them in writing decimals. If they find the correct answers they will probably have *written* the numbers correctly.

EXAMPLES.

2. Add 2 and 17 hundredths ; 13 and 6 thousandths ; 12 and 145 thousandths ; 10 and 93 thousandths ; 17 and 81 ten thousandths ; 75 and 708 hundred thousandths ; 16 and 456 thousandths.

3. Add 26 and 5001 ten thousandths ; 37 and 604 thousandths; 8 and 77 hundredths ; 15 and 708 thousandths; 98 and 7 tenths ; 1.99 hundredths ; 18 and 45 thousandths.

4. Add 61 and 4 thousandths ; 4 and 7 hundredths ; 329 and 8 tenths ; 47 and 39 hundredths ; 731 and 96 thousandths ; 5 and 5 ten thousandths ; 6 and 8 tenths.

5. Add 42 and 8 hundredths; ; 521 and 28 thousandths ; 63 and 125 ten thousandths ; 108 and 215 thousandths ; 14 and 25 hundredths ; 9 and 5 tenths ; 18 and 23 hundredths ; 110 and 11 thousandths.

6. Add 17 and 55 thousandths ; 4 and 81 hundredths ; 90 and 1935 ten thousandths ; 77 and 85 hundred thousandths ; 24 and 106 ten thousandths ; 35 and 7 tenths.

7. Add 12 and 5 tenths ; 13 and 65 hundredths ; 114 and 25 thousandths ; 46 and 121 ten thousandths ; 64 and 7 tenths ; 127 and 18 thousandths ; 43 and 9 hundredths ; 102 and 6 tenths.

8. Add 6 and 157 thousandths ; 18 and 225 ten thousandths ; 172 and 16 hundredths ; 27 and 81 thousandths ; 9 and 23 hundred thousandths ; 13 and 27 thousandths ; 6 and 12 hundredths.

9. Add 12 and 9 thousandths ; 125 and 8 tenths ; 245 ten thousandths ; 249 ; 63 and 63 hundredths.

10. Add 17 ; 17 hundreds ; 17 thousandths ; 17 hundredths ; 1 and 7 tenths.

11. Add 16 thousand ; 16 hundreds ; 160; 16 ; 1 and 6 tenths; 16 hundredths ; 16 thousandths.

12. What is the sum of 8 and 75 hundredths ; 60 and 7 tenths ; 12 and 5 thousandths ; 180 and 27 hundredths ; 29 and 21 thousandths ; 3 and 15 hundredths ?

13. What is the sum of 3 thousand and 6 hundredths ; 2

hundred and 45 thousandths; 10 and 1 hundredth; 4 thousand and 6 hundredths; 4 hundred and 6 thousandths?

14. What is the sum of 10 and 1 tenth; 100 and 1 hundredth; 1000 and 1 thousandth; 200 and 2 thousandths; 20 and 2 hundredths?

15. What is the sum of 25 tenths; 126 hundredths; 1354 thousandths; 13579 ten thousandths; 2468 thousandths; 357 hundredths?

Art. 114. Subtraction of Decimals.

EXAMPLE 1.—From 123 and 56 hundredths, subtract 12 and 156 ten thousandths.

Process.—	123.56
	12.0156
Ans.	111.4444

RULE.—*Write the numbers so that the decimal points will be under each other, subtract as in whole numbers, and place the decimal point in the remainder under the others.*

If there is no figure directly above the one to be subtracted, consider the place as filled with a cipher.

EXAMPLES.

2. From 1. subtract 1 tenth.
3. From 20. subtract 2 hundredths.
4. From 250. subtract 25 thousandths.
5. From 1356. subtract 356 ten thousandths.
6. From 23464. subtract 3464 hundred thousandths.
7. From 100,000. subtract 100 thousandths.
8. From 1,000,000. subtract 1 millionth.
9. From 8 tenths subtract 4 hundredths.
10. From 75 hundredths subtract 75 thousandths.
11. From 56 thousandths subtract 56 ten thousandths.
12. From 5 tenths subtract 5 thousandths.
13. From 5 hundredths subtract 5 hundred thousandths.
14. From 5 thousandths subtract 5 millionths.
15. From 5 subtract 3 tenths.

What is the difference between—

16. 10. and .01?
17. 5. and .5?
18. 600 and 6 hundredths?
19. 7000 and 7 thousandths?
20. 80000 and 8 ten thousandths?
21. 3 and .3?
22. 3 tens and 3 tenths?
23. 3 hundreds and 3 hundredths?
24. 3 thousands and 3 thousandths?
25. 3 millions and 3 millionths?

Art. 115.—Multiplication of Decimals.

General Principle.—The denominator (understood) of any product of decimals is the product of their denominators; thus the product of tenths and tenths is hundredths ($\frac{1}{10}\times\frac{1}{10}=\frac{1}{100}$); the product of tenths and hundredths is thousandths, ($\frac{1}{10}\times\frac{1}{100}=\frac{1}{1000}$,) &c., &c.

Example 1.—Multiply 1.25 by .125.

Process.—The same as in whole numbers, except five figures are pointed off in the product for decimals, because the product of hundredths and thousandths is hundred thousandths. $1.25=1\frac{25}{100}=\frac{125}{100}$; $.125=\frac{125}{1000}$; and $\frac{125}{100}\times\frac{125}{1000}=\frac{15625}{100000}=.15625$, *Ans.*

$$\begin{array}{r} 1.25 \\ .125 \\ \hline 625 \\ 250 \\ 125 \\ \hline \textit{Ans. } .15625 \end{array}$$

Rule.—*Multiply as in whole numbers, and point off from the right of the product as many figures for decimals as there are decimals in both the multiplicand and multiplier; or so that the denominator (understood) of the product of the decimals shall be the product of their denominators.*

If there be not figures enough in the product, prefix ciphers.

Ex. 2.—Multiply .256 by 100.

Process.—$.256\times100=25.600$ or simply remove the decimal point two places to the right.

SPECIAL RULE.—*To multiply by* 10, 100, *&c., remove the decimal point as many places to the right as there are ciphers in the multiplier, annexing ciphers if necessary.*

EXAMPLES.

[Let pupils write the following examples with the respective products on their slates; also recite the answers without having them written, till they can do so readily and without mistakes.]

Multiplicands ..	3	3	.3	.03	.03	.3	.03
Multipliers	2	.2	.2	.2	.02	.3	.3
Products	6	.6	.06	.006	.0006	.09	.009

6	6	.6	.06	.06	7	7	.7	.07	.07
5	.5	.5	.5	.05	6	.6	.6	.6	.06
8	8	.8	.08	.08	9	9	.9	.09	.09
7	.7	.7	.7	.07	8	.8	.8	.8	.08
10	10	1.0	.10	.10	11	11	11	1.1	.11
9	.9	.9	.9	.09	10	1.0	.10	1.0	.10
12	12	12	.12	.12	12	12	12	.12	.12
11	1.1	.11	1.1	.11	12	1.2	.12	1.2	.12

3	4 tenths.	5 hund'ths.	6 hund'ths.	7 hund'ths.
2 tenths.	3 tenths.	4 tenths.	5 hund'ths.	6 thous'ths.
4	5 tenths.	6 hund'ths.	7 hund'ths.	8 hund'ths.
3 tenths.	4 tenths.	5 tenths.	6 hund'ths.	7 thous'ths.
5	6 tenths.	7 hund'ths.	8 hund'ths.	9 hund'ths.
4 tenths.	5 tenths.	6 tenths.	7 hund'ths.	8 thous'ths.
6	7 tenths.	8 hund'ths.	9 hund'ths.	10 hund'ths.
5 tenths.	6 tenths.	7 tenths.	8 hund'ths.	9 thous'ths.

12 thousandths.	12 ten thousandths.	12 millionths.
11 tenths.	11 hundredths.	11 thousandths.

EXAMPLES FOR THE SLATE, ETC.

3. Multiply 21 and 6 tenths by 3 and 6 hundredths.
4. 156 and 25 thousandths by 2 and 75 hundredths.
5. 50 and 5 hundredths by 2 and 16 thousandths.

6. 175 thousandths by 100.
7. 22 ten thousandths by 11 hundredths.
8. 18 by 256 thousandths.
9. 6 and 5 tenths by 65 hundredths.
10. 325 thousandths by 50.
11. 672 ten thousandths by 25.
12. 1 millionth by 1000.
13. 100 by 1 thousandth.
14. 5 thousandths by 4 thousandths.
15. 125 millionths by 1,000,000.
16. 275 and 275 thousandths by 25 and 25 hundredths.

In U. S. Money cents and mills are decimal fractions of a dollar; 1 cent=$.01; 25 cents=$.25; 50 cents=$.5; 1 mill=$.001; 5 mills=$.005, &c.

17. At 12 cts. a yard, how much will 16.5 yds of calico cost?

18. At 75 cts. a bushel, how much will 18.25 bushels of corn cost?

19. At 62 cts. 5 m. a gallon, how much will 20.5 gals. of molasses cost?

20. At $8.625 a ton, what cost 6.5 tons of coal?

21. At $100 an acre, what cost 63.75 acres of land?

22. At $9.625 a barrel, what cost 20 barrels of flour?

23. At $0.1875 a pound, what cost 37.5 lbs. of lard?

24. At $.025 a mile, how much will it cost to travel 100 miles?

Art. 116.—Division of Decimals.

General Principle. The denominator (understood) of the dividend divided by that of the divisor is the denominator of the quotient; thus thousandths divided by tenths are hundredths, and hundredths divided by tenths are tenths ($\frac{1}{1000}\div\frac{1}{10}=\frac{1}{100}$; $\frac{1}{100}\div\frac{1}{10}=\frac{1}{10}$) &c.

Example 1.—Divide 15.625 by .25.

Process.—The same as in whole numbers, except one figure is pointed off in the quotient because thousandths divided by hundredths, are tenths;

```
.25)15.625(62.5
    15 0       Ans.
      62
      50
      125
      125
```

$$15.625 = 15\frac{625}{1000} = \frac{15625}{1000};\ .25 = \frac{25}{100} \text{ and}$$

$$\frac{15625}{1000} \div \frac{25}{100} = \frac{15625}{10\cancel{0}\cancel{0}} \times \frac{\cancel{1}\cancel{0}\cancel{0}}{\cancel{2}\cancel{5}} = \frac{625}{10} = 62.5, \text{ Ans.}$$

Rule.—*Divide as in whole numbers, and from the right of the quotient, point off as many figures for decimals as the decimals in the dividend exceed those in the divisor; or, so that the denominator (understood) in the quotient, shall be the quotient of the denominator in the dividend divided by that of the divisor.*

If there are not figures enough in the quotient, prefix ciphers to the decimals, or annex them to whole numbers.

If there are not as many decimals in the dividend as in the divisor, annex ciphers. If other ciphers are annexed to the remainder, they must be considered as filling decimal places in the dividend.

Ex. 2.—Divide 3.25 by 100.

Process.—Remove the point two places to the left, prefixing a cipher where a figure is wanting, which is the same as dividing by 100. *Ans.* .0325

Special Rule.—*To divide decimals by* 10, 100, &c., *remove the decimal point as many places to the left as there are ciphers in the divisor, prefixing ciphers if necessary.*

EXAMPLES.

[Let the pupils write the following examples with the respective quotients, on their slates; also recite the answers without seeing them till they can do so readily and without mistakes.]

2)6 .2)6 .2).06 .2).006 .02).0006 .3).09 .3).009 .03).0009

4)24 .4)2.4 .4).24 .4).024. .04).024. .004).024

5)30 .5)30 .05)30 6)42 .6)42 .6)4.2

7)56 .7)56 .7)5.6 .07).56 .8)72 .8).72

9)90 .9)90 .09).90 8).96 .8)9.6 .08).96

11)132	.11)132	11)13.2	1.1)132	.11)1.32
12)144	.12)144	12)14.4	1.2)144	.12)1.44

Divide—

6 tenths by 2 tenths.	16 tenths by 4 tenths.
6 hundredths by 2 tenths.	16 thousandths by 4 hund'ths.
6 thousandths by 2 tenths.	24 hundredths by 4 hund'ths.
9 (whole number) by 3 hun'hs	32 tenths by 8 tenths.
9 hundredths by 3 hund'ths.	36 thousandths by 9 hund'ths.
9 thousandths by 3 tenths.	45 hundredths by 5 tenths.
12 thousands by 4 hund'ths.	56 millionths by 7 thous'ndths.
12 hundredths by 4 tenths.	64 millionths by 8 hundredths.

EXAMPLES FOR THE SLATE.

3. Divide 1728 by	12,	.12,	1.2,	.012
4. Divide .1728 by	12,	.12,	1.2,	.012
5. Divide 17.28 by	12,	.12,	1.2,	.012
6. Divide 1.728 by	12,	.12,	1.2,	.012
7. Divide 172.8 by	12,	.12,	1.2,	.012
8. Divide 1728 by	144,	.144,	1.44,	14.4
9. Divide 17.28 by	144,	.144,	1.44,	14.4
10. Divide 1.728 by	144,	.144,	1.44,	14.4
11. Divide 172.8 by	144,	.144,	1.44,	14.4

Divide—

12. 34 and 11 hundredths by 9 and 8 tenths.
13. 125 and 18 thousandths by 5 and 25 hundredths.
14. 40 and 215 ten thousandths by 8 and 5 tenths.
15. 25 ten thousandths by 25 hundredths.
16. 1 thousandth by 1000.
17. 63 and 63 hundredths by 7 tenths.
18. 125 millionths by 500.

19. At $5.75 a cord, how many cords of wood can be bought for $50 ?

20. If 7.5 cords of wood cost $38.775, what is the price of a cord ?

21. If 75.97 acres of land cost $2696.935, what is the price of an acre ?

22. At $35.50 an acre, how many acres of land can be bought for $1348.4625.

23. If 12.75 yards of cloth cost $97.5375, what is the price per yard ?

24. At 7.65 a yard, how many yards of cloth can be bought for $91.80 ?

25. At $7.375 a ton, how many tons of coal can be bought for $176.63125 ?

26. If 100 tons of coal cost $737.50, what is the price per ton ?

Art. 117.—Promiscuous Examples in Decimal Fractions, including U. S. Money.

EXERCISE I.

1. Bought tea for 1 dollar 25 cts. ; sugar, 3 dollars 37 cts. 5 m. ; starch, 50 cts. ; molasses, 2 dol. 9 cts. ; ginger, 18 cts. 5 m. What is the amount of the bill ?

2. A man having $10,000, paid $6,521.875 for a farm ; how much had he left ?

3. At $1.875 a bushel, how much will 12.5 bushels of wheat cost ?

4. If 16.25 bushels of wheat cost $28.4375, what is the price per bushel ?

5. At $1.75 a bushel, how many bushels of wheat can be bought for $30 ?

6. How many hundred weight of sugar in five barrels weighing 2 cwt. 75 hundredths ; 2 cwt. 875 thousandths ; 3 cwt. 4 hundredths ; 2 cwt. 5 tenths ; 3 cwt. 1275 ten thousandths ?

7. A merchant having a barrel of sugar weighing 2 cwt. has sold from it 1 cwt. 375 thousandths ; how much of it is left ?

8. If 2.5 yards of cloth will make a coat, how many yards will make 15 coats ?

9. If 2.375 yards of cloth will make a coat, how many coats will 28.5 yards make ?

10. If 37.5 yds. of cloth will make 15 coats, how many yards will make one coat ?

EXERCISE II.

11. Bought a barrel of flour for 10 dol. 50 cts. ; a bushel of timothy seed for 4 dol. 62 cts. 5 m. ; a cheese for 3 dol. 6 cts. 5 m. ; a box of soap for 8 dollars, and a broom for 38 cts. ; what is the amount of the bill ?

12. A lady having purchased a bill of goods amounting to 4 dollars 6 cts., gave the clerk a 10 dollar bill ; how much change was due her ?

13. At $1.37 a pound, how much will 3.25 pounds of tea cost ?

14. At $1.375 a pound, how many pounds of tea can be bought for $5 ?

15. If 4.75 pounds of tea cost $6.00, what is the price per pound ?

16. How many hundred weight of meal in 4 bags weighing 1 cwt. 1875 ten thousandths ; 1 cwt. 57 hundredths ; 1 cwt. ; .98 cwt. ?

17. A miller ground 6 cwt. of meal and sold 3 cwt. 125 thousandths ; how much of it was left ?

18. If a barrel contain 2.75 bushels of potatoes, how many bushels will there be in 12.5 barrels ?

19. If a barrel will contain 2.5 bushels of potatoes, how many barrels will contain 10.75 bushels ?

20. If 100 bushels of potatoes be put in 36.5 barrels, how many must each barrel contain ?

EXERCISE III.

21. Bought a load of potatoes for $15.75 ; of turnips for $7.375 ; of carrots for 9 dollars, 8 cts. 5 m. ; of beets for $10 ; to what did they all amount ?

22. Exchanged a horse worth $225, for a pair of oxen worth $180.625, and the rest in cash ; what was the amount of cash ?

23. At $6.375 a yard, how much will 3.75 yds. of cloth cost?

24. At $6.625 a yard, how much cloth can be bought for $25 ?

25. If 3.8 yds. of cloth cost $20.5, what is the price per yard ?

26. How many acres of land in five fields measuring 4 A. 27 hundredths; 6 A. 28 thousandths; 5 A.; 7 A. 7 hundredths; 6 A. 5 tenths?

27. If a field measures 10 A. 5 hundredths, and 5 A. 5 tenths be fenced off from it, how much of it will be left?

28. If a field contains 6.54 acres, how many acres will there be in 10 fields of the same size?

29. If 8 fields of equal size contain 45.75 acres, how many acres in each?

30. If a field contain 6.45 acres, how many such fields will contain 58.05 acres?

EXERCISE IV.

31. Bought a carpet for $12; matting, $6; carpet binding, 50 cts.; tacks, 6 cts.; what was the amount of the bill?

32. A clerk's salary is $1,000 a year, and his expenses $656.-625, how much can he lay up?

33. If a clerk receives $1,000 a year (313 working days), how much is it a day?

34. If a man earn $2.626 a day, how much will he earn in 300.5 days?

35. If a man earn $572.5 in 312.75 days, how much does he earn in a day?

36. How many pounds in 4 hams weighing 20 and 2 tenths pounds; 21 and 28 hundredths; 19 lbs.; 18 and 25 thousandths?

37. A ham weighed 22.5 lbs.; after being smoked it weighed 19 lbs. 8125 ten thousandths; how much weight had it lost?

38. If each firkin of butter contains 54.75 lbs., how many pounds will there be in 10.5 firkins?

39. If a firkin will hold 54.24 lbs. of butter, how many firkins will hold 1000 lbs?

40. If 512.25 pounds of butter be packed in 9.75 firkins, how many pounds will there be in each?

EXERCISE V.

41. If a man's income is $5,000 a year, and his expenses $10 a day, how much will he have left?

42. If a man's income is $5,000 a year (365 days), how much may he expend, and lay up $5.00 every day?

43. If a man lay up $2,000 a year (or 313 working days), how much will he lay up each day on an average?

44. How much butter in four tubs, weighing 16.5 lbs.; 20 lbs.; 18.25 lbs.; 19 and 25 thousandths pounds?

45. If each load of coal weigh 18.75 cwt., how much will 20 loads weigh?

46. If a man draw with his team 20.5 cwt. of coal in 24 loads, how much must he draw at a load?

47. If a man draw 16.75 cwt. at a load, how many loads will 251.25 cwt. make?

48. If a man chop 3.25 cords of wood in a day, how long will it take him to chop 30 cords?

Art. 118.—Reduction of Common Fractions to Decimals.

Example 1.—Reduce $\frac{3}{4}$ to a decimal.

Process.—Since $\frac{3}{4}=3\div 4$ and 4 is not contained in 3 a whole number of times, we find the value of 3 in the first place of decimals to be 3.0 (30 tenths) the same as the numerator, with a cipher annexed, which divided by 4, is .7+.2 remainder, but .2=.20 the same as annexing another cipher to the numerator, and .20÷4=.05, which added to .7=.75, *Ans.*

$$\begin{array}{r} 4)\underline{3.00} \\ \textit{Ans.}\ \ .75 \end{array}$$

Rule.—*Add ciphers to the numerator and divide by the denominator.*

Ex. 2. Reduce $\frac{1}{7}$ to a decimal.

Process.—$\begin{array}{r} 7)\underline{1.000000} \\ 142857+ \end{array}$ or $\begin{array}{r} 7)\underline{1.000} \\ .143 \end{array}$

In this Arithmetic, whenever a decimal, as in the above example, will extend to more than four places, three places will be considered sufficiently accurate if it is to be used afterwards, but if the figure in the fourth place would be more than five, 1 will be added to the figure in the third or thousandths place; as .143 for .14283.

Ex. 3. Reduce $\frac{1}{3}$ to a decimal.

Process.—$\begin{array}{r} 3)\underline{1.000000} \\ .333333 \end{array}$ or $\begin{array}{r} 3)\underline{1.0} \\ \dot{3} \end{array}$

2. A decimal, consisting of the same figure repeated (as in the above example), or several figures, is called a repetend or circulating decimal, and is distinguished as such by a (.) dot over the first and

last figures repeated. If only a part of the decimal is repeated it is called a mixed repetend.

Art. 119.—REDUCTION OF DECIMAL TO COMMON FRACTIONS.

EXAMPLE 1.—Reduce .375 to a common fraction.

Process.—$.375=\frac{375}{1000}$ which reduced to its lowest terms is $\frac{3}{8}$.

$$25)\frac{375}{1000} \quad 5)\frac{15}{40} \quad \frac{3}{8}$$

RULE.—*Erase the decimal point, and write, for the denominator of the common fraction,* 1 *with as many ciphers annexed as there are are decimal figures; then reduce the fraction to its lowest terms.*

Ex. 2.—Reduce $.\dot{3}$ or 333333, &c., to a common fraction

Process.—Since $.\dot{3}=.3\frac{1}{3}$ ($\frac{1}{3}$ reduced to a decimal) and 3 or $\frac{9}{3}$, is $\frac{1}{10}$ less than $3\frac{1}{3}$ (or $\frac{10}{3}$), .3 is $\frac{1}{10}$ less than $.\dot{3}$; but if the denominator of $\frac{3}{10}$ be diminished in the same proportion the value of the fraction is not altered. Therefore, $\dot{3}$ or $\frac{\dot{3}}{10}=\frac{3}{9}$, which equals $\frac{1}{3}$.

$$\dot{3}=\frac{3}{9}=\frac{1}{3}$$

If the repetend consist of more than one figure the denominator of the common fraction will be as many nines as there are figures repeated; thus, $.\dot{1}2\dot{3}=\frac{123}{999}=\frac{41}{333}$.

When the decimal is a repetend write for the denominator of the common fraction as many 9's as there are figures in the repetend.

Ex. 3. Reduce $.1\dot{6}$ to a common fraction.

When only a part of the decimal is repeated it is called a mixed decimal, and is the same as a mixed number in a complex fraction; thus:

$$.1\dot{6}=.1\frac{6}{9}=\frac{1\frac{6}{9}}{10}=\frac{15}{90}=\frac{1}{6},\ Ans.$$

EXAMPLES.

Reduce the common fractions to decimals, and the decimals to common fractions.

(1.)	$\frac{1}{2}$;	$\frac{2}{3}$;	$\frac{3}{4}$;	$\frac{4}{5}$	(5.)	$\frac{11}{12}$;	$\frac{12}{14}$;	$\frac{15}{20}$;	$\frac{20}{30}$
(2.)	.5;	.25;	.75;	.125	(6.)	.8;	.16;	.075;	.225
(3.)	$\frac{5}{6}$;	$\frac{7}{8}$;	$\frac{8}{9}$;	$\frac{9}{10}$	(7.)	$\frac{1}{9}$;	$\frac{2}{99}$;	$\frac{1}{6}$;	$\frac{12}{63}$
(4.)	.375;	.625;	.875;	.025	(8.)	$.\dot{6}$;	$\dot{0}2\dot{7}$;	$.\dot{1}2\dot{3}$;	$.1\dot{6}$

Fractional Compound Numbers.

Art. 120.—Fractional compound numbers are compound numbers in the form of fractions; as £$\frac{5}{8}$; $\frac{3}{4}$ cwt.; $\frac{2}{3}$ gal.

GENERAL RULE.—*Proceed, as in similar cases, in compound whole numbers, using the rules for fractions when necessary or convenient.*

Special rules will also be given in some cases, but it is better to understand and apply the general rule.

CASE I.

Art. 121.—*Compound numbers reduced to fractional compound numbers; reduction ascending.*

EXAMPLE 1.—Reduce 10s. 6d. to the fraction of a pound.

Process.—By common fractions, 6d. $\div 12 = \frac{6}{12}$s. $= \frac{1}{2}$s., to which add or prefix the 10s. The sum $10\frac{1}{2}$s. $\div 20 =$ £$\frac{21}{40}$. This common fraction reduced to a decimal is £.525.

$$\begin{array}{l} 12)\underline{6 \text{ d.}} \\ 20)\underline{10\frac{6}{12} \text{ s.}} \\ \pounds\frac{126}{240}=\frac{21}{40} \end{array} \quad \text{or} \quad \begin{array}{l} 20)\underline{10\frac{1}{2}} \\ \frac{21}{40}=.525 \end{array}$$

Process.—By decimal fractions, 6d. $\div 12 =$ 6.0d. $\div 12 =$ .5s., to which add or prefix the 10s. The sum 10.5s. $\div 20 =$ £.525 $=$ £$\frac{525}{1000}$ $= \frac{21}{40}$.

$$\begin{array}{l} 12)\underline{6.0} \\ 20)\underline{10.5} \\ .525=\frac{525}{1000}=\frac{21}{40} \end{array}$$

SPECIAL RULE.—*Beginning with the least denomination given, reduce it to the next greater by division of fractions (common or decimal), and add, or annex it to any given number of the same denomination or name. Proceed thus till the required fraction is found, which reduce to its lowest terms.*

For convenience write the least denomination first, and the others under it in order.

EXAMPLES.

[Let the answers be found in both common and decimal fractions.]

Ex. 2. What part of a cwt. is 2 qr. 10 lbs.?

3. What part of a mile is 26 rods 11 ft.?

4. What part of a hhd. is 15 gal. 3 qts.?

5. What part of a yard is 3 qr. 3 na.?
6. What part of a day is 6 h. 30 m.?
7. What part of an acre is 2 R. 10 rods?
8. What part of a yard is 3 qr. $3\frac{2}{3}$ na.?

Process.—$3\frac{2}{3}$ na. $\div$ 4 $=\frac{11}{3}$ na. $\div$ 4 $=\frac{11}{12}$ qr. $3\frac{11}{12}$ qr. $\div$ 4 $=\frac{47}{48}$ yds.

9. What part of £3 is 15 s. 6 d.?

Process.—First find what part of £1, and then $\frac{1}{3}$ of that.

CASE II.

Art. 122.—*Fractional compound numbers reduced to integral compound numbers; reduction descending.*

EXAMPLE 1.—Reduce £$\frac{5}{7}$ to whole numbers.

Process.—By common fractions. Since £$\frac{5}{7}$ is less than £1, reduce it to shillings. £$\frac{5}{7}\times 20=\frac{100}{7}$s. $=14\frac{2}{7}$s.; $\frac{2}{7}$ s. $\times 12=\frac{24}{7}$d. $=3\frac{3}{7}$d.; $\frac{3}{7}$d. $\times 4=\frac{12}{7}$far. $=1\frac{5}{7}$far. Therefore, £$\frac{5}{7}$=14s. 3d. $1\frac{5}{7}$far., *Ans.*

Ex. 2. Reduce £.625 to whole numbers.

Process (by decimal fractions), the same as in reduction descending of whole numbers, except pointing off, as in multiplication of decimals.

$$\begin{array}{r} .625\ £. \\ 20 \\ \hline 12.500\text{s.} \\ 12 \\ \hline \textit{Ans. } 12\text{s}\ \ 6.000\text{d.} \end{array}$$

SPECIAL RULE.—*Reduce the fractions to less denominations, and find the value of each in whole numbers.*

EXAMPLES.

How much in whole numbers is

(3.) £$\frac{2}{3}$	(10.) £.325.
(4.) $\frac{12}{25}$ ton.	(11.) .675 cwt.
(5.) $\frac{7}{12}$ mile.	(12.) .75 rod.
(6.) $\frac{5}{8}$ yard.	(13.) .6 yard.
(7.) $\frac{8}{9}$ acre.	(14.) .25 sq. mile.
(8.) $\frac{4}{9}$ hogshead.	(15.) .0025 tun.
(9.) $\frac{1}{20}$ year.	(16.) .0785 day.

CASE III.

Art. 123.—*Reduction of Fractional Compound Numbers to fractions of other denominations.*

EXAMPLE 1.—Reduce £$\frac{1}{360}$ to the fraction of a penny.

Process.—Reduction descending by multiplication of common fractions and cancellation. $£\frac{1}{360}=\frac{1}{360}\times20\times12=\frac{2}{3}$d., *Ans.*

Ex. 2. Reduce $\frac{3}{7}$ oz. to the fraction of a cwt.

Process.—Reduction ascending by division of common fractions.

$$\frac{3}{7}\text{ oz.}=\frac{3}{7}\div 16,\ 25\text{ and }4=\frac{3}{7\times16\times25\times4}=\frac{3}{11200}\text{ cwt., } \textit{Ans.}$$

Ex. 3. Reduce .00125 ton to pounds.

Process.—Reduction descending by multiplication of decimals.

```
     .000125 ton.
          20
     .002500 cwt.
         100
Ans. .250000 lb.
```

Ex. 4. Reduce .5 rod to the fraction of a mile.

Process.—Reduction ascending by division of decimals.

```
     40).500 rods.
      8).0125 fur.
Ans. .0015625 mile.
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EXAMPLES.

5. What part of a pound is $\frac{3}{4}$ of an oz. Av.?
6. What part of a pound is $\frac{1}{1200}$ of a cwt.?
7. What part of a pound is .5 of an oz. Troy?
8. What part of a pound is .0004 of a ton?
9. What part of a gallon is $\frac{2}{3}$ of a gill?
10. What part of a gallon is $\frac{1}{126}$ of a hhd.?
11. What part of a gallon is .25 of a pint?
12. What part of a gallon is .003 of a tun?
13. What of an hour is $\frac{1}{2}$ of a minute?
14. What of an hour is $\frac{1}{48}$ of a day?
15. What of an hour is .8 of a second?
16. What of an hour is .04 of a day?
17. What part of a rod is $\frac{11}{12}$ of a foot?
18. What part of a rod is $\frac{1}{640}$ of a mile?
19. What part of a rod is .165 of a yard?
20. What part of a rod is .003 of a mile?

Art. 124.—Addition and Subtraction of Fractional Compound Numbers.

EXAMPLE 1.—Add £$\frac{2}{3}$ to $\frac{5}{6}$ shilling.

Process.—

	s.	d
£$\frac{2}{3}$ =	12	8
$\frac{5}{6}$s. =		10
	13s.	6d.

Ex. 2. From $\frac{5}{6}$ cwt. subtract $\frac{3}{4}$ lb.

Process.—

$\frac{5}{6}$ cwt. =	3 qr.	8 lb.	5$\frac{1}{3}$	oz.
$\frac{3}{4}$ lb. =			12	
Ans.	3 qr.	7 lb.	9$\frac{1}{3}$	oz.

Ex. 3. Add .6 acre and .06 rod.

Process.—.6 acre = 96 rods, to which add .06 rods, and the sum will be 96.06 rods.

	.6 acre.	
	160	
	96.0	rods
	.06	"
Ans.	96.06	"

RULE.—*Reduce the fractions to whole numbers, and add or subtract them as other compound numbers.*

EXAMPLES.

Ex. 4. Add $\frac{3}{5}$ of a day to $\frac{2}{3}$ of an hour.

5. From $\frac{7}{8}$ of a mile subtract $\frac{3}{4}$ of a fur.

6. Add .525 of a gal. to .9 of a qt.

7. Add $\frac{5}{16}$ lb. to $\frac{7}{8}$ oz.

8. From $\frac{1}{6}$ hhd. take $\frac{1}{3}$ of a barrel.

9. From .625 of an hour take 3.5 minutes.

Art. 125.—Promiscuous Examples in Fractional Compound Numbers.

EXERCISE I.

1. How much in whole numbers is £$\frac{3}{7}$?
2. How much in whole numbers is .375 of a pound (Troy)?
3. What part of a ton is 5 cwt. 2 qr. 15 lbs.?

4. What part of a hundred weight is 3 qr. 12 lbs. 8 oz. (in decimals).
5. What part of a drachm is $\frac{1}{144}$ of a pound (Apo.)?
6. What part of a yard is $\frac{8}{9}$ of a nail?
7. What part of a rod is .825 of a foot?
8. What part of a square rod is .0025 of an acre?
9. What is the sum of $\frac{2}{3}$ cord and $\frac{2}{3}$ foot?
10. How much more is .25 of an hour than 5.5 minutes?

EXERCISE II.

11. How much in whole numbers is $\frac{6}{11}$ of a hogshead?
12. How much in whole numbers is .625 of a bushel?
13. What part of a year is 125 days 12 hours?
14. What part of a degree is 25′ 36″ (in decimals)?
15. What part of a farthing is $\frac{1}{144}$ of a shilling?
16. What part of a pound is $\frac{3}{4}$ of a pwt.?
17. What part of an ounce is .84 of a scruple?
18. What part of a yard is .64 of a nail?
19. What is the sum of .375 of an acre and 75.5 rods?
20. How much more is $\frac{2}{3}$ of a rod than $3\frac{1}{2}$ yards?

EXERCISE III.

21. How much in whole numbers is $\frac{1}{4}$ of a mile?
22. How much in whole numbers is .865 of a cord?
23. What part of a barrel is 21 gal.?
24. What part of a bushel is 3 pks. 6 qts. (decimal)?
25. What part of a second is $\frac{1}{180}$ of a minute?
26. What part of a degree is $\frac{1}{2}$ of a circular minute?
27. What part of a farthing is .0025 of a shilling?
28. What part of an ounce is .9 of a pwt.?
29. What is the sum of $\frac{1}{8}$ ton and $\frac{7}{8}$ cwt.?
30. How much less is .26 cwt. than 28.5 lbs.?

EXERCISE IV.

31. How much in whole numbers is $\frac{5}{6}$ of a pound (A.)?
32. How much in whole numbers is 2.5 of a yard?
33. What part of a yard is 2 ft. 9 in.?
34. What part of a cord is 21 cu. ft. 576 in.?
35. What part of a quart is $\frac{1}{32}$ of a gal.?

36. What part of a peck is $\frac{8}{9}$ of a pint ?
37. What part of a minute is .006 of an hour ?
33. What part of a circle is .72 of a degree ?
39. Add $\frac{2}{5}$ s. $6\frac{3}{5}$ d. $2\frac{1}{5}$ far.
40. How much more is .3 s. than 2.25 d. ?

EXERCISE V.

41. How much in whole numbers is $\frac{4}{5}$ of a pound (Troy) ?
42. How much in whole numbers is .0025 of a ton ?
43. What part of an ounce is 4 dr. 2 scr. ?
44. What part of a yard is 1 qr. 3 na. (decimal) ?
45. What part of an inch is $\frac{1}{180}$ of a yard ?
46. What part of an acre is $\frac{1}{4}$ of a rood ?
47. What part of a cord is 512 of a foot ?
48. What part of a gill is .0056 of a gall. ?
49. What is the sum of $.\dot{6}$ of a bushel and .8 of a peck ?
50. How much more is $\frac{1}{4}$ bushel than $\frac{7}{8}$ peck ?

Art. 126.—Promiscuous Examples in Fractions, Common and Decimal, and Fractional Compound Numbers.

EXERCISE I.

1. How many yards are there in three pieces of cloth, measuring as follows: $30\frac{4}{5}$, $37\frac{1}{2}$, $38\frac{3}{4}$ yards ?

2. From a piece of cloth which contained $33\frac{1}{3}$ yds., $16\frac{5}{6}$ yds. have been cut off ; how many are left ?

3. How much will $4\frac{1}{5}$ tons of iron cost at $\$18\frac{3}{4}$ a ton ?

4. At $\$18\frac{3}{4}$ a ton, how many tons of iron can be bought for $60 ?

5. If $4\frac{1}{5}$ tons of iron cost $75, what will a ton cost ?

6. How many are 3 and 7 tenths, 44 and 41 hundredths, 73 and 9 thousandths, 12 and 305 thousandths ?

7. From 5 hundredths subtract 476 ten thousandths.

8. At $.1875 a yard, what cost 12.25 yds. of muslin ?

9. At $.1875 a yard, how many yds. of muslin can be bought for $5.75 ?

10. If 19 yds. of muslin cost 5.377, what is the price per yd.?

11. At $18.875 a 1000, what will 12.500 feet of pine boards cost?

12. At $18.875 a 1000, how many feet of pine boards can be bought for $12.50?

13. If 1200 ft. of pine boards cost $20, what is the price per 1000 ft.?

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14. What part of a lb. Av. is $\frac{4}{7}$ of an oz.?

15. What part of a nail is $\frac{1}{20}$ of a yard?

16. What part of a mile are 3 fur. 8 rods?

17. What part of a bushel is .5 of a peck?

18. What part of a pound Troy is .75 of an oz.?

19. What part of a pound Troy are 8 oz. 8 pwt. (decimal)?

20. What decimal fraction is equal to $\frac{1}{4}$?

21. What common fraction is equal to .8?

22. What whole numbers are equal to $\frac{9}{16}$ of a day?

23. What whole numbers are equal to .5625 of a day?

24. How much are $\frac{2}{3}$ of a week, $1\frac{3}{4}$ days, $5\frac{1}{2}$ hours?

25. At $.625 a bushel, what cost 15 bus. 3 pks. 4 qts. of rye?

26. At $.625 a bushel, how much rye can be bought for $10?

EXERCISE II.

27. How many pounds are $20\frac{1}{2}$, $21\frac{1}{3}$, $22\frac{1}{4}$, $23\frac{1}{5}$, $24\frac{1}{6}$ lbs.?

28. From a cask containing $64\frac{2}{5}$ gallons of molasses, $30\frac{4}{7}$ gals. have been used; how many are left?

29. At $\$\frac{3}{20}$ a yard, how much ribbon can be bought for $\$\frac{1}{4}$?

30. At $\$\frac{3}{20}$ a yard, how much will $3\frac{1}{3}$ yards of ribbon cost?

31. If $1\frac{2}{3}$ yds. of ribbon cost $\$\frac{1}{4}$, what is the price per yd.?

32. How many are 44 and 19 thousandths, 8 and 71 hundred thousandths, 83 and 3327 ten thousandths, 60 and 301 ten thousandths?

33. From 14 and 15 tenths subtract 7 and 37 thousandths?

34. At $12 a ton, how much coal can be bought for $5.64?

35. At $12 a ton, what costs 3.047 tons of coal?

36. If 1.047 ton of coal cost $12.564, what is the price per ton?

37. At \$3.875 a 100, how many bricks can be bought for \$23.25?

38. At \$3.875 a 100, how much will 3,750 bricks cost?

39. If 4,575. bricks cost \$161.5625, what is the price per 100?

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40. What part of a pound (Troy) is $\frac{3}{4}$ of an oz.?

41. What part of an oz. is $\frac{1}{1800}$ of a cwt.?

42. What part of a pound (Troy) are 9 oz. 12 pwt.?

43. What part of a yard is .8 of a nail?

44. What part of a bushel are 3 pks. 2 qts. (decimal)?

45. What part of a peck is .175 of a bushel?

46. What decimal fraction is equal to $\frac{3}{8}$?

47. What common fraction is equal to .16?

48. What whole numbers are equal to $\frac{22}{64}$ of a mile?

49. What whole number is equal to .03515625 of a lb. Av.?

50. How much are $\frac{1}{12}$ hhd., $\frac{11}{12}$ gal., and $\frac{1}{2}$ qt.?

51. At \$4.80 a cord, how much wood can be bought for \$70.80?

52. At \$4.80 a cord, what cost 15 c. 96 ft. of wood?

53. If 13 cords 96 ft. of wood cost \$61.875, what is the price per cord?

EXERCISE III.

54. How many are $28\frac{3}{4}$, $\frac{3}{5}$ of $18\frac{3}{4}$, 32, $\frac{2}{3}$ of $18\frac{3}{4}$?

55. From 60 subtract $\frac{1}{3}$ of $100\frac{1}{6}$.

56. At \$$\frac{1}{2}$ a pound, how many pounds of feathers can be bought for \$$8\frac{1}{4}$?

57. At \$$\frac{1}{2}$ a lb., how much will $15\frac{1}{2}$ lbs. of feathers cost?

58. If $14\frac{1}{4}$ pounds of feathers cost \$$7\frac{1}{8}$, what is the price per pound?

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59. How many are 28 and 45-thousandths, 3 and 91-hundredths, 80 and 219 ten-thousandths, 17 and 7 tenths?

60. From 900 and 9-hundredths subtract 99 and 9 thousandths.

61. At \$10.375 a barrel, how many barrels of flour can be bought for \$$72\frac{5}{8}$?

62. At \$10.625 a bbl., how much will 10 barrels of flour cost?

63. If 9 bbls. of flour cost $96.75, what is the price per bbl.?

64. At $7.50 a 100, how many cabbages can be bought for $5.625 ?

65. At $7.50 a 100, how much will 44 cabbages cost?

66. If 56 cabbages cost $4.20, what is the price per 100 ?

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67. What part of a yard is $\frac{4}{5}$ of a nail ?

68. What part of an inch is $\frac{1}{75}$ of an E. English ?

69. What part of a pound Av. are 9 oz. $2\frac{2}{7}$ dr.?

70. What part of a yard is .5 of a quarter ?

71. What part of an acre is .875 of a square rod ?

72. What part of an acre are 1 rood, 14 rods (decimal)?

73. What whole numbers are equal to $\frac{5}{8}$ of a yard ?

74. What whole numbers are equal to .000175 of an acre ?

75. From $\frac{3}{5}$ of an ounce subtract $\frac{7}{8}$ of a pennyweight.

76. At $.625 a bushel, how much corn can be bought for $9.00 ?

77. At $.625 a bushel, what cost 15 bu. 3 pks. 4 qts. of corn ?

78. If 15 bu. 3 pks. 4 qts. of corn cost $9.525, what is the price per bushel ?

EXERCISE IV.

79. How many yards in six pieces of cloth measuring as follows : $18\frac{3}{4}$, $19\frac{5}{6}$, $21\frac{2}{3}$, $20\frac{5}{8}$, $22\frac{1}{2}$, $24\frac{1}{8}$?

80. From a piece of silk which contained $31\frac{7}{8}$ yards, $8\frac{5}{6}$ yds. have been cut off; how many remain ?

81. At $\$\frac{4}{5}$ a pound, how much tea can be bought for $2.00 ?

82. At $\$\frac{4}{5}$ a pound, how much will $3\frac{1}{4}$ lbs. of tea cost ?

83. If $2\frac{1}{2}$ lbs. of tea cost $2, what is the price per pound?

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84. Add 4 and 35 ten thousandths, 10 and 35 hundred thousandths, 6 and 35 millionths, 100 and 35 ten millionths.

85. From 113 and 5 tenths subtract 8 and 37 thousandths.

86. At $45.625 a barrel, how many barrels of molasses can be bought for $136.875 ?

87. At $.375 a gallon, how much will 28.625 gal. of vinegar cost ?

83. If 25.5 gals. of vinegar cost $8.925, what is the price per gallon ?

89. At $21 a 1000, how much will 150 shingles cost ?

90. If 200 shingles cost $3.50, what is the price per 1000 ?

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91. What part of an acre is $\frac{7}{8}$ of a square rod ?

92. What part of a minute is $\frac{1}{1620}$ of a day ?

93. What part of an A. are 2. R. 20 square rods ?

94. What part of a pint is .03125 of a gal. ?

95. What part of a cwt. are 1 qr. 8 lbs. 10 oz. (decimal)?

96. What part of a nail is .05 of a yard ?

97. What common fraction is equal to .05 ?

98. What decimal fraction is equal to $\frac{4}{25}$?

99. What whole numbers are equal to $\frac{5}{8}$ of a bushel ?

100. What whole numbers are equal to to .07 of a hhd. ?

101. From $\frac{2}{9}$ of a mile subtract $\frac{7}{11}$ of a fur.

102. At 27.25 an acre, how much land can be bought for $3500 ?

103. At $105. an acre, how much will 26 A. 2 R. 25 rods of land cost ?

104. If 112 A. 3 R. 20 rods of land cost $11287.50, what is the price per acre ?

EXERCISE V.

105. How many dollars are $5\frac{7}{9}$, $7\frac{1}{4}$, $4\frac{5}{8}$, $11\frac{1}{2}$, $12\frac{2}{3}$, $11\frac{5}{6}$ dollars ?

106. From $\frac{2}{7}$ of $16\frac{11}{12}$ subtract $4\frac{7}{16}$.

107. At $4\frac{4}{5}$ cts. a pound, how many pounds of lead can be bought for 1.30\frac{4}{5}$?

108. At $4\frac{4}{5}$ cts. a pound, how much will $28\frac{1}{4}$ lbs. of lead cost ?

109. If $23\frac{1}{3}$ lbs. of lead cost 1.45\frac{5}{6}$, what is the price per lb. ?

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110. How many are 10 and 19 thousandths ; 106 and 3 hundredths ; 17 and 16 millionths ; 9 and 9 tenths ; 71 and 63 ten thousandths ?

111. From 51.004 subtract 31 and 8 hundredths.

112. At \$.347 a pound, how much will 9 lbs. of tea cost ?

113. At \$.375 a pound, how many pounds of tea can be bought for \$5.25 ?

114. If 11.25 pounds of tea cost \$6.75, what is the price per pound ?

115. At \$5.625 a 100 feet, how many feet of boards can be bought for \$36.5625 ?

116. At \$5.625 a 100 feet, how much will 1000 feet of boards cost ?

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117. What part of a pound is $\frac{14}{15}$ of a pennyweight ?

118. What part of a grain is $\frac{1}{75}$ of a dram ?

119. What part of a bushel are 1 peck, 5 qts., 1 pint ?

120. What part of a pint is .025 of a gallon ?

121. What part of a mile are 110.4 rods ?

122. What part of a mile are 5 fur. 20 rods (decimal)?

123. What common fraction is equal to .625 ?

124. What decimal fraction is equal to $\frac{7}{8}$?

125. What whole numbers are equal to $\frac{4}{5}$ of a mile ?

126. From $\frac{2}{3}$ of a hhd. subtract $\frac{1}{2}$ of a barrel.

127. At \$3.50 per gal., how much will 37 gals., 2 qts., 1 pt. of wine cost ?

128. If 25 gals., 1 qt., 1 pt. of wine cost \$78.6625, what is the price per gallon ?

EXERCISE VI.

129. How many gals. are $8\frac{3}{4}$, $11\frac{2}{3}$, $9\frac{4}{5}$, $6\frac{7}{12}$, $10\frac{9}{20}$, $6\frac{5}{6}$ gals. ?

130. A man having $137\frac{3}{4}$ acres, sold $25\frac{4}{5}$, how many acres had he left ?

131. At \$$26\frac{1}{4}$ an acre, how many acres of land can be bought for \$1000 ?

132. At \$$26\frac{1}{4}$ an acre, what will $33\frac{1}{3}$ acres of land cost ?

133. If $33\frac{1}{3}$ acres of land cost \$$1666\frac{2}{3}$, what is the price per acre ?

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134. What is the sum of 9 and 9 tenths ; 10 and 12 thousandths ; 100 and 1 hundredths ; 1000 and 1 thousandths ; 10000 and 1 ten thousandths ?

135. From 300. subtract 3 hundredths.

136. At $1.75 a bushel, how many bushels of peaches can be bought for $25 ?

137. At $1.75 a bushel, how much will $20\frac{1}{2}$ bushels of peaches cost ?

138. If $20\frac{1}{2}$ bushels of peaches cost $36.90, what is the price per bushel ?

139. At $7.20 a 1000, what cost 19.625 bricks ?

140. At $7.20 a 1000, how many bricks can be bought for $148.50 ?

141. If 20000 bricks cost $140, what is the price per $1000 ?

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142. What part of a barrel is $\frac{63}{125}$ of a gill ?

143. What part of a nail is $\frac{1}{18}$ of a yard ?

144. What part of a cubic foot is .0005 of a cord ?

145. What part of an acre are 3 R. 14 sq. rds. 16 sq. yds. 4 sq. ft. and 72 sq. in. ?

146. What part of a cwt. are 3 lbs. 11 oz. 3.2 dr. (decimal)?

147. What common fraction is equal to .35 ?

148. What decimal fraction is equal to $\frac{5}{8}$?

149. What whole numbers are equal to .375 of an ounce ?

150. What whole number is equal to $\frac{3}{5}$ of a month ?

151. Add $\frac{1}{3}$ of a ton and $\frac{4}{5}$ of a cwt.

152. At $4 a yd., how much cost 4 yds. 3 qrs. 1 na. of cloth ?

153. At $5.25 a yard, how much cloth can be bought for $34.125 ?

EXERCISE VII.

154. How many miles are $7\frac{1}{2}$, $3\frac{1}{3}$, $6\frac{1}{4}$, $4\frac{1}{6}$, $8\frac{1}{12}$, $12\frac{1}{8}$ miles ?

155. Sold a carriage for $$175\frac{1}{2}$, and gained $$14\frac{3}{4}$; what did it cost ?

156. At $$4\frac{1}{2}$ a bushel, how much will $7\frac{5}{6}$ bushels of clover seed cost ?

157. At $$4\frac{1}{2}$ a bushel, how many bushels of clover seed can be bought for $$39\frac{3}{4}$?

158. If $6\frac{5}{6}$ bushels of clover seed cost $$30\frac{3}{4}$, what is the price per bushel ?

159. What is the difference between 3 hundredths and 3 thousandths?

160. At \$.1575 a pound, how much sugar can be bought for \$15.04125?

161. At \$.15 a pound, how much will .375 of a pound of sugar cost?

162. If 21.5 pounds of sugar cost \$3.225, what is the price per pound?

163. At \$5.50 a 100 lbs., how much flour can be bought for \$22?

164. At \$5.75 a 100 lbs., how much will 637.5 pounds of flour cost?

165. If 531.75 pounds of flour cost \$35.159, what is the price per 100 lbs.?

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166. What part of a rod is $\frac{14}{15}$ of an inch?

167. What part of a dram is $\frac{1}{280}$ of a pound?

168. What part of a barrel are 8 gals. 1 qt. $3\frac{3}{7}$ gills?

169. What part of a grain is .00075 of an oz., Aps.?

170. What part of a pound, Troy, is .432 of a grain?

171. What part of a degree are 41′ 51″ (decimal)?

172. What common fraction = .625?

173. What decimal fraction = $\frac{3}{11}$?

174. What whole numbers = .037625 of ton?

175. What whole numbers = $\frac{1}{4}$ of a tun?

176. From $\frac{7}{8}$ ton subtract $\frac{3}{5}$ cwt.

177. At \$.08 a pound, how much will 12 lbs. 5 oz. of soap cost?

178. At \$075 a pound, how much soap can be bought for \$3.00?

179. If 75 pounds of soap cost \$6., what is the price per lb.?

EXERCISE VIII.

180. How many acres are $38\frac{3}{4}$, $45\frac{1}{5}$, $50\frac{7}{8}$, $35\frac{3}{10}$ acres?

181. Bought a piece of cloth containing $27\frac{1}{2}$ yards. I have sold $\frac{1}{4}$ of it, how much is left?

182. At $2\frac{2}{7}$ cents a foot how many feet of lumber can be bought for \$1?

183. At $2\frac{2}{7}$ cents a foot, how much will $19\frac{1}{2}$ feet of lumber cost ?

184. If $15\frac{1}{2}$ feet of lumber cost $35\frac{3}{7}$ cents, what is the price per foot ?

185. What is the difference between 7 thousandths and 7 millionths ?

186. At $19.875 a barrel, how many barrels of pork can be bought for $139.125 ?

187. At $19.875 a barrel, how much will 10 barrels of pork cost ?

188. If 8 barrels of pork cost $159, what is the price per barrel ?

189. At $11.50 a 100 pounds, how much will 210 lbs. of sugar cost ?

190. If 256 pounds of sugar cost $29.44, what is the price per 100 lbs.

191. What part of a yard is $\frac{3}{13}$ of a nail ?

192. What part of a grain is $\frac{1}{5184}$ of a lb. Troy ?

193. What part of a cord are 26 cubit feet 1152 inches ?

194. What part of a quart is .0005 hhd. ?

195. What part of a mile are 4 furlongs 30 rods 2 yards 2 feet, 3 inch, decimal ?

196. What common fraction=.075 ?

197. What decimal fraction=$\frac{21}{24}$?

198. What whole numbers=.15 of a bushel ?

199. What whole numbers=.$\frac{2}{7}$ of an hour ?

200. From $\frac{11}{12}$ of £1, subtract 7s. 8d.

201. At $13 a ton, how much hay can be bought $75 ?

202. At $13 a ton, how much will 18 cwt. 1 qr. 21 lbs. of hay cost ?

203. If 18 cwt. 1 qr. 21 lbs. cost $11.50, what is the price per ton ?

EXERCISE IX.

204. How many yards in three pieces of calico, measuring $24\frac{1}{4}$, $21\frac{1}{3}$, $20\frac{7}{11}$ yds. ?

205. At $\$\frac{3}{7}$ a bushel, how many bushels of potatoes can be bought for $\$5\frac{4}{7}$?

206. At $\$1\frac{1}{4}$ each, how much will 9 sheep cost ?

207. A man having $9\frac{1}{2}$ cords of wood, has sold $3\frac{7}{9}$ cords ; how much has he left ?

208. At $\$1\frac{2}{5}$ a day, how much will a man earn in $26\frac{1}{2}$ days ?

209. From 1 hundred and 1, subtract 1 hundred and 1 hundredths ?

210. At \$.1875 a pound, what will 10.4 lbs. of beef cost ?

211. At \$2.75 a hundred feet, how many feet of timber can be bought for \$736.945 ?

212. At \$2.25 a hundred, what will 175.28 feet of timber cost ?

213. At \$2 a bushel, how many peaches can be bought for $37\frac{1}{2}$ cents ?

214. At \$2 a bushel, what will 3 pecks, 5 quarts, of peaches cost ?

215. At \$5 a square rod, what will a village lot cost, containing $\frac{1}{3}$ acre ?

216. If a man can earn \$1 in $\frac{1}{9}$ of a week, how much does he earn in a day ?

217. If $\frac{3}{8}$ of an ounce of nutmegs cost 10 cents, what is the price per pound ?

218. Three village lots contain respectively $1\frac{1}{3}$ acres, $3\frac{1}{4}$ roods, $30\frac{1}{2}$ rods ; how much do they all contain ?

219. At \$8.95 a cwt., how much sugar can be bought for \$43.855 ?

220. At \$8.75 a cwt., how much will 2 cwt. 1 quarter 15 lbs. of sugar cost ?

221. If 3 cwt. 2 quarters 10 lbs. of sugar cost \$27, what is the price per cwt ?

EXERCISE X.

222. At $\frac{7}{8}$ of a cent each, how many apples can be bought for 56 cents ?

223. If 65 apples cost $56\frac{7}{8}$ cents, how much does 1 apple cost ?

224. At $\frac{7}{8}$ cents each, how much cost 60 apples ?

225. How many yards are left in a piece of cloth which con-

tained 37 yards, after cutting off at different times $5\frac{1}{4}$, $3\frac{7}{8}$, $8\frac{2}{3}$, $7\frac{1}{6}$ yards?

226. At \$$6\frac{3}{4}$ a ton, what cost $8\frac{4}{9}$ tons of hay?

227. If $9\frac{4}{9}$ tons of hay cost \$$63\frac{3}{4}$, what is the price per ton?

228. At \$$6\frac{3}{4}$ a ton, how many tons of hay can be bought for \$$62\frac{3}{4}$?

229 From one thousand and seven hundredths, subtract five hundred and five hundredths.

230. What cost 7.625 cords of wood at \$6.50 a cord?

231. At \$6.50 a cord, how many cords of wood can be bought for \$61.75?

232. If 7.625 cords of wood cost \$61, what is the price per cord?

233. At \$9.50 a ton what will 12 cwt. 40 lbs. of coal cost?

234. At \$12 a barrel, how much oil can be bought for \$9.75?

235. If $3\frac{1}{2}$ pints of oil cost 50 cts. what is the price per gal.?

236. If .5 hhd. of molasses cost \$15.75 what is the price of a quart?

EXERCISE XI.

237. A lady bought $\frac{1}{2}$ of a piece of muslin containing $30\frac{1}{4}$ yds.; $\frac{1}{3}$ of another containing $33\frac{1}{3}$ yds.; $\frac{1}{4}$ of another containing $44\frac{1}{2}$ yds.; and a whole piece containing $31\frac{5}{8}$ yds.; how many yards did she buy?

238. At $5\frac{3}{4}$ cts. a pound, how much will a calf weighing $137\frac{1}{2}$ lbs. cost?

239. At $7\frac{3}{4}$ cts. a pound how much veal can be bought for $87\frac{1}{2}$ cts.?

240. If a calf weighing $118\frac{3}{4}$ lbs. cost \$$12.41\frac{5}{8}$ what is the price per pound?

241. Bought a horse for \$112.5625 and sold him for \$125, how much was gained?

242. At \$2.6875 a yard, how much cassimere can be bought for \$43.671875?

243. At \$2.6875 a yard, what cost 16.25 yds. of cassimere?

244. If 16.25 yds. of cassimere cost \$40, what is the price per yard?

245. At \$4.75 a 100, how many shingles can be bought for \$36.29?

246. At \$25 a 1000 what cost 9875 shingles?

247. If 1200 shingles cost \$24.75 what cost 100?

248. At \$.125 a yd., what cost 11 yd. 2 qr. 3 na. of muslin?

249. If 10 yd. 1 qr. of muslin cost \$1.64, what is the price per yard?

250. At \$.125 a yard, how many yards of muslin can be bought for \$5.25?

EXERCISE XII.

251. How much carpeting ⅔ yd. wide will cover a room 22½ ft. long, 18 ft. wide?

252. How many pieces of paper 1¼ ft. wide and 8½ yds. long must be bought to cover the sides of a room 18¾ ft. long, 16½ ft. wide, and 11¼ ft. high; there being one door 6¼ ft. by 2¾, and two windows 5½ by 2¼ ft.?

253. How much will it cost to pave a street ½ mile long and 2¼ rods wide, at \$12 a square rod?

254. If a piece of land containing 5.5 acres be divided into building lots 4 rods long and 2.2 rods wide, what would they all be worth at \$200 each?

255. How many shingles will cover the roof of a house 32 ft. long the rafters on each side being 16¾ ft. long, allowing one shingle for every 24½ sq. in.?

256. How much will it cost to dig a ditch around a garden 6.5 rods square, the ditch to be 3.25 feet deep and 2.5 feet wide, at 1 ct. a cubic foot?

DUODECIMALS.

Art. 127.—Duodecimals are a species of fractional compound numbers sometimes used in measuring lumber, &c. They arise from successive divisions of 1 foot by 12. (Latin, *duodecim*).

TABLE.

12'''' (fourths) make	........1''' (third)		$=\frac{1}{1728}$ ft.
12''' (thirds)	........1'' (second)		$=\frac{1}{144}$ ft.
12'' (seconds)	........1' (prime, or inch)		$=\frac{1}{12}$ ft.
12' (inches)	........1 (foot)		

The marks used to distinguish the different denominations are called **Indices.**

Duodecimals may be added, subtracted, multiplied and divided, like other compound numbers.

In multiplication of duodecimals by duodecimals—

Feet multiplied by feet give square feet.

Square feet multipled by feet give cubic feet.

Feet multiplied by inches (as $1\times\frac{1}{12}$) give sq. inches, $\frac{1}{12}$ sq. ft.

Square feet multiplied by inches (as $1\times\frac{1}{12}$) give inches, $\frac{1}{12}$ cubic foot.

Inches multiplied by inches (as $\frac{1}{12}\times\frac{1}{12}$) give square inches, or seconds, $\frac{1}{144}$ square foot.

Square inches multiplied by inches (as $\frac{1}{12}\times\frac{1}{12}$) give cubic sq. in. or seconds, $\frac{1}{144}$ cu. ft.

Inches multiplied by seconds (as $\frac{1}{12}\times\frac{1}{144}$) give thirds ($\frac{1}{1728}$) sq. foot.

Square inches multipled by seconds give thirds ($\frac{1}{1728}$) cu. ft.

Seconds multiplied by seconds give fourths, &c.

EXAMPLE 1.—What are the contents of a board 10 feet 6 inches long, and 2 feet 3 inches wide?

ft.	in.	
10	6	
5	3	
2	7	6
52	6	
55 ft.	1'	6''

Process.—$6'\times3'=\frac{6}{12}\times\frac{3}{12}=\frac{18}{144}=18''=1'\ 6''$. Write 6'' and carry 1'. Next 10 feet $\times\ 3'=10\times\frac{3}{12}=\frac{30}{12}$ and 1' makes $\frac{31}{12}=2\frac{7}{12}=2$ feet 7 inches. Then $6'\times5$ ft., &c.

RULE.—*Multiply each term in the multiplicand by each term in the multiplier, giving each product an index equal to the indices of both its factors; then after reducing and carrying, as in compound numbers, add the like terms of the products.*

EXAMPLES.

(2.) Multiply 8 feet four inches by 3 feet 9 inches.

(3.) 9 ft. 6 in. by 2 ft. 8 in.	(7.) 13 ft. 7 in. by 6 ft. 5 in.
(4.) 12 ft. 10 in. by 4 ft. 3 in.	(8.) 14 ft. 8 in. by 7 ft. 1 in.
(5.) 10 ft. 8 in. by 5 ft. 2 in.	(9.) 11 ft. 9 in. by 9 ft. 3 in.
(6.) 15 ft. 5 in. by 3 ft. 4 in.	(10.) 16 ft. 4 in. by 3 ft. 3 in.

11. How many square feet are there in a board 14 feet 9 inches long, and 2 feet wide?

12. How many square feet in a board 16 feet 8 inches long, and 1 foot 10 inches wide?

13. How many square feet in a door 6 ft. 6 inches long, and 3 feet 4 inches wide?

14. How many square feet in a floor 18 feet 10 inches long and 15 feet wide?

15. How many square feet in a piece of molding 20 feet long and 3 inches wide?

16. How many square feet in 20 boards, each 12 feet long and 9 inches wide?

17. How many cubic feet in a stick of timber 10 feet long, 1 foot 3 inches wide, and 4 inches thick?

18. How many cubic feet in a load of wood 8 feet long, 4 feet 6 inches high, and 3 feet 10 inches wide?

19. How many cubic feet in a block of marble 6 feet 8 inches long, 2 feet 6 inches wide, and 2 feet thick?

20. How many cubic feet in a wall 21 feet 6 inches long, 6 feet 3 inches high, and 2 feet thick?

21. What will it cost to plaster a room 24 feet 6 inches long, 15 feet 5 inches wide, and 8 feet 4 inches high, at 36 cents a square yard?

22. How many bricks 8 inches long, 4 inches wide, and 2 inches thick, will it take to build a wall 72 feet long, 4 feet 6 inches high, and 1 foot thick, supposing the bricks not to be separated by mortar?

ANALYSIS.

Art. 128.—Analysis in Arithmetic is a method of solving questions without formal rules. Rules are derived from analysis. The process consists generally in reasoning from a given number to 1 of the same kind, and from 1 to the required number.

EXAMPLE 1.—If 7 pounds of sugar cost $1.12, how much will 42 pounds cost?

Process.—If 7 lbs. of sugar cost $1.12, 1 lb. will cost $\frac{1}{7}$ of $1.12, or 16 cts., and 42 lbs. will cost 42 times 16 cts., or $6.72, *Ans.*

Or since 42 lbs. is 6 times 7 lbs., 42 lbs. will cost 6 times $1.12 (the price of 7 lbs.), and $\$1.12\times 6=\$6.72$, *Ans.*

Ex. 2. If $\frac{3}{4}$ of a yard of cloth cost $\$4\frac{1}{2}$, how much will $\frac{7}{9}$ of a yard cost?

Process.—If $\frac{3}{4}$ yard cost ($\$4\frac{1}{2}$) $\$\frac{9}{2}$, $\frac{1}{4}$ yard will cost $\$\frac{3}{2}$, and $\frac{4}{4}$, or 1 yard, will cost $6. Then $\frac{1}{9}$ yard will cost $\$\frac{2}{3}$, and $\frac{7}{9}$ will cost $\frac{14}{3}$, or $\$4\frac{2}{3}$, *Ans.*

In examples like the last it is better to express each multiplication by writing the multiplier as a factor in the numerator of a fraction, and each division by writing the divisor as a factor in the denominator, then cancel, &c., thus:

$$\$\frac{\cancel{9}\times\overset{2}{\cancel{4}}\times 7}{\cancel{2}\times 3\times\cancel{9}}=\frac{14}{3}=\$4\tfrac{2}{3},\ Ans.$$

Ex. 3. *Barter.*—How many loads of wood at $\$4\frac{1}{2}$ will pay for 3 barrels of flour at $\$8\frac{3}{4}$?

Process.—If 1 bbl. of flour costs $\$8\frac{3}{4}$, 3 bbls. will cost $\$26\frac{1}{4}$, and if $\$4\frac{1}{2}$ will pay for 1 load of wood, $\$26\frac{1}{4}$ will pay for as many loads as $\$4\frac{1}{2}$ is contained times in $\$26\frac{1}{4}$, which is 6. *Ans.*, 6 loads.

Ex. 4. *Aliquot parts or Practice.*—What cost 5 cwt. 65 lbs., at £2 5s. 6d. per cwt.?

	£	s.	d.		
Process.—50 lbs. $=\frac{1}{2}$ cwt.	£2	5s.	6d.		
			5		
	11	7	6	price of 5 cwt.	
10 lbs. $=\frac{1}{5}$ of 50 lbs.	1	2	9	"	50 pounds.
5 lbs. $=\frac{1}{2}$ of 10 lbs.		4	$6\frac{3}{5}$	"	10 "
		2	$3\frac{3}{10}$	"	5 "
Ans.,	12	17	$0\frac{9}{10}$		

Ex. 5. A general lost $\frac{1}{4}$ of his army in battle, $\frac{1}{3}$ were taken prisoners, $\frac{1}{5}$ deserted, and he had 2600 men left; how many had he at first?

Process.—$\frac{1}{4}\times\frac{1}{3}\times\frac{1}{5}=\frac{47}{60}$, and the remainder $\frac{13}{60}=2600$. $\frac{1}{60}=200$, $\frac{60}{60}=12000$, the army at first.

Analysis may also be applied to questions under rules to be hereafter given, such as Proportion, Partnership, Reduction of Currencies, Alligation, &c.

EXAMPLES.

6. If 20 barrels of apples cost \$50, what will 35 bbls. cost?

7. If 33 tons of coal cost \$198, how much will 11 tons cost?

8. If 15 pounds of butter cost \$4.50, how much will 70 lbs. cost?

9. If 12 pairs of shoes cost \$30, how much will 48 prs. cost?

10. How much will 65 sheep cost if 5 sheep cost \$17.50?

11. How much are 50 cows worth, if 10 cows are worth \$450?

12. If $\frac{2}{3}$ of an acre of land cost \$$66\frac{2}{3}$, how much will $6\frac{3}{4}$ acres cost?

13. How much will $\frac{3}{7}$ of a ton of hay cost, if $\frac{6}{7}$ of a ton costs \$12?

14. How much will $\frac{8}{11}$ of a cord of wood cost, if $\frac{3}{11}$ costs \$$1.12\frac{1}{2}$?

15. If $\frac{2}{5}$ of a pound of tea costs \$$\frac{3}{16}$, how much will $\frac{8}{9}$ of a pound cost?

16. If $\frac{4}{5}$ of a yard of cloth costs \$4.80, how much will $\frac{7}{8}$ of a yard cost?

17. If $\frac{2}{3}$ of a cord of wood cost \$1.10, how much will $\frac{6}{7}$ of a cord cost?

18. How much will $\frac{9}{10}$ of a ton of plaster cost if $\frac{2}{5}$ of a ton cost \$3?

19. How much will $\frac{11}{12}$ of an acre of land be worth if $\frac{3}{10}$ of an acre is worth \$30?

20. How much will $\frac{14}{15}$ of a chain 30 feet long cost if $\frac{2}{3}$ of a like chain 36 feet long is worth \$24?

21. How many eggs at 20 cents a dozen must be given for 9 pounds of butter at 30 cents a pound ?

22. How many pounds of lard, at 15 cents a pound, will pay for 14 pounds of sugar, at $12\frac{1}{2}$ cents a pound ?

23. How many bushels of oats, at $37\frac{1}{2}$ cents a bushel, will pay for 5 yards of cloth, at $4.50 ?

24. How many yards of calico, at $18\frac{3}{4}$ cents a yard, can be bought for 10 pounds of butter, at $31\frac{1}{4}$ cents ?

25. At $87.50 an acre, what cost 8 acres 110 rods ?

26. At $6.75 a ton, what cost 7 tons 12 cwt. 60 lbs. of hay ?

27. At $\$5.37\frac{1}{2}$ a yard, what cost 3 yards 3 quarters 3 nails of cloth ?

28. At £2 11s. 6d. a bushel, what cost 5 bushels, 1 peck, 4 quarts of timothy seed ?

29. At £10 12s. 6d. an acre, what cost 9 acres 60 rods ?

30. At £2 8s. $6\frac{1}{4}$d. a cwt., what cost 7 cwt. 30 pounds of flour ?

31. A regiment of soldiers was diminished $\frac{1}{3}$ by sickness, $\frac{1}{4}$ captured by the enemy ; $\frac{1}{6}$ killed and missing, and then consisted of 250 men ; how many did it number at first ?

32. A young man spent $\frac{1}{2}$ of his property in 3 years, $\frac{1}{3}$ of it the next 2 years, and then he had $600 left ; how much had he at first ?

33. Paid $2,100 for $\frac{3}{20}$ of a vessel, what was the whole vessel worth ?

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34. If 18 lbs. of cheese cost $2.70, what will a cheese weighing 72 lbs. cost ?

35. If $\frac{2}{5}$ of hogshead of sugar cost $34, what will $\frac{5}{7}$ of a hogshead cost ?

36. If $\frac{5}{6}$ of a gallon of alcohol cost $\$3\frac{2}{3}$, what will $\frac{3}{11}$ of a gallon cost ?

37. How many bushels of corn at 90 cents a bushel, will pay for 63 lbs. of beef at 14 cents a pound ?

38. At $5.38 a cwt., how much will 60 lbs. of flour cost ?

39. If a barrel of ale cost £7 14s. 4d., what will 24 gallons 1 quart cost ?

40. There is a town in which $\frac{1}{2}$ the men are farmers, $\frac{1}{4}$ mechanics, $\frac{1}{6}$ laborers, and the rest 26 without employment; how many men are there in the town?

---

41. How many pounds of pork at $12\frac{1}{2}$ cents a pound, wlll pay for 7 days labor, at $\$1.12\frac{1}{2}$ a day?

42. If 30 pounds of coffce cost $7.50, what will 9 lbs cost.

43. If $\frac{2}{3}$ of a cord of wood is worth $\$3\frac{1}{6}$, how much is $\frac{11}{12}$ of a cord worth?

44. How much will $\frac{7}{8}$ of a pound of soap cost, if $\frac{3}{4}$ of a pound cost 12 cents?

45. At $94 an acre, what will 50 square rods cost?

46. How much will 29 gals. of vinegar cost at $\$10\frac{1}{2}$ a barrel?

47. A man left his elder son $\frac{1}{2}$ of his property, the younger $\frac{1}{3}$, and the elder son had $1000 more than the younger; what was the whole property?

---

48. If $\frac{1}{2}$ of a ship is worth $56,000, what is $\frac{7}{16}$ of it worth?

49. What is $\frac{4}{5}$ of a bushel of clover seed worth, if $\frac{2}{7}$ of a bushel is worth $\$2\frac{3}{7}$?

50. If a man earn $42 in 12 days, how much will he earn in 50 days?

51. How many days' labor at $\$.87\frac{1}{2}$ a day will pay for 7 bushels of buckwheat at 75 cents a bushel?

52. At $\$21.31\frac{1}{4}$ a cwt., how much will 86 lbs. of honey cost?

53. If a year's labor is worth £100, how much will it be for 7 months and 20 days?

54. A benevolent lady gave $\frac{1}{4}$ of her income to the Bible Society, and $\frac{3}{5}$ of it to the poor, reserving only $600 for herself, what was her income?

55. If $\frac{3}{5}$ of a ton of hay cost $\frac{2}{3}$ of $\$9\frac{3}{4}$, how much will $\frac{7}{10}$ of a ton cost?

56. How many barrels of apples at $\$2\frac{1}{4}$ a barrel, will pay for 4 bbls. of flour at $\$9\frac{3}{8}$?

57. What will $\frac{7}{9}$ of a yard of velvet cost, if $\frac{2}{7}$ of a yard cost $1.08?

58. What will 15 yards of calico cost, if 9 yds. cost $1.62 ?

59. At $4.50 a yard, what will 3 yds. 1 qr. 3 na. cost ?

60. A lady gave $\frac{1}{2}$ of her property to her son, $\frac{1}{3}$ to her daughter, and the rest, amounting to $1,200, to benevolent objects, what was the amount of her property ?

PERCENTAGE.

Art. 129.—Percentage is calculating numbers by hundredths, or parts of a hundred.

Per cent. (derived from the Latin words *per centum*, meaning by the hundred) is used in expressing hundredths, or parts of a hundred ; thus, 5 per cent. is 5 hundredths, or five for every hundred (dollars, pounds, &c.); 6 per cent. is 6 hundredths.

The sign % is often used for per cent.

Art. 130.—In **Percentage** three things are chiefly considered.

The **Principal,** the number on which percentage is calculated.

The **Rate** per cent., the number of hundredths.

The **Percentage,** the number which the principal produces at a given rate.

Any two of these being known, the other may be found.

The term *Principal* thus used includes, but is not limited to money at interest.

The rate per cent. is expressed by a fraction, usually a decimal fraction, thus:

1 per cent. is written.................. .01 $= \frac{1}{100}$
5 per cent. is written.................. .05 $= \frac{5}{100}$ or $\frac{1}{20}$
10 per cent. is written.................. .10 $= \frac{1}{10}$
25 per cent. is written.................. .25 $= \frac{1}{4}$

100 per cent. is written..................1.00 = the whole.
$\frac{1}{2}$ per cent. is written.................. .005 or .00$\frac{1}{2}$.
$\frac{1}{4}$ per cent. is written.................. .0025 or .00$\frac{1}{4}$.

Some rates per cent. cannot be exactly expressed by decimals; as $\frac{1}{3}$ per cent. must be written .00$\frac{1}{3}$; 33$\frac{1}{3}$ per cent. .33$\frac{1}{3}$.

Write the following rates per cent.

3 per cent.; 6, 4, 12, 7, 8, 15, 9, 20, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{3}$, 2$\frac{1}{4}$, 3$\frac{1}{2}$, 18$\frac{3}{4}$, 3$\frac{3}{5}$, $\frac{3}{8}$, 37$\frac{1}{2}$, $\frac{1}{6}$, 33$\frac{1}{3}$, $\frac{2}{3}$, 16$\frac{2}{3}$, 6$\frac{1}{4}$, 12$\frac{1}{2}$, 10$\frac{2}{5}$, 75, 110, 125, 137$\frac{1}{2}$.

Case I.

Art. 131.—*To find the percentage.*

MENTAL EXERCISES.

How much is 3 per cent. of \$5 ?

Process.—Since 3 % is .03, 3 % of \$5. is .03 times \$5, which is .15 cents. Therefore 3 % of \$5. is 15 cents.

How much is 4 % of \$1 ? \$10 ? \$12 ? \$8 ? 50 cents ?

How much is 5 % of 4 pounds ? 20 lbs.? 50 lbs.? 100 lbs.?

If a miller take 4 % toll for grinding wheat, how much will it be for grinding 100 bushels ? 200 ? 50 ? 25 ?

EXAMPLES FOR THE SLATE.

Example 1. What is 6$\frac{1}{4}$ % of 568 pounds ?

Process.—6$\frac{1}{4}$ % = .0625, therefore 6$\frac{1}{4}$ % of 568 lbs. = 568 × .0625 = 35.5000 = 35$\frac{1}{2}$ lbs.

```
      568
    .0625
    -----
     2840
    1136
   3408
   ------
Ans. 35.5000 lbs.
```

Or $6\frac{1}{4}\% = \frac{6\frac{1}{4}}{100} = \frac{1}{16}$ and $\frac{1}{16}$ of $568 = 35\frac{1}{2}$.

Rule.—*Multiply the principal by the rate per cent.*
Percentage = Principal × Rate.

2. What is 3 % of \$8750 ? 6 % ? 7 % ?
3. What is 5 % of 364 gallons ? 7 % ? 12 % ?
4. What is 6 % of 576 pounds ? 8 % ? 16 % ?
5. What is 7 % of 368 bushels ? 9 % ? 4$\frac{1}{2}$ % ?
6. What is 8$\frac{1}{3}$ % of 261 gallons ? $\frac{3}{5}$ % ? 16$\frac{2}{3}$ % ?

7. A farmer has 320 bushels of wheat, 25 % more of oats, and $12\frac{1}{2}$ % less of corn; how many bushels of oats has he? Of corn?

8. A regiment consisted of 840 soldiers, of whom $16\frac{2}{3}$ were killed and missing; how many were left?

9. A merchant having \$6400 capital, gained $18\frac{3}{4}$ %; how much did he gain?

10. Another merchant, having \$7200 capital, lost $12\frac{1}{2}$ % of it; how much did he lose?

11. A young man, having \$1240, spent $6\frac{1}{4}$ % of it for clothes and board, 10 % of it for a horse, and $12\frac{1}{2}$ % of it in traveling; how much had he left?

12. A grocer bought 450 pounds of coffee, and found that 10 % of it was damaged; how much of it was good?

13. A flock of 175 sheep increased 20 %, how large was it afterwards?

Case II.

Art. 132.—To find what per cent. one number is of another.

Example 14.—What per cent. of \$10 is \$2.50?

Process.—Since \$2.50 is a certain per cent. of \$10, the same per cent. of \$1. is $\frac{1}{10}$ of \$2.50, which is \$.25=25 %. Therefore \$2.50 is 25 % of \$10.

$$10)\underline{2.50}$$
Ans., .25 = 25 %.

Rule.—*Divide the number which is the percentage by the other number.*

Rate = Percentage ÷ Principal.

15. What % of \$75. is \$5? \$10? \$20? \$25?

16. What % of \$87.50 is \$5.25? \$7.87½?

17. What % of \$60. is \$6? \$9? \$12?

18. What % of \$56. is \$7? \$14? \$21?

19. What % of \$150 is \$9? \$15? \$50?

20. What % of \$200 is \$8? \$16? \$24? \$50?

21. What % of \$1000 is \$50? \$60? \$75?

22. A farmer raised 500 bushels of wheat and kept 50 bushels for family use; what per cent. of it did he keep?

23. A regiment consisting of 900 soldiers lost 75 of them in a battle; what per cent. was it?

24. A young man having $350, has spent $50 of it; what per cent. of it has he spent?

25. A grocer bought 560 gallons of molasses, and found that 56 gallons had leaked out; what was the percentage?

26. A miller took 15 bushels of corn for grinding 300 bush.; what per cent. was it?

Case III.

Art. 133.—To find the principal when a certain per cent. of it is known.

Example 27.—A man gave $50 to benevolent objects, which was 10 % of his income; what was his income?

Process.—Since $50 is 10 % of his income, his income was as many dollars as .10 is contained times in 50, which is 500. Therefore his income was $500.

Or, since $50 is $\frac{1}{10}$ of his income, the whole of it is $50×10=$500.

$$\begin{array}{r} \$\ \text{cts.} \\ .10\overline{)50.00} \\ \hline \textit{Ans.}\ \$500. \end{array}$$

Rule.—*Divide the given percentage by the rate per cent.*

Principal = percentage ÷ rate.

28. $5 is 6 % of how many dollars?

29. 12 pound is 10 % of how many pounds?

30. 15 gallons is 25 % of how many gallons?

31. 37½ bushels is 18¾ % of how many bushels?

32. $75 is 5 % of how many dollars?

33. A man pays an income tax of 4 % amounting to $84; what is his income?

34. A debtor, whose property is worth $4,500 is able to pay only 75 % of what he owes; how much does he owe?

35. A man wishes to leave his daughter an income of $1,000 a year; what sum must he invest for her at 7 %.

36. A miller has taken 1 barrel of flour for toll at 2¼ %; how many barrels of flour has he ground?

37. A regiment of soldiers lost 60 men in a battle which was 10 % of their whole number; how many belonged to the regiment before the battle?

CASE IV.

Art. 134.—To find the principal, when being increased or diminished a certain per cent., the sum or remainder is known.

EXAMPLE 38.—A man, whose property has increased 50 % is now worth $15,000; what was he worth before?

Process.—Since his property has increased 50 % it was formerly as many dollars as it is now times $1.50, *i. e.* $10,000.

$1.50)15000.00
Ans. $10000.

Or it was formerly $\frac{100}{150}$ or $\frac{2}{3}$ of $15,000=$10,000.

Ex. 39.—A man who has lost 50 % of his property, is now worth $10,000; what was he worth before?

Process.—Since his property is 50 % less than formerly, it was then as many dollars as it is now times $.50 *i. e.* $20,000.

.50)10000.00
Ans. $20000.

Or it was formerly $\frac{100}{50}$ or 2 times $10,000=$20,000.

RULE.—*Divide the given number by* 1, *with the rate per cent. added or subtracted accordingly as the percentage has been added to or subtracted from the required number.*

Principal=principal $\overset{+}{\underset{-}{\text{or}}}$ *percentage* ÷ *by* 1 $\overset{+}{\underset{-}{\text{or}}}$ *rate.*

40. A farmer has raised 1,200 bushels of potatoes this year, which is 25 % more than he raised last year; how many did he raise last year?

41. A drover has purchased 800 sheep which is 20 % less than he expected to purchase; how many did he expect to purchase?

42. My tailor told me it would require 6 yards of cloth to make me a suit of clothes. Supposing it would shrink as much as 5 % of its length, how many yards should I have purchased?

43. A young man after spending 18¾ % of his pocket money had $81.25 left; how much did he have at first?

44. A lady wishing to purchase a cloak found the price ($60) to be 20 % more than she expected to pay; how much did she expect to pay?

45. A lady expecting to pay $48 for a silk dress found that to be 20 % more than the price; what was the price?

Art. 135.—Promiscuous Examples in Percentage.

46. A farmer has sold 118 tons of hay this year which is 12½ % more than he sold last year, and 33⅓ % less than he expects to sell next year; how much did he sell last year and how much does he expect to sell next year?

47. A farmer having 150 tons of hay expects to sell 90 tons; what per cent. is that of the whole?

48. A farmer has sold 75 tons of hay which is 60 % of all he had; how much did he have?

49. A liquor dealer bought a hogshead of rum and mixed 33⅓ % of water with it; how much did it make?

50. At another time he mixed 75 gals. of water with 225 gals. of brandy; what per cent. of the mixture was water?

51. At another time he filled up a cask containing ale with 20¼ gals. of water which was 37½ % of what the cask would contain; how much did the cask hold?

52. At another time he mixed some wine with 15 % of water and then had 57½ gallons; how much wine was there?

53. In a certain town the population is 2750, and 4 % are colored; how many colored persons live in the town.

54. In another town there are 2040 whites and 360 blacks; what per cent of the whole population are black?

55. In another town 8⅓ % of the population are colored, of whom there are 136; what is the whole population?

56. In another town the population has increased in 10 years 127½ %, and is now 2275; what was it ten years ago?

57. The expenses of a family are 18¾ % greater this year than the last, when they amounted to $963.75; how much are they this year?

58. A debtor whose property is worth $6,750, is able to pay only 67½ % of what he owes; how much can he pay?

59. A man worth $10,000 has invested $2,520 in government bonds; what per cent of his property is thus invested?

60. A gentleman traveling, having spent 75 % of his money found that he had $75 left; how much had he at first?

Applications of Percentage.

Art. 136.—Percentage is applicable to **Commission and Brokerage, Stocks and Gold at a Premium, Insurance, Profit and Loss, Interest, Discount, Taxes, Duties, Partnership, Bankruptcy, Exchange, &c.**

Some of these are so much like percentage that they scarcely need to be separately treated except in a few particulars.

Art. 137.—Commission is the percentage paid to a commission merchant or agent doing business for another. It is calculated the same as percentage.

A *consignment* consists of goods sent to a person to sell on commission. The *gross proceeds* are the whole amount of the sales. The *net proceeds* are what is left after deducting the expenses.

Art. 138.—An **Account of Sales** is a written statement of goods sold on commission, with the prices, gross and net proceeds, &c.; as

SALES OF PRODUCE CONSIGNED BY THOS. FAY & CO., DETROIT.

1867.	Sold to	Produce.	Price.	$ cts.
Aug. 1	Rogers & Son	Flour, 20 bbls..	$11.00	220.00
" 8	C. Jones & Co......	Wheat, 300 bu...	2.50	750.00
" 25	T. Agnew & Co....	Corn. 500 bu...	90	450.00
				$1420.00

Charges.

Freight on 20 bbls...........@ 50 cts.	$10.00	
" 800 bu.............@ 10 cts.	80.00	
Cartage and Storage..................	12.50	
Commission on $1240 @ 2¼ %........	31.95	124.45
Net proceeds....................		$1295.55

SMITH, MYGATT & CO.

New York, *Aug.* 31, 1867.

Art. 139.—Brokerage is the percentage paid to brokers. It is sometimes called *discount*.

A *Broker* is one who exchanges or loans money, buys and sells stocks, also goods not in his own possession.

Art. 140.—Stocks are money or property invested in Banks, incorporated or chartered Companies, Bonds, &c. They are divided into shares, usually of $100 each, for which Certificates or Scrip is issued, liable to be bought or sold.

When stocks sell for what they originally cost, they are *at par;* when they sell for more, they are *above par*, at a *premium* or *advance;* and *below par*, or at a *discount*, when they sell for less.

The premium or discount is a certain percentage on the par value, to be added or subtracted from it, in finding the *market* value.

The market value of any number of shares is found by multiplying it by the market value of a single share.

Stocks are quoted: at par, 100; at a premium of 1%, 101; 2½%, 102½; 18¾, 118¾; at a discount of 5%, 95; 12½%, 87½; 25%, 75, &c.

Stockholders are the owners of stock.

A *Dividend* is what is paid to stockholders as their part of the profit or gain.

Bonds are securities for money loaned, bearing interest, issued by Corporations or Governments.

The United States have issued the following bonds:

U. S. 5's, paying 5% interest in gold, and payable in 1871 and 1874.

U. S. 6's, paying 6% interest in gold, and payable in 1867, 1868, and 1881.

U. S. 5-20's, paying 6% interest in gold, and payable in 5 to 20 years.

U. S. 10-40's, paying 5% interest in gold, and payable in 10-40 years.

U. S. 7-30's, paying $7\frac{3}{10}$, or 7.30% interest, in currency, and payable in three years from their date.

Art. 141.—Gold, at a premium, is bought and sold the same as stocks.

Art. 142.—Insurance is security against loss.

Fire Insurance is security against loss by fire; *Marine Insurance*, against loss on the ocean, &c. Insurance, also, secures a certain allowance in case of *accident, sickness*, or *death*. The last is called *Life Insurance.*

The *Policy* is the written contract.

The *Premium* is a certain percentage on the amount insured.

Art. 143. — *Promiscuous Examples in Commission, Brokerage, Stocks, Gold, and Insurance.*

[These examples are to be done the same as others in Percentage.]

The *amount* bought or sold, collected, invested or insured, or the *par value* of stocks and gold, is the *principal;* to be found, if required, by Case III. in Percentage; or Case IV. when the percentage is to be deducted from the given sum, or has been deducted from the required sum.

The *per cent.* is the *rate;* to be found, if required, by Case II. in Percentage.

The *commission, brokerage, dividend, premium*, or *discount*, is the *percentage;* to be found, if required, by Case I. in Percentage.

Ex. 1. What is the commission for selling goods amounting to $2500, at 3½ per cent.?

Process, the same as in Percentage, Case I.

2. A commission merchant received \$87.50 for selling goods amounting to \$2500; what per cent. was his commission?

Process.—Percentage, Case II.

3. A commission merchant received \$87.50 for selling goods at $3\frac{1}{2}$ per cent.; what was the amount he sold?

Process.—Percentage, Case III.

4. A commission merchant received \$2587.50 for the purchase of goods, after deducting $3\frac{1}{2}$ per cent. commission; what was the amount of the goods he purchased?

Process.—Percentage, Case IV.; commission to be deducted.

5. A commission merchant, after deducting $3\frac{1}{2}$ % commission from the whole sum he had received, had a balance of \$2500 for the purchase of goods; what was the whole sum he received?

Process.—Percentage, Case IV.; commission already deducted.

6. A broker in New York exchanged \$1500, uncurrent money, at $\frac{1}{4}$ per cent. discount; how much was his brokerage?

7. A broker has $\$5012\frac{1}{2}$ to invest in bank stock, after deducting $\frac{1}{4}$ % for brokerage; how much is to be invested?

8. A merchant gave a broker \$1000 uncurrent money, and received from him \$990 current money; what per cent. was the brokerage?

9. A merchant gave a broker \$10 for exchanging some uncurrent money, at $\frac{1}{2}$ %; what was the amount?

10. A broker, after deducting $\frac{3}{4}$ % for brokerage, paid back \$1985 current money; how much uncurrent money had he received?

11. What is the value of 12 shares of railroad stock, at a premium of 5 %?

12. What is the value of the same at 5 % discount?

13. How much stock, at 5 % premium, can be bought for \$5250?

14. How much stock, at 5 % discount, can be bought for \$4750?

15. When gold is 125, how much is $500 in gold worth in paper currency?

16. When gold is 125, how much is $500 in paper currency worth in gold?

17. What must be paid for insuring a house valued at $2500, at 1½ % premium?

18. Paid $31.25 for insuring a house valued at $2500; what per cent. was the premium?

19. Paid $37.00 for insuring a house, at 1 % premium; what was the amount of the insurance?

20. For how much must a house, valued at $2500, be insured, at 1½ %, to cover both its loss by fire and the cost of the insurance?

21. What is the commission for selling goods amounting to $6500, at 3 %?

22. What is the brokerage on $1500 uncurrent money, at ⅓ %?

23. An agent has received $1230 for the purchase of goods, after deducting 2½ % commission; what will be the amount of the goods, and what his commission?

24. How much money at 1 % discount will pay a note for $1500?

25. What is the value of 10 shares of bank stock at a premium of 12 %?

26. How many shares of railroad stock 10 % below par will pay a debt of $1800?

27. What must be paid for the insurance on a store and goods valued at $18000, at 3 per cent?

28. How much current money should a broker give in exchange for $560, at ½ per cent. discount?

29. What will be the amount of a bill of goods to be purchased with the balance of $5616, received by remittance, after deducting 4 % commission?

30. How much are 15 shares in an Insurance Company worth at 6¼ % advance?

31. What is the premium on an insurance amounting to $5620, at 1¼ %?

32. When gold is $118\frac{3}{4}$, how much in currency should be received for \$60 interest on a Government bond, payable in gold?

33. An agent in Chicago has bought grain for a flouring mill in New York amounting to \$3500; what is his commission, at $3\frac{1}{4}$ per cent.?

34. What should a broker give in exchange for \$640 uncurrent money, at $3\frac{1}{4}$ % discount?

35. A Southern merchant has remitted to his agent in Philadelphia \$$2370\frac{1}{2}$, for the purchase of goods, after deducting his commission, at $3\frac{1}{2}$ %; what will be the amount of the goods to be purchased?

36. What should a broker deduct from the amount of a draft for \$850, taken in exchange for currency, at $\frac{3}{8}$ % dis.?

37. What are 18 shares of bank stock worth at $16\frac{2}{3}$ % adv.?

38. What is the premium on the insurance of a house valued at \$2800, at $4\frac{1}{2}$ %?

39. When gold is 125, how much of it will pay a debt of \$600?

40. A gentleman wishes a horse-dealer to purchase for him a span of horses worth \$900, and is willing to allow him $3\frac{1}{3}$ % commission; what will the horses cost him?

41. A gentleman gave a horse-dealer \$806 for a span of horses and his commission at 4 %; what was paid for the horses alone?

42. What is a draft for \$480 worth at $\frac{7}{8}$ % discount?

43. What are 6 shares in a steamboat company worth at a premium of 100 per cent.?

44. When gold is $127\frac{3}{8}$, how much currency will pay the duties (payable in gold) on a bill of imported goods amounting to \$3750.50?

45. What must be paid for insuring a steamboat valued at \$75000, at $6\frac{1}{4}$ per cent.?

46. The insurance of a steamship, valued at \$250000, costs \$10000; what per cent. is the premium?

47. A merchant in San Francisco has remitted to his agent in New York \$4888 in gold for the purchase of goods, after

deducting 4 % commission ; gold at the time being 125 ; what is the commission ?

48. What are 10 shares of the Corn Exchange Bank worth at a premium of $16\frac{2}{3}$ % ?

49. Bought a house lot for $10000 when gold was at par ; what ought it to be worth, gold being $130 ?

50. A merchant had paid $3\frac{1}{2}$ % for insurance on his stock of goods at $15000, for 10 years. After that time they were destroyed by fire ; what did he gain by having them insured ?

PROFIT AND LOSS.

Art. 144.—*Profit* (or Gain) and *Loss* are usually estimated at a certain *percentage* on the *cost* (principal.)

The *profit* or *loss*, at any rate per cent. is found by Case I. in Percentage.

The *per cent.* of profit or loss by Case II.

The *cost* by Case III. or IV.

The *profit* or *loss* = cost $\times$ per cent.

The *per cent.* of *profit* or *loss* = the profit or loss $\div$ cost.

$$\text{The cost} = \begin{cases} \text{the profit or loss} \div \text{per cent. or} \\ \text{the selling price} \div 1 \overset{+}{\underset{-}{\text{or}}} \text{ per cent.} \begin{smallmatrix}\text{gain}\\ \text{or}\\ \text{loss.}\end{smallmatrix} \end{cases}$$

The profit or loss is the difference between the cost and selling price.

The selling price is the cost with the profit added or the loss subtracted.

When the rate per cent. is an aliquot part of 100, it is often more convenient to express it in that form ; as 25 % $= \frac{1}{4}$; $12\frac{1}{2}$ % $\frac{1}{8}$; $33\frac{1}{3}$ % $\frac{1}{3}$.

EXAMPLES.

1. When cloth costs $4, and is sold at 25 % profit, what is the *profit* and selling price ?

Process as in Percentage, Case I. $4 \times .25$, or $\frac{1}{4} = \$1.00$ profit, which added to the cost is \$5.00 the selling price.

2. When the cost of cloth is \$4 a yard and the selling price \$5.00, what *per cent.* is the profit?

Process.—\$5 — \$4 = \$1. (gain) then as in Percentage, Case II. \$1 ÷ \$4 = .25, or 25 %.

3. When cloth is sold at 25 % profit and the profit is \$1 a yard, what did it cost?

Process as in Percentage, Case III. \$1.00 ÷ .25 or $\frac{1}{4}$ = \$4.00 (the cost.)

4. When cloth is sold for \$5.00 a yard and 25 % is thus gained, what did it cost?

Process as in Percentage, Case IV. \$5.00 ÷ \$1.25 = \$4.00 the cost.

The prices in the following examples may be considered as the prices per yard, pound, &c., &c.

5. Cost 10 cents; profit 20 %; selling price is what?
6. Cost 15 cents; selling price 18; gain per cent. is what?
7. Profit 5 cents; per cent. 20; cost is what?
8. Selling price 20 cents; loss 25 %; cost is what?
9. Cost \$1.; selling price \$1.25; gain per cent. is what?
10. Profit 20 cents; per cent. 15; cost is what?
11. Selling price 35 cents; profit 16⅔ %; cost is what?
12. Cost 32 cents; profit 12½ %; selling price is what?
13. Cost \$3; profit 50 cents; per cent. is what?
14. Cost \$300; loss \$75; per cent. is what?
15. Selling price \$75; loss 25 %; cost is what?
16. Profit \$625; per cent. 12½; cost is what?
17. Cost \$150; profit 16⅔ %; profit is what?
18. Cost \$5.20; profit 10 %; selling price is what?
19. Selling price \$3612½; loss 6¼ %; cost is what?
20. Cost \$875; selling price \$1050; per cent. is what?
21. Cost \$1000; profit \$70; gain per cent. is what?
22. Cost \$1000; loss 7 %; selling price is what?
23. Loss \$120; per cent. 6; cost is what?
24. Profit \$300; per cent. 6; cost is what?
25. Selling price 6¼ cents; cost 6 cents; per cent. is what?

26. Selling price \$5.25 ; cost \$6; loss per cent. is what ?

27. Cost \$2000; profit 18¾ %; selling price is what ?

28. Bought a horse for \$175 and sold him for \$210 ; what was the profit per cent. ?

29. Sold a house for \$2850, 5 % less than cost ; what did it cost ?

30. Bought a piece of land for \$1500 and sold at an advance of 33⅓ % ; what was the selling price ?

31. Gained in trade \$1600, which 16⅔ % of the capital employed ; what was the capital ?

32. A wool merchant bought 36000 lbs. of wool at 56 cents a pound, the expenses on it were \$108, and he sold it for 62½ cts. a pound ; how much per cent. did he gain ?

33. If 16 cwt. 31 lbs. of sugar cost \$203.875, for what must it be sold per pound to gain 24 % ?

34. A grocer sold tea for \$.75 a pound, and lost 12½ % ; what did it cost ?

35. A grocer sold tea for \$1 a pound, and gained 33⅓ % ; what did it cost ?

36. A grocer gained 12½ cts. a pound on tea, which was 25 % ; what did the tea cost per pound ?

INTEREST.

Art. 145.—Interest is a certain percentage paid for the use of money *for a specified time.*

The **principal** is the money lent, or loaned.

The **rate** is the per cent. paid annually or per annum.

The **amount** is the principal with the interest added to it.

Simple interest is the interest on the principal only. **Compound interest** is on the principal and interest already due. When only the word interest, is used, simple interest is meant.

Legal interest is at the rate fixed by law. In all the United States it is 6 per cent. except New York, Michigan, Wisconsin, Minnesota, S. Carolina, and Georgia, 7 %; Louisiana, 5 %; Florida, Alabama, Mississippi, and Texas, 8 %; Kansas and California, 10 %; Oregon, $12\frac{1}{2}$ %.

EXAMPLE 1.—What is the interest of $125.50 for 2 years 5 months and 21 days, at 6 %?

Process.—Since the interest of $1.00 for a year is $.06 (6 cts.) the interest of $125.50 for 1 year is (.06 times $125.50 or 125.50 times $.06) $7.53; for 2 years ($7.53×2) $15.06; for 5 months ($\frac{5}{12}$ of $7.53) $3.138 and for 21 days ($\frac{21}{30}$ or $\frac{7}{10}$ of $.628 the interest for 1 month or $\frac{1}{12}$ of a year) $.439. Therefore, the interest for the whole time is $18.637.

It is sufficiently accurate to count $\frac{1}{2}$ mill or more as 1 mill, and reject less fractions.

	$125.50	
	.06	
	7.53	int. 1 year.
	2	
	15.06	" 2 years.
$\frac{5}{12}$	3.138	" 5 months.
$\frac{7}{10}$	.439	" 21 days.
Ans.	$18.637	" whole time.

	$125.50	
	.06	
	7.53	=1 year.
	2	
	15.06	=2 years.
$\frac{1}{3}$ year's	2.51	=4 months.
$\frac{1}{4}$ 4 month's	.628	=1 month.
$\frac{1}{2}$ month's	.314	=15 days.
$\frac{1}{3}$ 15 days	.105	=5 days.
$\frac{1}{5}$ 5 days	.021	=1 day.
	$18.638	whole time.

RULE.—*To find the interest for one year, multiply the principal by the rate.*

For two or more years, multiply this product by the number of years.

For months take aliquot parts of a year's interest.

For days take aliquot parts of a month's interest.

The above rule is applicable to any rate; but at 6 % the following method is preferable:

Art. 146.—At 6 % the interest of $1 being $.06 a year or 12 months, it is half as many cents as months. Hence the interest of $1 for

1 month......	$.005	6 months.....	$.03
2 months.....	$.01	9 months.....	$.045
3 months.....	$.015	11 months.....	$.055

Also, the interest of \$1 for one month (30 days) being \$.005, it is 1 mill for every 6 days, or $\frac{1}{6}$ as many mills as days.

Example 1.—By second method.

Process.—The interest of \$1. for

2 years=......	\$.12
5 months=....	\$.025
21 days=......	\$.0035
whole time=...	\$.1485

Therefore

\$125.50
.1485
62750
100400
50200
12550
6)18.636750 at 6 %
3.106125 at 1 %
7
\$21.742875 at 7 %

The interest at 6 % divided by 6 is the interest at 1 %, and this multiplied by 7 is the interest at 7 %; or if multiplied by any other rate it would be the interest at that rate.

Rule.—*Find the interest of* \$1 *for the given time, allowing* 6 *cents for every year*, half *as many cents as months and as many mills as there are times* 6 *days; then multiply it by the given number of dollars.*

EXAMPLES.

In the following examples, pupils should use both methods of calculating interest till they become familiar with them, then either as they may prefer, or the teacher may direct. There may be a slight difference in the answers obtained by the different methods. In business, interest is more generally computed by tables.

What is the interest of \$1 for

(1.)	1 year	5 months	18 days,	at 6 %?	also of \$125.00?
(2.)	2 "	4 "	12 "	" 6 %?	" \$75.50?
(3.)	3 "	7 "	15 "	" 6 %?	" \$210.25?
(4.)	4 "	8 "	20 "	" 6 %?	" \$87.625?
(5.)	5 "	9 "	21 "	" 6½ %?	" \$10.12½?
(6.)	6 "	1 "	3 "	" 6 %?	" \$256.00?
(7.)	7 "	0 "	4 "	" 6 %?	" \$160.50?
(8.)		9 "	25 "	" 6 %?	" \$62.375?
(9.)	10 "	10 "	10 "	" 6 %?	" \$100.00?
(10.)			27 "	" 6 %?	" \$1000.00?

What is the interest of

(11.)	$45.00	for 1 year	3 months	6 days,	at 7 % ?
(12.)	$130.00	" 2	" 1	" 3	" 7 % ?
(13.)	$234.50	" 3	" 8	" 25	" 7 % ?
(14.)	$15.87½	" 4	" 5	" 15	" 7 % ?
(15.)	$112.56¼	" 5	" 10	" 9	" 4½ % ?
(16.)	$215.00	" 3	" 11	" 13	" 5 % ?
(17.)	$321.18¾	" 2	" 6	" 17	" 7 % ?
(18.)	$400.	" 4	"	" 10	" 8 % ?
(19.)	$500.	" 1	" 4	" 22	" 5 % ?
(20.)	$650.	" 3	" 3	" 3	" 4½ % ?
(21.)	$36.50	"	6 "		at 6 % ?
(22.)	$118.75	"		19	" 7 % ?
(23.)	$90.00	" 2	" 1	" 6	" 4½ % ?
(24.)	$124.80	" 3	" 5	" 7	" 5 % ?
(25.)	$84.30	" 1	" 6	" 12	" 6 % ?
(26.)	$100.	" 4	" 7	" 18	" 7 % ?
(27.)	$72.70	" 2	" 2	" 2	" 7 % ?
(28.)	$95.50	" 1	" 1	" 1	" 6 % ?
(29.)	$100.	"	4	" 8	" 7 % ?
(30.)	$1000.	"		29	" 4½ % ?

31. What is the *amount* of $10 for 1 year 6 months at 6 % ?

32. What is the amount of $25 for 6 months 12 days at 7 % ?

33. What is the amount of $50 for 2 years 20 days at 7 % ?

34. What is the amount of $60 for 2 months 15 days at 6 % ?

35. What is the amount of $75 for 1 year 10 days at 7 % ?

36. What is the amount of $100 for 7 months 18 days at 5 % ?

37. What is the interest of $120 for 2 years 9 months at 6 % ?

38. What is the amount of $150 for 3 months 3 days at 7 % ?

39. What is the amount of $1 for 100 years at 7 % ?

40. What is the amount of $10 for 10 days at 6 % ?

41. What is the interest of $125.50 from July 1, 1858, to Jan. 1, 1859, at 6 % ?

42. What is the amount of $200 from Aug. 1, 1860, to Nov. 16, 1860, at 7 % ?

43. What is the amount of $137.50 from May 16, 1865, to January 1, 1866, at 6 % ?

44. What is the interest of $300 from June 1, 1864, to Feb. 10, 1865, at 7 %?

45. What is the amount of $250 from July 10, 1865, to March 1, 1866, at 6 %?

46. What is the amount of $320 from Sept. 15, 1867, to April 1, 1868, at 5 %?

47. What is the interest of $250 from July 15, 1866, to Jan. 1, 1867, at 6 %?

48. What is the amount of $560 from January 1, 1867, to June 16, 1867, at 7 %?

49. What is the amount of $67.50 from July 21, 1866, to Sept. 1, 1866, at 6 %?

50. What is the amount of $100 from June 15 to July 1, at 5 %?

Exact Interest.

Art. 147.—In the preceding examples 30 days have been allowed to each month, which makes a year to consist of 360 instead of 365 days. To find the exact interest we must count the exact number of days in each month (see Table of Time, Art. 50,) and consider each day $\frac{1}{365}$ part of 1 year.

EXAMPLE 51.—What is the exact interest of $36 from March 1 to June 9, at 6 %?

Process.—The number of days are 31 in March, 30 in April, 31 in May, and 8 in June (31+30+31+8,) 100. And $2.16, the interest for a year, divided by 365, gives the interest for 1 day, which multiplied by 100, gives the interest for 100 days. For convenience we multiply by 100 first, and then divide by 365.

$$\begin{array}{r}
\$36 \\
\underline{06} \\
2.16 \\
\underline{100} \\
365)216.00(\$.59+ \\
\underline{1825} \\
3350 \\
\underline{3295} \\
55
\end{array}$$

52. What is the exact interest of $50 from July 1 to Sept. 16, at 6 %?

53. What is the exact interest of $1000 from Oct. 10 to Jan. 1, at 7 %?

54. What is the exact interest of \$100 from July 1 to Aug. 1, at 6 %?

55. What is the exact interest of \$100 from Feb. 10 to March 10, at 6 %? if it is leap-year?

Art. 148.—*Interest of Sterling Money.*

Example 56.—What is the interest of £50 10s. 6d. for 1 year 4 months, at 5 per cent.?

Process.—£50 10s. 6d. = (Article 121) £50.525, on which the interest 4 m. = $\frac{1}{3}$ year, found as in Federal Money, is £3.36833, or (Art. 122) £3 7s. 4d. 1.5968far.

```
   £50,525
         5
 ---------
 3)2.52625
     84208
 ---------
  £3.36833
        20
 ---------
s  7.36660
        12
 ---------
   4.39920
         4
 ---------
   1.59680
```

Rule.—*Reduce the shillings, &c., to the decimal of pound. Then proceed as in Federal Money, and reduce the decimals to shillings, &c.*

What is the interest of

Ex. 57. £325 12s. 3d. for 5 years, at 6 %?

58. £174 10s. 6d. for 3 years 6 months, at 6 %?

59. £150 16s. 8d. for 4 years 7 months, at 6 %?

60. £45 10s. for 2 years, at 4 %?

PARTIAL PAYMENTS.

Art. 149.—A **partial payment** is the payment of part of a note or debt bearing interest.

A **Note** is a written promise to pay a debt; as

\$325.38.

New York, Sept. 1, 1867.

Thirty days after date I promise to pay J. V. Peck, or order, three hundred and twenty-five $\frac{38}{100}$ dollars, for value received.

Sam'l A. Rogers.

Sam'l A. Rogers is the *drawer* or *maker*.
J. V. Peck is the *payee*.
$325.38 is the *face* of the note.

United States Rule.

When partial payments have been made, apply the payment, in the first place, to the discharge of the interest then due.

If the payment exceeds the interest, the surplus goes toward discharging the principal, and the subsequent interest is to be computed on the balance of principal remaining due.

If the payment is less than the interest, the surplus of the interest must not be taken to augment the principal, but interest continues on the former principal, until the period when the payments taken together exceed the interest due, and then the surplus is to be applied towards discharging the principal, and interest is to be computed on the balance as aforesaid.

Example 1.— Hartford, Jan. 1, 1860.

For value received, I promise to pay ——— Five Hundred Dollars on demand, with interest at 6 per cent.

Indorsements :—May 1, 1861, $175. Sept. 16, 1862, $25. Jan. 1, 1864, $100. July 25, 1864, $120.

What was due Jan. 1, 1865.

Dates.				Time between dates.			
				Years.	Months.	Days.	Int. of $1,
Note	1860..	1	1				
1st Payment,	1861..	5	1	1	4		.08
2d Payment,	1862..	9	16	1	4	15	.0825
3d Payment,	1864..	1	1	1	3	15	.0775
4th Payment,	1864..	7	25		6	24	.034
Settlement,..	1865..	1	1		5	6	.026

Principal..		$500.
Interest to May 1, 1861		40.
Amount..		540.
1st Payment		175.
Balance and new principal May 1, 1861.............		365.
Interest to Sept. 16, 1862....................	$30.11	
2d Payment (less than interest)..............	25.00	
Surplus interest...........................	5.11	

Interest of the same principal from Sept. 16, 1862 to Jan. 1, 1864	28.29	33.40
Amount		398.40
3d payment (to be deducted)		100.00
Balance and new principal		298.40
Interest to July 25, 1864		10.146
Amount		308.546
4th Payment (to be deducted)		120.000
Balance and new principal		188.546
Interest to Jan. 1, 1865		4.902
Amount due Jan. 1, 1865		$193.448

Ex. 2. A note of $450 is dated June 16, 1860. Interest at 7 per cent.

Indorsements :—Aug. 1, 1861, $20. Jan. 16, 1863, $220.

What was due May 16, 1864 ?

Ex. 3. A note of $300 is dated July 1, 1861. Interest at 6 per cent.

Indorsements :—Jan. 1, 1863, $15. July 1, 1865, $150.

What was due May 1, 1866 ?

Ex. 4. A note of $620 is dated Sept. 9, 1863. Interest 5 per cent.

Indorsements :—Dec. 21, 1863, $75. Sept. 8, 1864, $200. June 20, 1865, $20.

What was due Sept. 20, 1865.

Ex. 5. A note of $750 is dated May 1, 1862. Int'rst 6 per cent.

Indorsements :—Nov. 16, 1863, $50. Sept. 1, 1864, $175.

What was due May 1, 1865 ?

Ex. 6. A note of $500 is dated Sept. 7, 1860. Interest 7 per cent.

Indorsements :—Jan. 1, 1861, $100. June 19, 1861, $10. Jan. 1, 1862, $200.

What was due Sept 1, 1862 ?

Ex. 7. $600. NEW HAVEN, Jan. 1, 1863.

On demand I promise to pay D—— P—— & Co., or order, six hundred dollars, for value received, with interest.

P—— W——.

Indorsements :—Jan. 1, 1864, $100. July 1, $10.

What was due Jan. 1, 1865 ?

Ex. 8. $1500. New York, Dec. 25, 1863.

On demand, we promise to pay G—— M—— & Co., or order, fifteen hundred dollars, for value received, with interest.

P—— W—— & Co.

Indorsements :—Jan. 7, 1865, $200. July 7, 1865, $25.

What was due Aug. 1, 1865 ?

Mercantile Rule.

Art. 150.—*Find the amount of the principal for one year, and from it subtract the amount of each payment during the year, to the end of it ; the remainder will be a new principal, with which proceed as before.*

If the time of settlement is less than a year, find the amount of the principal for such a portion of a year, and subtract from it the amount of the payments to the same date.

Example 9.—A note of $1000 is dated Jan. 1, 1864 interest 6 per cent.

Indorsements :—March 12, 1864, $200. October 25, $350. April 18, 1865, $100.

What was due June 18, 1865 ?

Principal		$1000.
Interest for 1 year		60.
Amount		1060.
1st Payment (March 10)	$200.00	
Interest till Jan. 1, 1865	9.60	
2d Payment (Oct. 25)	350.00	
Interest till Jan. 1, 1865	3.85	563.45
Balance for new principal		496.55
Interest till settlement		13.82
Amount		510.37
3d Payment (April 18, 1865)	100.00	
Interest till settlement	1.00	101.00
Balance due June 18, 1865		$409.37

Ex. 10. A note of $1000 is dated Jan. 1, 1866. Interest 6 per cent.

Indorsements :—March 1, $100. May 25, $100. Sept. 1, $100. July 1, 1867, $500.

What was due Sept. 1, 1867 ?

Ex. 11. A note of $500 is dated July 1, 1865. Interest 7 per cent.

Indorsements :—Jan. 1, 1866, $100. April 1, $100. May 16, $100.

What was due July 1, 1866 ?

Ex. 12. A note of $800 is dated May 1, 1866. Interest 6 per cent.

Indorsements :—Sept. 1, $200. Jan. 1, 1867, $200. March 1, $200.

What was due May 1, 1867 ?

Connecticut Rule.

Art. 151.—*The Connecticut rule is the same as the Mercantile when a payment is made in less than a year; otherwise it is the same as the U. S. Rule.*

Find by the Connecticut Rule what was due in Example 1, page 204.

Process.—The same as by U. S. Rule till the third payment has been deducted, leaving

Balance for a new principal		$298.40
Interest for 1 year		17.90
Amount (Jan. 1, 1865)		316.30
4th Payment	$120.00	
Interest to Jan. 1, 1865	3.12	123.12
Amount due Jan. 1, 1865		$193.18

By the same rule find the amounts due in Examples 2, 3, 4, &c.

If the answers are very nearly the same as by the U. S. Rule they may be considered correct.

Rate.

Art. 152.—*To find the rate when the principal, interest, or amount and time are given.*

EXAMPLE 1.—At what *rate* will \$200 yield \$30 interest in 2 years 6 months ?

Process.—At 1 % the interest of \$200 for 2 years 6 months is \$5. Therefore if the interest is \$30 the rate must be as many times 1 % as \$5 is contained times in \$30, or 6 %.

	\$200	
	.01	
6 mos. ½)	2.00	1 year.
	2	
	4.00	2 yrs.
	1.00	8 mos.
	5.00	30.00
	Ans.	6%

RULE.—*Divide the given interest by the interest of the principal at* 1 %.

Ex. 2. At what rate will \$320 yield \$72.80 in 3 yrs. 3 mos.?

TIME.

Art. 153.—*To find the time when the principal, interest, or amount and rate are given.*

Ex. 3. In what time will \$200 yield \$30 interest, at 6 % ?

Process.—The interest of \$200 for 1 year, at 6 %, is \$12. Therefore if the interest is \$30, the time must be as many years as \$12 is contained times in \$30, or 2½ years.

\$	\$
12)	30
Ans.	2½ years.

RULE.—*Divide the given interest by the interest of the principal for* 1 *year.*

Ex. 4. In what time will \$320 yield \$72.80, at 7 per cent.?

PRINCIPAL.

Art. 154.—*To find the principal when the interest or amount, time, and rate are given.*

Ex. 5. What principal, at 6 %, will yield \$30 interest, in 2 years 6 mos.?

Process.—\$1 at 6 % will yield in 2 years 6 months \$.15 interest. Therefore \$30 interest will require as many dollars as \$.15 is contained times in \$30, or \$200.

	\$
.15)	30.00
Ans.	\$200.

Rule.—*Divide the given interest or amount by the interest or amount of* \$1, *for the given time.*

Ex. 6. What principal at 7 % will yield \$72.80 interest in 3 years 3 months?

EXAMPLES.

7. At what rate will \$500 yield \$34 interest in 1 year 1 month 18 days?

8. At what rate will \$300 amount to \$366 in 3 yrs. 8 mo.?

9. In what time will \$560 yield \$106.40 at 8 per cent.?

10. What principal will yield \$192 interest in 4 yrs. 3 mos. 6 days, at 6 per cent.?

11. At what rate will \$1200 yield \$3 interest in 15 days?

12. In what time will \$360 yield \$10.50 interest at 5 %?

13. What principal will yield \$37.50 interest in 4 years 2 months, at 6 %?

14. In what time will \$360 amount to \$360.66, at 6 %?

15. At what rate will \$350 yield \$101.50 interest in 7 years 3 months?

16. What principal will yield \$9 interest in 1 year 2 mos. 12 days, at 6 %?

17. At what rate will \$500 yield \$62.50 interest in 2 years 1 month?

18. In what time will \$65 yield \$2.60 interest at 6 %?

19. What principal will amount to \$1245 in 3 years and 6 months, at 7 %?

20. At what rate will \$1000 amount to \$1150, in 2 years and 6 months?

COMPOUND INTEREST.

Art. 155.—Compound Interest is interest on the principal and interest already due.

Example 1.—What is the compound interest of \$500 for 3 years 6 months, at 6 %?

\$500	
.06	
30.00	Interest 1st year.
500	
530	Amount " "
.06	
31.80	Interest 2d "
530	
561.80	Amount " "
.06	
33.7080	Interest 3d "
561.80	
595.508	Amount " "
.03	
17.86524	Interest 6 months.
595.508	
613.37324	Amount "
500	
\$113.37324	Compound int'st for 3 y'rs 6 mos.

RULE.—*Find the interest for a year or the time till it is due, and add it to the principal for a new principal; on which find the interest as before. Proceed thus till the last interest is due, and from the amount subtract the first principal.*

EXAMPLES.

What is the compound interest of—

(2.) \$200 for 3 years, at 7 %?

(3.) \$300 for 4 years, at 6 %?

(4.) \$400 for 5 years, at 5 %?

(5.) \$200 for 2 years, at 6 % (payable semi-annually?)

(6.) \$100 for 1 year, at 6 % (payable quarterly?)

7. What is the amount of 700 for 3 years, 9 months, and 24 days, at 7 %, compound interest?

8. What is the amount of \$740, at 6 %, compound interest, (semi-annually,) from Dec. 20, 1866, to Nov. 2, 1869?

9. What is the compound interest of \$1000 for 2 years, 8 months, 15 days, at 6 %?

10. What is the amount of \$500 for 2 years at 8 %, compound interest, payable quarterly?

TABLE,

Showing the amount of \$1, *or* £1, *at* 3, 4, 5, 6, *and* 7 *per cent. compound interest, for any number of years from* 1 *to* 10.

Years	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.030,000	1.040,000	1.050,000	1.060,000	1.07,000
2	1.060,900	1.081,600	1.102,500	1.123,600	1.14,490
3	1.092,727	1.124,864	1.157,625	1.191,016	1.22,504
4	1.125,509	1.169,859	1.215,506	1.262,477	1.31,079
5	1.159,274	1.216,653	1.276,282	1.338,226	1.40,255
6	1.194,052	1.265,319	1.340,096	1.418,519	1.50,073
7	1.229,874	1.315,932	1.407,100	1.503,630	1.60,578
8	1.266,770	1.368,569	1.477,455	1.593,848	1.71,818
9	1.304,773	1.423,312	1.551,328	1.689,479	1.83,845
10	1.343,916	1.480,244	1.628,895	1.790,848	1.96,715

Multiply the amount of \$1, by the given number of dollars.

Find the answers to the above examples by the table.

The semi-annual interest of \$1 is the same as the annual interest at ½ the rate per cent.

DISCOUNT.

Art. 156.—Discount is a certain percentage deducted for the payment of money a specified time before it is due.

The **present worth** of a sum, or debt, payable at some future time without interest, is the sum which put at interest till it becomes due, will amount to the given sum.

EXAMPLE 1.—What is the present worth of \$500, payable in one year at 6 per cent.?

```
1.06)500(471.698
     424
     ---
      760
      742
      ---
       180
       106
       ---
        740
        636
        ---
        1040
         954
        ----
          860
          848
          ---
```

Process.—At 6 % the present worth of \$1.06 due in 1 year, is \$1.00. Therefore the present worth of \$500 is as many dollars as it contains times \$1.06, or \$471.698+. \$500—471.698= \$28.302 discount.

RULE.—*Divide the given sum by the amount of* \$1 *for the given time, at the given rate; the quotient will be the present worth.* (Percentage, Case IV.)

To find the discount, subtract the present worth from the given sum or debt.

EXAMPLES.

What is the

2. Present worth of \$130, due in 5 years, at 6 %?

3. Discount of \$115, due in 2 years 6 months, at 6 %?

4. Present worth of \$334, due in 1 year 1 month 18 days, at 6 %?

5. Discount of \$666.40, due in 2 years 4 months 15 days, at 8 %?

6. Present worth of \$942, due in 4 years 3 months 6 days, at 6 %?

7. Present worth of \$534.04, due in 3 years 5 months 18 days, at 5 %?

8. Discount of \$366, due in 3 years 8 months, at 6 %?

9. Present worth of 273.75, due in 1 year 7 months, at 6 %?

10. What was the discount of \$263.04, due April 27, 1859, but paid Feb. 15, 1858, at 8 % ?

11. What is the value May 10, 1863, of a debt of \$200, due Aug. 28, 1865, at 7 %?

12. How much must be paid July 3, for a debt of \$142.45, due Nov. 27, at 9 %?

13. How much should be deducted from a debt of \$170.50, due April 19, 1869, if paid Jan. 9, 1867, at 6 %?

14. What is the discount, Feb. 5, 1862, on a note of \$407.088, payable Aug. 20, 1864, at 7 %?

15. How much should be paid, May 18, 1867, on a note of \$5783.09⅓, payable Sept. 25, 1870, at 8 %?

BANK DISCOUNT.

Art. 157.—Bank Discount is a certain percentage paid to banks, or bankers, for the use of money paid on notes before they are due. It is the same as simple interest paid in advance.

Bank discount is greater than *true discount*, because it is computed on the amount or face of the note, which includes the interest with the money lent, instead of only the principal.

In computing bank discount *three days of grace* are allowed, in addition to the specified time.

EXAMPLE 1.—What is the bank discount of a note of \$150, payable in 90 days, at 6 % ?

Process.—The interest of \$1 for 93 days is \$.0155, and of \$150, 150×.0155=\$2.325.

```
      $
    150.
   .0155
   -----
     750
    750
   150
   ------
Ans. $2.3250
```

RULE.—*Find the interest for three days more than the specified time.*

If the note bears interest, find the interest on the amount that will be due on it at maturity.

The discount subtracted from the given sum gives the present worth.

EXAMPLES.

What is the bank discount

2. On a note of \$300 for 6 months, at 6 % ?
3. On a note of \$450 for 4 months, at 5 % ?
4. On a note of \$500 for 3 months, at 7 % ?
5. On a note of \$750 for 9 months, at 5½ % ?
6. On a note of \$1000 for 3 months, at 7 % ?

What is the present worth

7. Of a note of \$120 for 4 months, at 7 % ?
8. Of a note of \$360 for 30 days, at 6 % ?
9. Of a note of \$340 for 6 months, at 8 % ?
10. Of a note of \$480 for 1 month, at 6 % ?
11. Of a note of \$1950 for 2 months, at 6 % ?

Art. 158.—*To find for what amount a note must be given that it may be worth a given sum when discounted.*

EXAMPLE 12.—For what amount due in 60 days must a note be given that its present worth may be \$500, at 6 % ?

Process.—The bank discount of \$1 for 63 days is \$.0105, and the present worth is \$.9895. Therefore \$500 is the present worth of as many dollars for the same time as \$.9895 is contained times in \$500.

$$\$500 \div \$.9895 = \$505.30.$$

RULE.—*Divide the required sum by the present worth of* **\$1.**

13. For what amount must a note payable in 90 days be given that its present worth may be \$300, at 6 % discount ?

14. For what amount must a note payable in 10 months be given that its present worth may be \$500, at 7 % discount ?

15. For what amount must a note payable July 1 be given that it may be worth Jan. 1, the same year, \$730, at 6 % dis. ?

16. A man wishes to procure from a bank \$1000. At 5 % discount what will be the amount for which he must give his note, payable in six months ?

Promiscuous Examples in True and Bank Discount.

If bank discount is not specified, discount means true discount.

1. What is the discount of \$100 for six months at 6 % ?

2. What is the bank discount of the same?

3. What is the present worth of the same at true discount? bank discount?

4. A debt of $300 is due Oct. 1; if paid June 1, the same year, how much should be paid?

5. Bought $100 worth of goods on 6 months' credit; how much should be deducted for cash, at 7 % discount?

6. A note of $3600, payable in 60 days, was discounted at a bank at 6 %; how much was received for it?

7. A speculator wished to procure from a bank $10,000 for 4 mos. For what amount should he have given a note for it at 7 % discount?

8. A merchant bought goods amounting to $3000 on 6 mos. credit, but was allowed 5 % of the amount for cash; money being worth 7 %, how much did he gain by paying cash?

9. A merchant bought 75 barrels of flour for $500 and sold it for $640, receiving for it a note payable in 8 months, which he had discounted at 6 %, bank discount; how much did he gain?

10. A drover wishes to procure from a bank $2000 for 2 mos. 15 days; for what amount must he give his note, at 7 % dis.?

TAXES.

Art. 159.—A **Tax** is money required by law to be paid for the support of the government and its institutions, or public improvements.

A **poll tax** is a certain sum on male citizens, called polls.

An **income tax** is a certain percentage on incomes.

Taxable property is either Personal Property or Real Estate.

Real Estate is that which is not movable; as land, houses, &c.

Personal Property is such as is movable, as money, notes, furniture, &c.

Taxes are commonly a certain percentage on property, of all which an *inventory*, or list of articles, is first made.

EXAMPLE 1.—In a certain town \$5150 is to be raised by tax. The number of polls is 300, each taxed 50 cents. The real estate is valued at \$800,000, personal property \$200,000. What is the rate per cent. on \$1, and what is the amount of A's tax, who pays for 5 polls, and whose real estate is valued at \$4000 and personal property \$1500?

Process.—300 polls $\times$ 50 cents = \$150 poll tax; \$5050 — \$150 = \$5000 property tax; \$5000 $\div$ 1000000 (800000 + 200000) = .005 mills the per cent. or tax on \$1. (Percentage, Case II.) A's property, \$5500 (4000 + 1500) $\times$ .005 = \$27.50 A's property tax, to which add his poll tax (5 polls $\times$ 50 cents) \$2.50. *Ans.* \$30.00.

RULE.—*Subtract the poll tax from the whole tax; find the per cent. of the remainder on all the property, and then each man's property tax, to which add his poll tax.*

After finding the tax on one dollar, it will be convenient in practice to make a tax table, as follows:

TABLE.

Tax on \$1.=\$.005	Tax on \$8.=\$.04	Tax on \$60.=\$.30
2.= .01	9.= .045	70.= .35
3.= .015	10.= .05	80.= .40
4.= .02	20.= .10	90.= .45
5.= .025	30.= .15	100.= .50
6.= .03	40.= .20	1000.=5.00
7.= .035	50.= .25	&c.

By this table A's property tax in the above example is on

\$5000	\$25.00
500	2.50
	\$27.50 property tax.

Ex. 2. The tax of a certain town is \$4500. The number of polls is 500, each taxed \$1. The real estate is valued at \$600,000, and personal property \$200,000. What per cent. is the tax, and what is B's tax, whose property is valued at \$3500, and who pays for two polls?

When district schools are supported by families in proportion to the attendance, divide the whole expense by the whole number of days' attendance, and multiply the quotient by the days' attendance from each family.

DUTIES.

Art. 160.—Duties are taxes on goods imported or exported.

A **Port of Entry** is a place where duties are collected.

A **Custom-House** is an office where duties are collected.

Ad valorem duty is a certain percentage on the cost of the goods.

Specific duty is a certain price per weight or measure.

An **invoice** is a list of goods, with the prices.

Tare is an allowance for weight of boxes, casks, &c., containing the goods.

Draft is an allowance of weight for waste.

Leakage is an allowance for the waste of liquors in casks or barrels; *breakage*, for the same in bottles.

Gross weight is the weight of goods, including the packages.

Net weight is the weight after deducting the tare, &c.

In all these allowances reject fractions less than ½, and add 1 for ½ or more.

Example 1.—What is the duty, at 30 per cent., on 100 gals. of oil, invoiced at 75 cts. a gallon, allowing 2 % for leakage?

Process.—$100 \times .02 = 2.$ gallons leakage.
$100 - 2 = 98$ " net.
$98 \times .75$ cts. $= \$73.50$ net value.
$\$73.50 \times .30 = \$22.05$ duty.

Ex. 2. What is the duty on 10 bbls. of sugar, each weighing 215 lbs. gross, at 2 cts. a pound—draft 2 lbs. each; tare 12½ per cent.?

Process.—$215 - 2 = 213$ pounds.
$213 \times 10 = 2130$ "
$2130 \times .12\frac{1}{2} = 266$ lbs. tare.
$2130 - 266 = 1864$ lbs. net.
1864×2 cts. $= \$37.28$ duty.

RULE.—*Deduct all allowances, then to find the ad valorem duty multiply the cost by the given rate per cent.; to find the specific duty multiply the net weight or quantity by the duty on* ONE *of the same.*

Ex. 3. What is the duty, at 18 per cent., on 200 bags of coffee, each weighing 150 lbs., invoiced at $12\frac{1}{2}$ cts. a pound—draft 2 lbs. each; tare 3 per cent.?

Ex. 4. What is the duty, at $33\frac{1}{3}$ per cent., on 36 pieces of silk, each containing 45 yards, invoiced at \$2 a yard?

Ex. 5. What is the duty, at 10 cts. a pound, on 34 chests of tea, each weighing 118 lbs.—draft on each 1 lb.; tare 8 per ct.?

Ex. 6. What is the duty, at 40 per cent., on 500 yards of satin, at $\$1.62\frac{1}{2}$?

Ex. 7. What is the duty, at 25 cts. a gallon, on 18 casks of wine, each containing 68 gals., allowing 2 per cent. leakage?

Ex. 8. What is the duty, at $12\frac{1}{2}$ cts. a pound, on 12 boxes of tobacco, each weighing 130 lbs.; draft 1 pound on each box; tare 6 per cent.?

Ex. 9. What is the duty, at 30 per ct., on 9 cases cf broadcloth, each case containing 20 pieces, and each piece 36 yards, at \$4 a yard?

EXCHANGE.

Art. 161.—Exchange is a means employed by persons in one place of making payments in another, such as *drafts*, or *bills* of Exchange. Exchange in the same country is *Domestic;* from one country to another, *Foreign.*

Form of a Draft.

$500 New Haven, Oct. 15, 1867.

At sight, pay Ezra Jones, or order, Five Hundred Dollars, for value received, and charge the same to the account of

R. Clark.

To S. Staples, New York.

R. Clark is the Drawer.

S. Staples is the Drawee.

Ezra Jones is the Payee.

S. Staples *accepts*, or promises to pay the above, by writing "Accepted" over his name, on the back.

If Ezra Jones *indorses* it, by writing his name on the back, any person who has it is entitled to the amount.

Drafts, or Bills of Exchange, like Stocks, may be at par, at premium, or discount, and their value found in the same way. If they are not payable at sight they are subject to *bank discount*, the same as notes.

EXAMPLES OF DOMESTIC EXCHANGE.

1. What is the cost of the following draft, at a premium of ¼ %?

$300 Augusta, Ga., Aug. 20, 1867.

At sight pay A. T. Stewart & Co., or order, Three Hundred Dollars, for value received, and charge to the account of

O. Whetmore & Co

Brown Brothers & Co., New York.

Process.—\$1 costs \$1.0025. Therefore \$300 cost $\$1.0025 \times 300 =$ \$300.75, *Ans.*

2. What is the cost of the following draft, at $2\frac{1}{2}$ % discount?

\$480 New York, Oct. 10, 1867.

Thirty days after sight, pay to John Rose, or order, Four Hundred and Eighty Dollars, for value received, and charge to the account of

Benedict & Son.

To Wm. S. Jerome, Macon, Ga.

Process.—Present worth of \$1, by bank discount=\$.9936; \$.9936—.025=\$.9686 the cost \$1 of the draft, $\$.9686 \times 480 = \$464.928$, *Ans.*

3. A merchant in St. Louis wishes to remit a draft for \$1000 to New York; what will it cost, exchange being at $2\frac{1}{2}$ per cent. premium?

4. A merchant in New Orleans wishes to remit to Philadelphia a draft for \$1500, at thirty days' sight; what will it cost, exchange being at 4 per cent. premium at sight.

Drafts are usually drawn at 30, 60, or 90 days' sight, and at a certain per cent. premium or discount, including allowance for the time.

Exchange on England.

Art. 162.—Exchange on England is always at a premium in this country, because in making it, a pound sterling is valued at $\$4.44\frac{4}{9}$, instead of its true value, \$4.84.

Ex. 1.—Bill of Exchange on England.

£300 New York, Dec. 1, 1866.

Thirty days after sight of this First of Exchange (second and third of same date and tenor unpaid,) pay to Samuel J. Nichols, or order, Three Hundred Pounds, value received.

Wm. J. Staples.

To Brown Brothers & Co., Liverpool, Eng.

What is the cost of the above bill, when exchange is 10½ % premium?

Process.—£1=\$$4\frac{4}{9}$ (old value) £300$\times 4\frac{4}{9}$=\$$1333.33\frac{1}{3}$, \$1 at $10\frac{1}{2}$% premium=\$1.105. \$$1333.33\frac{1}{3}\times 1.105$=\$$1473.33\frac{1}{3}$, *Ans.*

Art. 163.—*To Find the Cost of a Bill of Exchange on England.*

Rule.—*Reduce the pounds and decimal of a pound, at the old value, (\$$4\frac{4}{9}$,) to dollars, &c.; which multiply by the cost of* \$1.

Ex. 2. A gentleman, about to visit England, wishes to buy a bill exchange for \$2000; what will be the amount of it at 9½ premium?

Process.—\$1.095 = cost of \$1.
\$$1.095 \times 4\frac{4}{9}$ = \$$4.86\frac{2}{3}$ the cost of £1.
\$$2000 \div 4.86\frac{2}{3}$ = £410.959 + = £410 19s. 2d. +

Art. 164.—*To find the amount of a Bill on England which can be bought with U. S. Money.*

Rule—*Divide the given number of dollars by the cost of* £1.

EXAMPLES IN EXCHANGE ON ENGLAND.

3. What will a bill of exchange on London, amounting to £500, cost in New York, exchange being $9\frac{1}{4}$ per cent. prem.?

4. A gentleman wishes to buy a bill of exchange on Liverpool with \$2500; what will be its amount at $12\frac{1}{2}$ % premium?

5. An importer wishes to pay for goods ordered from Manchester, England, amounting to £1000; what will a bill of exchange cost at $9\frac{1}{2}$ per cent. premium?

6. An importer wishes to remit to Leeds, England, \$3000; what will be the amount of a bill of exchange, at 10 per cent. premium.?

Exchanges on other foreign countries are made by reduction of currencies.

Promiscuous Examples in Exchange.

1. What is the cost of a draft on Memphis for \$3600, at $1\frac{3}{4}$ per cent. premium?

2. What is the amount of a draft which costs \$1012.50, at $1\frac{1}{2}$ per cent. premium?

3. What will a draft on New York for \$600 cost, at 2 per cent. discount?

4. What will be the cost of a bill of exchange on Liverpool, England, of £150 10s., at $9\frac{1}{2}$ per cent. premium?

5. What will be the amount of a bill of exchange on London, bought in Boston for \$3000, at $9\frac{1}{2}$ per cent. premium?

6. What will be the cost of a draft for \$750, on Hartford, at $\frac{1}{2}$ per cent. discount?

7. What is the amount of a draft bought for \$1250, at a premium of $2\frac{1}{2}$ per cent.?

PARTNERSHIP.

Art. 165.—Partnership is a company of two or more persons in the same business. They are called a *firm*, or *house*, and each member a *partner*.

The **capital**, or stock, is the money or property employed in their business, of which the profit or loss is a certain part or percentage.

In Bankruptcy the creditors are the same as partners in business, and the property of the bankrupt as profit or loss.

Case I.

Art. 166.—*To find each partner's share of the gain or loss when their capital has been used the same length of time.*

Example.—Peck, Staples & Clark are partners in business. P.'s capital is \$4800 ; S.'s \$2400, and C.'s \$1800. The whole gain is \$3000 ; what is each one's share ?

Process.—P.'s capital,	\$4800	The whole gain is $\frac{3000}{9000} = \frac{1}{3}$, or $33\frac{1}{3}$ per cent. of the whole capital. Therefore each partner's share is $\frac{1}{3}$, or $33\frac{1}{3}\%$ of his capital. Therefore,
S.'s "	\$2400	
C.'s "	\$1800	
Whole capital,	\$9000	

$$\left.\begin{array}{lll} \$4800 \times \frac{1}{3}, \text{ or } .33\frac{1}{3} = & \$1600 & \text{P.'s gain,} \\ \$2400 \quad\quad " & \$\ 800 & \text{S.'s} \quad " \\ \$1800 \quad\quad " & \$\ 600 & \text{C.'s} \quad " \end{array}\right\} \textit{Ans.}$$

Rule.—*Take such a part of each partner's capital as the whole gain or loss is of the whole capital; or*

Multiply each partner's capital by the gain or loss per cent.

EXAMPLES.

2. Messrs. Staples & Clark are in partnership ; S.'s capital is \$3000, C.'s \$2000 ; they have gained \$1500 ; what is each one's share ?

3. Messrs. Howland & Son shipped a cargo of goods, amounting to \$20,000, ⅔ of which belonged to H. The profits were \$4000 ; what was each one's share ?

4. The capital of a Railroad Company is \$2,000,000, the

annual earnings are \$400,000, the expenses \$240,000. I own 25 shares; what is my annual dividend?

5. A factory owned by three men was damaged by fire to the amount of \$3000 more than the insurance. S. owned $\frac{1}{3}$, R. $\frac{2}{5}$, and W. the remainder; what was each man's share of the loss?

6. A merchant having failed in business, owed J. Strong \$825, T. Williams \$700, S. Vernon \$1175. He can pay the three only \$900; what is each one's share?

Case II.

Art. 167.—*To find each partner's share of the gain or loss, when their capital has been used unequal portions of time.*

Example 7.—Mead, Rogers & Smith have furnished capital as follows: Mead, \$5000 for 2 months; Rogers, \$4000 for 4 months, and Smith \$3000 for 3 months. They have gained \$3500; what is each one's share?

Mead's capital,	$\$5000 \times 2 = \$10000$	for 1 month.
Rogers' "	$\$4000 \times 4 = \$16000$	"
Smith's "	$\$3000 \times 3 = \$\ 9000$	"
The whole "	$= \$35000$	"

The whole gain is $\frac{3500}{35000} = \frac{1}{10}$ of the sum of the products of each one's capital multiplied by the time it was used; therefore each partner's share is $\frac{1}{10}$, or 10 per cent. of his capital, multiplied by its time.

$$\left.\begin{array}{l}\text{Mead's share is } \frac{1}{10} \text{ of } \$10000 = \$1000 \\ \text{Rogers' share is } \frac{1}{10} \text{ of } \$16000 = \$1600 \\ \text{Smith's share is } \frac{1}{10} \text{ of } \$\ 9000 = \$\ 900\end{array}\right\} \textit{Ans.}$$

Rule.—*Multiply each partner's capital by the time it was used, and treating the product as his capital, proceed as in Case I.*

EXAMPLES.

8. Messrs. Hoyt & Lane formed a partnership as drovers; H. furnished \$3200 for 2 months, and L. \$2000 for 4 months. They gained \$1350; what was each one's share?

9. Two carpenters, Adams & Nelson, contracted to build a

house for $600; A. furnished 9 men for 100 days, and Nelson 12 men for 75 days; what was each one's share?

10. Three men performed a piece of work in 28 days altogether, for which they were paid $42. A. received $15, B. $12, and C. the remainder; how many days did each work?

11. Messrs. Todd & Rowe engaged in business with a capital of $3600. T.'s capital was in the business 4 months, and his share of the profits was $160; R.'s capital was in the business 6 months, and he received as his share $192; how much capital did each furnish?

Promiscuous Examples in Partnership.

1. Messrs. Tappan, Edwards & Kimball were partners. T. furnished $800 capital, E. $700, K. $1300; they lost $560; what was the loss of each?

2. A man bequeathed $8400 to his three sons, in proportion to their ages, 10, 14, and 18 years; how much did each receive?

3. Messrs. Hawley, King & Stebbins were partners. H. furnished $10,000 capital for 15 months, K. $12,000 for a year, and S. $15,000 for 9 months. They gained $9652.50; what was each man's share?

4. Messrs. Benedict & Coe were partners three years. B. furnished $6150 capital, C. $8100; B.'s share of the gain was $1250; what was the whole gain?

5. C. Jenkins commenced business with a capital of $2500; after 6 months he took into partnership A. Reed, with $3500 capital; 3 years afterwards their joint capital was doubled; how much had each gained?

6. I agreed to pay three men $72 for delivering to families, at different distances, 36 tons of coal. A drew 9 tons 6 miles, B 12 tons 4 miles, C the remainder, 2 miles; how much is due to each?

7. Messrs. Scott & Taylor were partners three years; S. furnished $\frac{2}{3}$ as much capital as T.; they gained $5000; what was each one's share?

8. Three drovers hired a pasture for \$36. A had 24 head of cattle, and paid \$12; B paid \$16, and C the remainder, as their portions; how many cattle did B and C each have in the pasture?

9. Three drovers had 500 sheep each, for which they hired a pasture and paid \$56, each agreeing to pay in proportion to the number of weeks his sheep were in the pasture. A paid \$14 for 2 weeks, B paid \$17½, and C \$24½; how long were B and C's sheep in the pasture?

10. A bankrupt owes \$18,000, and his property is worth \$3600; how much will a creditor receive, whom he owes \$1500?

11. Messrs. Paige, Cassidy & Warren are partners. P.'s capital, \$20,000, has been in the business 3 years; C.'s capital, \$15,000, 2 years 6 months; W.'s capital, \$12,000, 1 year 9 months; they have gained \$2962½; what is each man's share?

Art. 168.—Promiscuous Examples in the various applications of Percentage.

EXERCISE I.

1. At 2½ % commission, how much will an agent receive for selling goods amounting to \$920?

2. A commission merchant has received \$624 for the purchase of goods after deducting 4 % commission; what amount must he expend?

3. At 3 % commission, what amount of goods must be sold in a year to realize an income of \$2400?

4. What is the premium on an insurance policy of \$4500, at 1½ %?

5. For what must a house valued at \$5000, be insured at 2 %, to cover the cost of insurance?

6. What is the value of 15 shares of bank stock, at 3 % below par?

7. What will a broker receive for exchanging \$578 in bank notes, at 1¼ % discount?

8. What will a broker give for \$720 in bank-notes at $\frac{3}{8}$ % discount?

9. What is the interest of \$1500 for 4 years 3 months 6 days, at 6 %?

10. What is the amount of \$360 from June 14, 1863, to Sept. 28, 1865, at 7 %?

11. A note of \$300 was dated Jan. 1, 1863. Interest 6 %. Indorsed July 1, 1863, \$109. Jan. 1, 1864, \$100. What was due July 1, 1864?

12. A man has paid in 3 years, \$341.75 interest, at 5 %; what was the principal?

13. A man pays \$800 rent for a house valued at \$10,000; what per cent. interest does he pay?

14. What is the compound interest of \$600 for 2 years, 6 months, at 6 %?

15. What is the present worth of \$399.60, due in 1 year 10 months, at 6 % discount?

16. What is the bank discount on \$750, payable in one month?

17. What is the present worth of a note for \$360, payable in 1 month, at 6 % bank discount?

18. For what sum must a note at 6 % for 90 days, be given at a bank to obtain \$393.80?

19. A merchant sold goods amounting to \$300, and gained 20 %; what did he gain?

20. If silk cost \$1.80 a yard, for what must it be sold to gain 25 %?

21. A man paid \$75 for a wagon, and sold it for \$100; what per cent. did he gain?

22. A man sold a wagon for \$112, and gained 40 %, what did it cost?

23. A town is taxed \$6250 on its property valued at \$1,200,000, and there are 500 polls taxed 50 cts. each; what per cent is the tax on the property?

24. What is the duty, at 20 % ad valorem, on 80 bales of imported wool, each weighing 400 lbs., invoiced at 25 cts. per pound; tare 5 %?

25. Mc Rae & Oakley shipped goods in partnership. Mc Rae furnished \$5000; Oakley, \$3000; they gained \$2320; what was each one's share?

26. When gold is 130, what is \$100 in currency worth?

27. What will be the cost of a draft at sight on St. Louis for \$1000, at 1 % premium?

28. What must be paid for a bill of exchange on Liverpool, Eng., for £400, at 9 % premium?

EXERCISE II.

29. A man having \$8,000 lost 12½ % of it; how much had he left?

30. A steamboat is insured for \$30,000 at 2⅛ %; what is the premium?

31. What will 20 shares of the Central Railroad cost at 18¾ % advance?

32. A bank has failed whose circulation is \$75,000, and is able to pay only 66⅔ %; what amount can it pay?

33. What is the duty on 840 bags of coffee, each weighing 120 lbs. at 3 cts. a pound; tare 3 per cent.?

34. The expenses of a school district are for teacher's salary \$600; fuel, &c., \$37. The whole attendance has been 5600 days; what must a man pay for 80 days' attendance?

35. Bought 195 cords of wood at \$4.12½ a cord, and sold it for \$4.87½ a cord; what was the gain per cent.?

36. A gentleman sold a carriage for \$150, which was 75 % less than it cost; what did it cost?

37. If I pay \$960 for wheat, for what must I sell it to gain 25 per cent.?

38. What is the interest of \$691.20 from March 5, 1862, to Sept. 20, 1864, at 7 per cent.?

39. How much money at 6 % interest will yield a semi-annual income of \$650?

40. What is the interest of \$350, at 6 %, for 1 year 6 mos. 12 days? Also, the discount and bank discount of the same for the same time?

41. $87. 25 NEWBURGH, N. Y., Aug. 14, 1857.

Six months after date I promise to pay H. S. Barnes Eighty-seven $\frac{25}{100}$ dollars, value received, with interest.

ROBT. LIVINGSTON.

Indorsed, Dec. 8, 1858, $5.00. May 20, 1859, $45.50.

How much was due August 26, 1859?

42. What is the compound interest of $200 for 3 years at 7 per cent.?

43. What is the value of a draft on New Orleans for $1500, at 3 % discount?

44. When gold is 125, how much of it is equal to $500 in currency? how much currency is equal to $500 in gold?

45. For what amount will 2427.77\frac{7}{9}$ purchase a bill of exchange on Liverpool, at 9¼ per cent.?

46. Messrs. Taylor & Olmsted are partners in business. T. furnished $5000 capital for 6 months, O. $4000 for 5 months. They have gained $2500; what is each one's share?

EXERCISE III.

47. A merchant having shipped flour amounting to $1260, paid 2¼ % for insurance; what was the whole cost?

48. An auctioneer sold goods amounting to $956.20, at 2½ per cent. commission; how much did he receive?

49. A merchant paid a broker $3090 for a draft on Charleston, allowing him 3 % brokerage; what was the amount of the draft?

50. What is the amount due on a note of $391, dated Sept. 7, 1864, and payable Feb. 1, 1865, at 6 per cent.?

51. $1000. UTICA, May 1, 1862.

For value received I promise to pay H. Low & Co., or order, One Thousand Dollars, with interest at 7 %.

Indorsed:—Nov. 1, 1862, $150. March 13, 1863, $200.

What was due Sept. 19, 1863?

52. A man at his death bequeathed $8000 to an asylum, to be paid after it amounted to $10000, at 7 %; how long was the time?

53. What is the interest of £113 10s. for 1 year 6 months, at 6 per cent.?

54. What is the compound interest of $250 for 3 years, at 6 per cent.?

55. What is the present worth of $500.76 payable in 8 mos., at 6 per cent. per annum?

56. What is the present worth of the same by bank discount?

57. What is the difference between the discount and the interest of $1000 for 1 year, at 6 per cent.?

58. For what amount must a note for 90 days be given at a bank to obtain $600, at 7 %?

59. Shipped 5 loads of furniture, worth $1200, to Mobile, and paid 2½ % for insurance; what was the premium?

60. For what amount must the same furniture have been insured to have covered the whole loss if the ship had sunk?

61. If a man pays $90 premium at 2½ % for insurance, for what amount is he insured?

62. A gentleman bought a harness for $60, and afterwards sold it for 12 % less; how much did he lose?

63. Bought a firkin of butter for $26.63; for how much must it be sold to gain 20 %?

64. A man bought a farm at $85 an acre, and afterwards sold it for $100 an acre; what per cent. did he gain?

65. A gentleman sold his watch for $72, which was 28 per cent. less than it cost; what did it cost?

66. What is the duty, at 2 cents a pound, on 10 boxes of sugar, each weighing 125 lbs., allowing 6 lbs. a box for tare?

67. What is the duty on a cargo of coffee, invoiced at $3560, at 30 per cent.?

68. What is the amount of tax due from J. Carpenter, whose property is assessed at $1800, and who pays for three polls, the whole tax of the town being $3600; the number of polls 602, paying 75 cents each, and the amount of property assessed is $250,000?

69. How many shares of the Erie Railroad stock can be bought for $7500, when it is 37½ % below par?

70. When gold is $129\frac{1}{4}$, how much of it must be paid for \$750 in currency? How much currency, also, is equal to \$750 in gold?

71. What is the value in Memphis of a draft on New York for \$3200, at $2\frac{1}{2}$ % premium?

72. What must be paid for a bill of exchange on England for £300 10s., at $9\frac{1}{2}$ per cent. premium?

73. A bankrupt's debts amount to \$30,000; the property in his possession \$18,000; how much can be paid a creditor whom he owes \$1251.37½?

74. Messrs. Halsey & Coe are partners in business; H.'s capital is \$5000, and has been employed 3 years; C.'s capital is \$4500, but was not paid in till 4 months after H.'s; they have gained \$5400; what is each one's share?

EXERCISE IV.

75. The population of a city has increased 250 per cent. in 20 years, and now contains 175,000; what was its population 20 years ago?

76. A lawyer charges 4 % commission for collecting a debt of \$625.25; how much will he receive?

77. A country merchant has forwarded \$3681 to his agent in the city for the purchase of goods, after deducting $2\frac{1}{4}$ % commission; how much will be the commission?

78. What is the premium for insuring a mill valued at \$4620, at $1\frac{3}{4}$ per cent.?

79. For what sum must a house, valued at \$3800, be insured, at 2 % to cover the property and premium?

80. What is the value of 16 shares in a Manufacturing Company, at $12\frac{1}{2}$ % premium?

81. What must be paid for a draft for \$1275.25, at $\frac{7}{8}$ % premium?

82. What is the amount of \$1475.28 from March 29, 1866, to July 4, 1867, at 7 per cent.?

83. \$200. NEW HAVEN, July 18, 1861.

Three months after date I promise to pay J. Atwater, or order, Two Hundred Dollars, with interest, at 6 %, value received. R. CLARK.

Indorsed :—Jan. 18, 1864, $20. July 1, 1867, $60.
What was due Nov. 7, 1867 ?

84. A man wished to leave his daughter an income of $600 a year ; what principal would be required at 7 % interest ?

85. What is the amount, at compound interest, of $500 for 2 years, at 5 % ?

86. What is the present worth of $800, due in 4 years and 8 months, at 6 % ?

87. What is the discount of $800, due in 10 years, at 6 % ?

88. What is the bank discount of $800, due in 10 years, at 6 % ?

89. For what sum must a note, payable in 60 days, be given to obtain from a bank $800, at 6 % discount ?

90. Bought a span of horses for $400, and sold them at 25% profit; for what were they sold ?

91. Bought a span of horses for $400, and sold them for $450 ; what per cent. was gained ?

92. Sold a span of horses for $400, which was 25 % less than they cost ; what did they cost ?

93. Sold a span of horses for $400, which was 25 % more than they cost ; what did they cost ?

94. A man whose property was valued at $6240, and who paid for three polls at $1 each, was taxed $34.20 ; what per cent. tax did he pay ?

95. What is the duty, at 12 %, on 20 boxes of tobacco, each weighing 250 lbs., and costing 20 cents a pound ; tare $6\frac{1}{4}$ % ?

96. Three men bought a farm of 300 acres, and agreed to divide it according to the amount each one could pay. A paid $12,000, B $10,000, C $8,000 ; how many acres did each one have ?

97. When gold is 135, what is the value of $270 in currency? How much currency must be given for $270 in gold ?

98. For what amount can a bill of exchange on Liverpool, England, be purchased with $720, at 8 % premium ?

EXERCISE V.

99. If a city, with a population of 120,000, should increase 10 % a year, what would be its population after 10 years ?

100. An auctioneer having sold some goods, retained $108 and paid the owner $3492; what per cent. commission did he charge?

101. A broker received $2412 for the purchase of stocks, after deducting ½ per cent. brokerage; how many $100 shares did he buy?

102. What is the premium for the insurance of a factory, valued at $5400, at 2½ per cent.?

103. For what must a house, valued at $1987.50, be insured, at ⅜ %, to cover both the house and premium?

104. How many shares of bank stock, at 30 % premium, can be bought for $1300?

105. When gold is 130, how much of it can be bought for $650 in currency? How much currency is equal to $650 in gold?

106. What is the interest of $720.50 from Jan. 16, 1865, to July 31, 1865, at 7 %?

107. What is the discount, Jan. 16, 1865, of $720.50, due July 31, 1865?

108. What is the bank discount, Jan. 16, 1865, of $720.50, due July 31, 1865?

109. $500. NORWALK, July 25, 1865.

On demand, for value received, I promise Wm. Kellogg, or order, Five Hundred Dollars, with interest.

J. CARTER.

Indorsed:—Dec. 19, 1866, $62. Aug. 4, 1867, $48.

What was due Oct. 8, 1867?

110. What is the compound interest of $400, for 2 years 8 months, at 5 %?

111. If cloth cost $5.50 a yard, for what must it be sold to gain 20 per cent.?

112. If cloth cost $5.50 a yard, and is sold for $6, what per cent. is gained?

113. If cloth is sold for $6, and the gain is 25 %, what did it cost?

114. What is the duty on cutlery invoiced at $2400, at 30 %?

115. The tax levied on a town is $3440, the number of polls

is 1000, at \$1 each; the inventory of property is \$320,000; what is a man's tax whose property amounts to \$2400, and who pays for 2 polls?

116. What must be given for a bill of exchange on London for £600 12s., at $9\frac{1}{2}$ % premium?

117. A bankrupt owes \$17,000, and is able to pay only 9000; how much will a man lose whom he owes \$680?

118. Messrs. Carter, Rowe & Foster are partners in business. C. furnishes $\frac{1}{2}$ the capital, R. $\frac{1}{5}$, and F. the rest; they have gained \$3300: what is each one's share?

EXERCISE VI.

119. A merchant having a capital of \$6480, lost 25 % of it; how much had he left?

120. A farmer having purchased some land, spent 10 % of the price in improvements, and then found that the whole cost was \$8800; what did he pay for the land?

121. A merchant having lost 10 % of his capital, had \$1800 left; how much had he at first?

122. A merchant having \$1800 capital, lost \$300; what per cent. was his loss?

123. Bought 100 tons of iron for \$3865, and sold it at $3\frac{1}{8}$ % less than cost; what was the selling price?

124. Bought 500 bushels of wheat for \$1300, and sold it for \$1560; what per cent. was the gain?

125. Sold 500 bushels of corn, and gained \$100, which was 20 % of the cost; what was paid for it?

126. Borrowed, Jan. 1, in New York, \$1250, and returned it with interest the following Sept. 15; what was the amount returned?

127. What is the interest of \$340 from July 20, 1860, to July 2, 1861, at 7 per cent.?

128. What is the interest of £760 5s. 6d., at 6 %, for 2 years 4 months?

129. What is the amount of \$850 for 5 years 6 months 3 days, at 6 %?

130. \$500. HARTFORD, Jan. 16, 1855.

On demand, I promise to pay A. Terry, or order, Five Hundred Dollars, for value received, with interest. J. R. DAY.

Indorsed :—April 1, 1856, \$50. July 16, 1857, \$400. Sept. 1, 1858, \$60.

What was due Nov. 16, 1858 ?

131. A gentleman has a son 15 years old, and he wishes to invest for him such a sum as will amount to \$10,000 when he is of age ; what sum must be invested, at 7 per cent.?

132. A young man has received a legacy which yields a semi-annual income of \$750, at 6 per cent.; what is the amount of the legacy ?

133. A man left his son \$6000, possession to be given after it amounts to \$9000, at 6 per cent.; how long must the son wait for it ?

134. A young man has a legacy of \$622.75, to be paid in 3½ years, but he wishes to have it immediately ; what is it worth at 5 % discount ?

135. What is the present value of a note for \$900, payable in 6 months, at 6 per cent. bank discount ?

136. What is the present value of a note for \$900, payable in 6 months, with interest, at 6 %, provided the discount at a bank is 7 per cent.?

137. For what amount must a note, payable in 60 days, be given to obtain from a bank \$400, at 6 % ?

138. An auctioneer receives \$112.50 for selling goods, at 2% commission ; what was the amount sold ?

139. A commission merchant has received \$1656 for the purchase of goods, after deducting 3½ % commission ; what is the amount of the goods to be purchased ?

140. What is the value of 50 shares of the Illinois Central Railroad, at a premium of 9 per cent.?

141. How many shares of stock, 10 % below par, can be bought for \$9000 ?

142. When gold is 131, how much of it can be bought for \$360 in currency ? How much currency is equal to \$360 in gold ?

143. A factory having been insured 7 years for $21,000, at 1¾ %, was destroyed by fire? what was the actual loss to the Insurance Company?

144. A certain town is taxed $2328, and pays 3 % for collecting it; the property is valued at $419,568, and there are 300 polls taxed $1 each; what is H. Scott's tax, whose property is valued at $3100?

145. For what amount can a bill of exchange on England be bought for $3633,635, at 9 % premium?

EQUATION OF PAYMENTS.

Art. 169.—Equation of Payments is finding the time when several payments, due at different times, may be made at once, or the balance of an account may be paid, without loss to either debtor or creditor. The time for such payment is called *equated time.*

An **account current** is a record of what one person is debtor or creditor to another.

CASE I.

Art. 170.—*When the payments are due at different times to find the equated time.*

EXAMPLE 1.—I owe J. Brush & Co. $400, payable in 3 mos., $300 in 4 mos., and $200 in 6 months; when should I pay the whole debt at once?

Process.—$400 for 3 months = $1200 for 1 month.
$300 for 4 " = $1200 "
$200 for 6 " = $1200 "
$900 for — " = $3600 "

Now $3600 for 1 month = $900 for as many months as it contains times $900. $900)$3600 *Ans.* 4 mos.

When the times of different payments are reckoned from different dates, any date may be assumed from which to reckon them and the equated time. It is more convenient to assume the date of the earliest payment.

Ex. 2. What is the equated time of the following account?

T. A. Scott, Dr.

To Suydam & Jackson:

1860.	Aug. 10.	Mdse. (3 mos. credit)	$300
"	Sept. 15,	" (6 mos.)	$400
1861.	Jan. 1,	Cash	$500

Process.—These different sums will be due—

1860.	Nov. 10,	$\$300 \times 0$ days $= \$0$	
1861.	Mar. 15,	$\$400 \times 124$ " $= \$49600$	for 1 day.
"	Jan. 1,	$\$500 \times 51$ " $= \$25500$	"
		$\overline{\$1200} \times$ — " $=) \overline{\$75100}$	"
		Ans. 63 days.	

Rule.—*Multiply each payment by its time, and divide the sum of the products by the sum of the payments.*

This rule is according to bank discount.

If the date is required, reckon the equated time from the given or assumed date.

Count ½ day or more as 1 day.

Cash payments have no products when the date is given, but must be included in the sum of the payments.

EXAMPLES.

3. A merchant buys goods, pays $200 at the time, and agrees to pay $200 in 4 months, and $200 in 8 months; what is the equated time of payment?

4. A merchant bought goods on 4 months' credit, as follows: April 10, $200; May 15, $160; June 1, $440; what is the equated time of payment?

5. A man borrowed $500, and agreed to pay $100 in 2 mos., $200 in 4 mos., and the balance in 6 mos.; what is the equated time?

6. The following bills of goods were bought on six months' credit: Aug. 5, $62.50; Aug. 11, $24.50; Sept. 10, $32.60; Oct. 15, $50. If a note payable in 6 months was given for the whole amount, when ought it to have been dated?

7. A man bought a farm for $8000, and agreed to pay $2000 at the time, and the rest in three annual payments; what is the equated time of payment?

8. The following bills of goods have been bought on 60 days credit; May 1, \$150; June 16, \$200; July 20, \$320; Sept. 1, \$300; what is the equated time of payment?

9. A man owes \$1600, ¼ of which is now due; ½ of it in 4 months; ⅛ in 6 months, and the rest in 8 months; what is the equated time of payment?

10. A merchant has bought goods as follows: Aug. 10, \$160 on 60 days' credit; Sept. 1, \$250, 90 days' credit; Oct. 12, \$300, 60 days; Nov. 10, \$200, 90 days; what is the equated time of payment?

11. A merchant has sold goods amounting to \$1200, of which he received \$300 at the time; \$300 was to be paid in 3 months; \$300 in 6 months, and the remainder in 9 months; what is the equated time of payment?

12. Sold goods as follows: Sept. 5, \$500 on 2 months' credit; Oct. 10, \$400, 3 months; Nov. 15, \$600, 3 months; Dec. 1, \$500, 1 month; what is the equated time of payment?

Case II.

Art. 171.—*To find the equated time of the balance of a debt, when partial payments have been made before it is due.*

Ex. 13.—J. Rowe owed me \$500 payable in 6 mos., but at the end of 3 months he paid me \$100, and at the end of another month \$200; how long may the balance remain unpaid?

Process.—\$100 for 3 months before due = \$300 for 1 month.
\$200 for 2 months before due = \$400 "
\$300 for — " " " = \$700 "

Therefore the balance (\$500—\$300) \$200 may remain unpaid as many months as it is contained times in \$700. \$200)\$700

Ans. 3½ mos.

Rule.—*Multiply each partial payment by the time it was made before it was due, and divide the sum of the products by the balance unpaid.*

EXAMPLES.

14. J. King promised to pay \$800 in 10 months. At the end of 4 months he paid \$200, and after 3 more months \$100; how long may he wait before paying the balance?

15. A merchant owed $1000, payable in 6 months. At the end of two months he paid $200, and at the end of three more months $300; how long may he leave the balance unpaid?

16. A man gave his note for $1200, payable in 6 months; at the end of the first, third and fifth months he paid $100 each time; how long may he keep the balance?

17. A man bought a house for $2400, payable in 2 years, but at the end of 1 year he paid ½ of it; how long after the whole was due may he wait before paying the balance?

CASE III.

Art. 172.—*To find the equated time of an account current.*

EXAMPLE 18.—

DR. J. H. MILLER. CR.

1867.			1867.		
April 1	To Mdse.	$700	July 11	By Cash	$200
“ 16	“	200	Aug. 1	“	300
May 11	“	100	Sept. 21	“	100
June 16	Cash	400	Oct. 1	Mdse.	200

Process.—First find the equated time of each side.

Dr. $700×0 days =
 200×15 “ = $3000 for 1 d.
 100×41 “ = 4100 “
 400×77 “ = 30800 “
 1400)37900 “
 From April 1, 27 days.
Dr. April 28, $1400.

Cr. $200×0 days =
 300×20 “ = $6000 for 1 d.
 100×72 “ = 7200 “
 200×82 “ = 16400 “
 $800)$29600 “
 From July 11, 37
Cr. Aug. 17, $800.

Difference between April 28and August 17 = 111 days.

If the account is balanced April 28, credit is given for $800, 111 days before it is paid, which would be a loss to the creditor. Therefore the balance, ($1400—$800) $600, is due as many days before April 28 as it is contained times in ($800×111 days) $88800, or 148 days. Hence the balance was due Dec. 2, 1866.

RULE.—*Find the equated time of each side; multiply the amount of the smaller side by the number of days between the two dates of equated time, and divide the product by the balance; the quotient will be the number of days to be* ADDED *to* *the equated time of the larger side when its amount becomes due* LAST, *but* SUBTRACTED *from it when it becomes due first.*

The *cash value*, or true balance, at any time of settlement, is found by adding the interest up to the time of settlement, when the balance is due beforehand, and subtracting it when due afterwards.

EXAMPLES.

19. What must be the date of a note to balance the following account?

DR. J. WILSON. CR.

1867.					1867.			
May	1	To Mdse.	$900		April	1	By Cash	$200
"	16	"	700		"	16	"	400
June	1	Mdse.	400		May	15	Draft	500

20. What must be the date of a note to balance the following account?

DR. G. S. WOOD. CR.

1866.					1866.			
March	1	To Mdse. (4 m.)	$1000		June	16	By Cash	$500
April	10	" (3 m.)	800		July	10	"	400
June	11	"	600		"		Draft (10 d.)	600

CASE IV.

Art. 173.—*To find the true balance of an account bearing interest when the time of settlement is given.*

EXAMPLE 21.—What is the true balance of the following account, at the time of settlement, August 20, 1868, allowing 60 days' credit to each charge, and 7 per cent. interest?

DR. THOS. GOODWIN. CR.

1868.					1868.			
Jan.	2	To Mdse.	$200		Feb.	20	By Mdse.	$100
April	20	"	400		May	10	"	300

Process.--

Dr. Due Mar. 2, $200.			Cr. Due April 20, $102.		
Int. till Aug. 20,		AM'T.	Int. till Aug. 20,		AM'T.
(172 days)....	6.60	$206.61	(121 days)....	$2.36	$102.32
Due June 20....$400			Due July 10.... 300		
Int. till Aug. 20,			Int. till Aug. 20,		
(61 days).....	4.66	404.66	(41 days).....	2.37	302.37
		$611.27			$404.69
		404.69			
True balance, Aug. 20..		$206.58			

RULE.—*Find the interest on each entry up to the time of settlement. Add the several amounts on each side, and the difference between the sums will be the true balance.*

EXAMPLE 22.

DR.		HOWE & STEBBINS.			CR.
1863.			1863.		
July 1	To Mdse.	$250	July 21	By Consignm't	$130
Aug. 13	"	200	Oct. 1	Draft	160
Sept. 24	"	550	" 10	"	210

What is the balance due Jan. 1, 1864, allowing 6 % interest?

REDUCTION OF CURRENCIES.

Art. 174.—Reduction of Currencies is changing one kind of Money into another without altering its value.

State Currencies.—Sterling Money was formerly the currency of this country before its separation from England, and is still used to some extent. But its value is not the same as in England; nor the same in all the States, because its paper bills depreciated more in some than in others. Hence arose the following currencies :

$$\text{N. E. Currency in} \left\{ \begin{array}{l} \text{N. E. States,} \\ \text{Virginia,} \\ \text{Kentucky,} \\ \text{Tennessee,} \end{array} \right\} \$1 = 6\text{s.} = £\tfrac{3}{10}.$$

N. Y. Currency in	New York, Ohio, N. Carolina,	$\$1 = 8s. = £\frac{2}{5}$.
Penn. Currency in	Pennsylvania, New Jersey, Delaware, Maryland,	$\$1 = 7s.\ 6d. = £\frac{3}{8}$.
Geor. Currency in	Georgia, S. Carolina,	$\$1 = 4s.\ 8d. = £\frac{7}{30}$.

Hence, also, in

New England Currency . . £1=$\$\frac{10}{3}$=$3.33⅓ ; 1s.=16⅔ cts.
New York Currency . . . . . £1=$\$\frac{5}{2}$ =$2.50 ; 1s.=12½ "
Pennsylvania Currency . . £1=$\$\frac{8}{3}$ =$2.66⅔ ; 1s.=13⅓ "
Georgia Currency . . . . £1=$\$\frac{30}{7}$=$4.28$\frac{4}{7}$; 1s.=22$\frac{2}{9}$ "

Art. 175.—*To reduce U. S. Money to State Currencies.*

Example 1.—What is the value of $836.50 in Pennsylvania currency?

Process.—$1=(7s. 6d.) 7½s., or £$\frac{3}{8}$; therefore $836.50=7½ times as many shillings, or $\frac{3}{8}$ times as many pounds.

$836.50
7½
418.25
5855.50
20)6273.75$_{12}$
Ans. £313.13s. 9.00d.

Rule.—*Multiply the given sum by the value of* $1 *in the required currency.*

EXAMPLES.

2. Reduce $315.4375 to New York currency.
3. Reduce $490.38 to New England currency.
4. Reduce $325.00 to Pennsylvania currency.
5. Reduce $245.00 to Georgia currency.

Art. 176.—*To reduce State Currencies to U. S. Money.*

Example 6.—What is the value of £312 18s. 9d.?

Process.—7½s.=$1, therefore £312 18s. 9d. reduced to shillings, and divided by 7½=$834.50.

£312 18s. 9d.=.75s.
20
7½)6258.75
Ans., $834.50

Rule.—*Divide the given sum reduced to shillings, by the number of shillings in* $1 ; *or reduced to pounds, by the fraction of a pound, equal to* $1.

EXAMPLES.

7. Reduce £90 6s. New York currency to U. S. money.

8. Reduce £120 7s. 6d. Pennsylvania currency to U. S. money.

9. Reduce £35 14s. 9d. New England currency to U. S. money.

10. Reduce £57 12s. 6d. Georgia currency to U. S. money.

Art. 177.—Table of the Principal Foreign Coins and their Value in U. S. Money.

Austria, Florin of....	$0.48½	Italy, Dollar of Rome,	$1.05
Canada, Pound Ster..	4.00	" Ducat of Naples,	.80
China, Tael	1.48	" Lira of Sardinia,	$.18\frac{6}{10}$
Denmark, Dol., (sp.).	1.05	" Lira of Tuscany,	.16
England, Pound Ster.	4.84	" Livre of Genoa,	$.18\frac{6}{10}$
" Crown.....	1.06	Mexico, Doubloon...	15.60
France, Franc.......	$.18\frac{6}{10}$	Portugal, Milrea	1.12
" Livre	.18	Prussia, Florin......	.22¾
Germany, Florin	.40	Russia, Ruble.......	.75
" Rix Dollar,	.69	Spain, Real Plate....	.10
India, Pagoda.......	1.94	Switzerland, Livre...	.27
" Rupee........	.44½		

To reduce Sterling Money of England to U. S. Money, multiply pounds and decimal of a pound by $4.84. To reduce U. S. Money to pounds sterling divide by 4.84. Reduce the decimal to shillings, &c.

EXAMPLES.

11. Reduce 1500 francs to U. S. Money.

12. Reduce 3000 livres of Switzerland to U. S. Money.

13. Reduce £140 15s. 9d. to U. S. Money.

14. Reduce 100 dollars (specie) of Denmark, to U. S. Money.

15. Reduce 500 rupees of India to U. S. Money.

16. Reduce 400 rubles of Russia to U. S. Money.

17. Reduce $705.6115 to Pounds Sterling, &c.

Promiscuous Examples.

1. Reduce £89 18s. N. York Currency to U. S. Money.
2. Reduce $488.38 to New England Currency.
3. Reduce £36 9d. N. England Currency to U. S. Money.
4. Reduce $314.43¾ to New York Currency.
5. Reduce 500 francs to U. S. Money.
6. Reduce $810 to Swiss livres.
7. Reduce £60 15s. Pennsylvania Currency to U. S. Money.
8. Reduce $417.25 to Pennsylvania Currency.
9. Reduce 400 India rupees to U. S. Money.
10. Reduce £114 16s. Georgia Currency to U. S. Money.
11. Reduce $954.12½ to Georgia Currency.

RATIO.

Art. 178.—Ratio denotes the magnitude of one number compared with another of the same kind.

Arithmetical ratio denotes the difference between two numbers; **geometrical** ratio the number of times one contains the other. The word *ratio*, used alone, means the latter.

Ratio equals the quotient of one number divided by another, and is usually expressed by (:) written between them; thus the ratio of 12 to 3 is written 12 : 3.

The two numbers are called the *terms* of the ratio; written together they are called a *couplet*, of which the first is called the *antecedent*, and the second the *consequent*.

Ratio respects only things of the same name, or which may be reduced to the same; thus the ratio of 12 inches, or 1 foot to 6 inches is 2, but there is no ratio between 12 inches or 1 foot and 6 cents.

Art. 179.—A **Compound** ratio is the product of two or more simple ratios; thus the ratio of 2 : 6. and 3 : 9. make the compound ratio $2 \times 3 : 6 \times 9$, or 6 : 54.

EXERCISES.

Express the ratios of 4 to 8, 3 to 9, 5 to 15, 7 to 14, 12 to 24.

What is the ratio of

72 : 12	$4 : $12	50 lbs. : 1 cwt.
6 : 24	10 lb. : 30 lb.	3 qrs. : 4 yds.
40 : 10	12 gal. : 3 gal.	24 qts. : 3 gals.
5 : 25	8 rods : 32 rods	£1 : 2s. 6d.

A ratio may be reduced to its lowest terms by dividing both terms by their greatest common divisor; thus, 5 : 25 = 1 : 5.

PROPORTION.

Art. 180.—Proportion is an equality of ratios. The ratio of 4 : 2 = 12 : 6; hence 4 : 2 is in proportion as 12 : 6.

Proportion is usually expressed by (::) four dots placed between two ratios; thus, 6 : 3 :: 24 : 12; which is read 6 is to 3 as 24 to 12. The first and last terms are called the *extremes;* the second and third the *means.*

When the consequent of the first ratio is the same as the antecedent of the next, it is called a *mean proportional;* thus, 8 : 4 :: 4 : 2; 4 is a mean proportional.

Art. 181.—Proportion is often the most convenient method of solving arithmetical questions, when the required number is evidently as many times greater or less than another of the same kind, as is expressed by the ratio of two other numbers on which they respectively depend; thus,

Prices depending on quantities;

Times, &c., depending on distances, &c.,

are evidently in proportion.

1 lb. of coffee : 10 lbs. of coffee :: $0.45 : $4.50;

12 men : 4 men :: 6 days : 2 days (in doing a certain work.)

The proportion is called *direct* when a greater number of one kind requires a greater number of another kind, or less requires less; but *indirect* or *inverse* when a greater number requires a less, or a less requires a greater; thus the first proportion above is direct, because the greater quantity (10 lbs.) requires the greater price ($4 50;) but the other is inverse, because the greater number of men (12) requires the less number of days (2,) to do a certain work.

In every proportion the product of the extremes is equal to the product of the means; thus, $3:4::6:8$, $3 \times 8 = 4 \times 6$. Hence,

The product of the means divided by either extreme gives the other; and

The product of the extremes divided by either mean gives the other.

EXAMPLE 1.—If 5 yards of linen cost $6.00, what will 20 yards of linen cost?

Process.—This is an example in proportion, because it is evident that the *prices* must be in proportion to the given *numbers of yards*. Since, too, the price of 20 yards is unknown, it is usually considered as the fourth term of the proportion, which is to be found, and therefore the corresponding number or given price will be the third term. It is evident, also, that 20 yards will cost more than 5 yards; therefore, 20 yards must be the second term, and 5 yards the first term, in order to make the ratios equal, or a proportion. Hence,

5 yards : 20 : : $6 : *Ans*.

Reducing the first ratio to its lowest terms,

$$1 \text{ yard} : 4 \text{ yards} :: \$6 : Ans. = \frac{\$6 \times 4}{1} = \$24.$$

RULE.—*Consider the answer the fourth term, and make the given number of the same name or kind the third term; then, if the answer will evidently be greater than the third term, make the greater of the other two numbers the second term; but, if less, make the smaller number the second term. The remaining number will be the first term.*

Reduce the left hand ratio to its lowest terms, or cancel any factor common to the first term and either the second or third. Then multiply the second and third terms together, and divide by the first.

Compound Numbers must be reduced to the lowest denomination mentioned, and the first and second terms must be of the same name as well as kind.

EXAMPLES.

2. If 5 yards of cloth cost \$35, how much will 20 yds. cost ?

3. If 12 tons of coal cost \$72, how much will 3 tons cost ?

4. If 7 lbs. of coffee cost \$2.33$\frac{1}{3}$, how much will 4 lbs. cost ?

5. If 3 bbls. of flour cost \$22.50, how much will 50 barrels cost ?

6. If 44 lbs. of tea cost \$33.00, how much will 11 lbs. cost ?

7. If 10 acres of land produce 250 bushels of corn, how much will 45 acres produce ?

8. If a man travel 300 miles in 12 days, how far can he travel in 4 days ?

9. If 1 lb. 6 oz. of silver is worth \$12.50, what are 3 oz. 10 pwts. worth ?

10. If 124 men can build a mill-dam in 60 days, in how many days could 248 men build it ?

11. If a man can walk 3$\frac{1}{4}$ miles in 1 hour, how long will it take him to walk 12 miles 120 rods ?

12. How many yards of cambric, $\frac{3}{4}$ yd. wide, will it take to line 20 yards of cloth, 1$\frac{1}{4}$ yds. wide ?

13. If a quantity of provisions will last 315 men 56 days, how many days will the same last 45 men ?

14. If a quantity of provisions will last 316 men 56 days, how many men will it feed 14 days ?

15. If $\frac{3}{4}$ of a yard of cloth cost \$$\frac{9}{11}$, what will 3$\frac{7}{8}$ yds. cost ?

Art. 182.—Proportion may be used in solving questions under many of the preceding rules.

16. *Percentage.*—What is 7 % of \$256 ?

By Proportion, \$100 : \$256 : : 7 % : \$17.92, *Ans.*

17. What per cent. of \$400 is \$24 ?

\$400 : \$100 : : \$24 : 6 %, *Ans.*

18. *Problems in Interest*—How long must \$200 be at 6 % interest to gain \$36 ?

Interest 1 year is 12, therefore \$12 : \$36 : 1 year : 3 yrs., *Ans.*

19. If the interest of $600 for 1 year 8 mos. is $60, what is the rate per cent.?

Interest at 1 % is $10, therefore 10 : 60 : : 1 : 6 %, *Ans.*

20. What principal at 6 per cent. will yield $4.52 interest in 1 year 4 months be ?

Interest of $1 is 8 cents, therefore .08 : 4.52 : : $1 : $56.50, *Ans.*

21. *Discount.*—What is the present worth and discount of $306, due in 4 mos., at 6 % discount ?

The present worth of $1.02 is $1.00, therefore $1.02 : 100 : : $306 : $300, *Ans.*
The discount of $1.02 is $.02, therefore $1.02 : $.02 : : $306 : $6, *Ans.*

22. *Profit and Loss.*—Sold a horse for $150, and gained 25 % ; what did the horse cost ?

$125 : $100 : : $150 : $120, *Ans.*

23. *Partnership.*—Messrs. Platt, Wood & Torrey are in partnership ; P.'s capital is $5000, W.'s $4000, T.'s $3000 ; they have gained $3600 ; what is each one's share ?

The whole capital $12,000 : $5000 : : $3600 : $1500, P.'s share, &c.

Solve the following questions by proportion.

24. What is 6 % of $750 ?

25. What per cent. of $500 is $60 ?

26. What is $650 in currency worth when gold is 130 ?

27. What is the interest of $150 for 4 years 2 mos., at 6 % ?

28. What principal, at 6 %, will gain $60 in 4 years ?

29. What is the present worth of $1350, due in 5 years 10 mos., at 6 % ?

30. What was the cost of a yard of cloth, which being sold for $4.25 occasioned a loss of 20 per cent. ?

31. A bankrupt's debts amount to $9600, and he has property amounting to $7500 ; how much can he pay a creditor whom he owes $1600 ?

32. Messrs. Ray & Stearns are in partnership ; R.'s capital is $3000, S.'s capital $2000 ; they have gained $2500 ; what is each one's share ?

33. For what amount must a note for sixty days be given, to obtain from a bank $1800, at 6 % discount ?

34. What is the interest of $225 for 2 years 7 mos., at 7 per cent.?

COMPOUND PROPORTION.

Art. 183.—Compound Proportion is the equality of a compound and a simple ratio. It consists of two or more simple proportions.

EXAMPLE 1.—If 7 men can cut 42 acres of wheat in 3 days, working 10 hours a day, how many acres can 14 men cut in 4 days, working 9 hours a day?

Process.—Since the answer will be acres, 42 acres will be the third term. The other numbers, two of the same kind make the first and second terms of as many simple proportions, each couplet to be arranged as if the answer depended upon it alone; thus,

$$\begin{array}{l} \cancel{7} : \overset{2}{\cancel{14}} :: 42 \\ \cancel{3} : 4 \\ 5\ \cancel{10} : \cancel{9}\ 3 \end{array} \qquad \cancel{7} \times \cancel{3} \times \overset{5}{\cancel{10}})42 \times \overset{2}{\cancel{14}} \times 4 \times \overset{3}{\cancel{9}} \text{ by cancellation.}$$

$$5)\underline{(42 \times 4 \times 3)504}$$

Ans. $100\frac{4}{5}$ acres.

RULE.—*Make that number which is of the same name as the answer the third term. Arrange the other numbers in pairs of the same kind, as the first and second terms of as many simple proportions as there are pairs, and each couplet as if the answer depended on it alone.*

Cancel as in simple proportion, or reduce the ratios and divide the product of all the second and third terms by the product of all the first terms.

EXAMPLES.

2. If 6 men can dig a ditch 36 rods long in 8 days, how many men will it require to dig a ditch 72 rods long in 4 days?

3. If 90 lbs. of beef will supply 12 men 20 days, how long will 144 pounds last 36 men?

4. If 6 horses eat 48 bushels of oats in 12 days, how many horses will eat 96 bushels in 8 days?

5. If $100 gain $7 in 12 months, how long will it take $600 to gain $21?

6. If 6 men can dig 4 acres of potatoes in 10 days, in how many days will 18 men dig 24 acres?

7. If 8 men can dig a cellar 40 feet long, 27 feet wide, and 8 feet deep, in 12 days, how many men will dig a cellar 50 feet long, 36 feet wide and 9 feet deep in the same time?

CONJOINED PROPORTION.

Art. 184.—Conjoined Proportion is a proportion in which each antecedent of a compound ratio is equal in value to its consequent.

Example.—If 10 bushels of corn will pay for 5 loads of wood, and 20 loads of wood for 4 tons of hay, how many bushels of corn will pay for 25 tons of hay?

Process.—Since 10 bushels corn = 5 loads of wood, 20 loads of wood = $\frac{20}{5}$ of 10 bushels corn; and since it also equals 4 tons of hay, 25 tons of hay = $\frac{25}{4}$ of $\frac{20}{5}$ of 10 bushels corn = *Ans.* 250 bush.

10 bushels corn = 5 loads of wood,
20 loads wood = 4 tons of hay,
25 tons of hay =

$$\frac{\overset{2}{\cancel{10}}\times\overset{5}{\cancel{20}}\times 25}{\cancel{5}\times\cancel{4}}=250, \textit{Ans.}$$

Rule.—*Write the equal quantities, with the sign of equality between them, under one another, in pairs, so that each consequent shall be of the same name as the next antecedent, placing the odd quantity on the opposite side of that of the same name.*

Cancel the factors common to both sides, and divide the product of all the quantities, on the side containing the odd one, by the product of all on the other side.

Art. 185.—Arbitration of Exchange is finding the rate of exchange between two countries, through intermediate exchanges between other countries. This is done by Conjoined Proportion.

EXAMPLES.

2. If $9 in the United States are equal to 12 rubles at St. Petersburg, and 8 rubles in St. Petersburg are equal to 15 florins in Frankfort, and 9 florins in Frankfort are equal to 20 francs in Paris, how many dollars in the United States are equal to 75 francs in Paris?

3. If 25 yards of cloth are worth 22 barrels of flour, 11 bbls. of flour are worth 150 pounds of wool, and 15 lbs. of wool are worth 18 lbs. of butter, how many pounds of butter will pay for 10 yards of cloth?

ALLIGATION.

Art. 186.—Alligation is finding the prices or quantities of mixtures. It is Medial or Alternate.

The word *alligation* means connecting together, and is used in arithmetic because the prices of mixtures are connected one with another.

Art. 187.—Alligation Medial is finding the price of a mixture when the quantity and price of each ingredient are given.

Example 1.—A farmer has mixed 20 bushels of corn, at 75 cents, 30 bu. barley at 60 cts., 40 bu. oats at 50, and 10 bu. rye at $1; what is a bushel of the mixture worth?

Process.—20 bushels	corn	×	$.75	=	$15.00	
30 "	barley	×	60	=	18.00	
40 "	oats	×	50	=	20.00	
10 "	rye	×	1.00	=	10.00	
100 bu. of mixture				=	53.00	
1 "				=		53 cts., *Ans.*

Rule.—*Divide the price of the whole mixture by the whole quantity.*

Ex. 2. A grocer mixed 8 lbs. of tea at 75 cents, 12 lbs. at 60 cts., 15 lbs. at 50 cts., and 20 lbs. at 40 cts.; what was a pound of the mixture worth?

Art. 188.—Alligation Alternate is finding the quantities of different ingredients whose prices are known, to form a mixture worth a certain price.

Ex. 3. A grocer wishes to make a mixture of tea worth 56 cts. He has one kind at 40 cts. a pound, another 50 cts., another 60 cts., and another 75 cts.; how much of each may he take?

$$56\left\{\begin{array}{lr} 40 & 19 \\ 50 & 4 \\ 60 & 6 \\ 75 & 16 \end{array}\right.$$

The whole mixture=40 lbs.

Process.—Since the gain and loss must be equal, we connect a less price than that of the mixture with one greater. On every pound, at 40 cents, there is gained 16 cts., and on every pound at 75 cts. there is lost 19 cents. Therefore, since the gain is to the loss as 16 to 19, the quantities must be as 19 to 16. For the same reason the other ingredients at 50 and 60 must be as 4 to 6. Hence the mixture must consist of 19 lbs. at 40 cents, 4 lbs. at 50 cts., 6 lbs. at 60 cts., and 16 lbs. at 75 cts.

These relative quantities are found by writing the difference between the price of the mixture and that of each ingredient opposite the one with which it is connected.

Art. 189.—*The quantity of one of the ingredients is sometimes given.*

In the above example, let the quantity at 75 cts. be 8 lbs. Then since 8 is only ½ of 16, only ½ of the other quantities must be taken to form the mixture.

Art. 190.—*Again, the quantity of the whole mixture may be given.*

In the same example, let the quantity of the mixture be 100 lbs. instead of 40, as found. Then, since $100 = 2\frac{1}{2}$ times 40, the quantity of each ingredient as first found must be multiplied by $2\frac{1}{2}$.

Rule.—*Write the prices under one another, and the mean price on the left. Connect each price less than that of the mixture with one greater, and each greater with one less. Write the difference between the price of the mixture and that of each ingredient opposite the price with which the latter is connected. The relative quantity of each ingredient will thus be found opposite its price.*

If one of the quantities, or the whole quantity, is given, and is greater or less than that found, all the others must be increased or diminished in the same proportion.

EXAMPLES.

Ex. 4. A grocer mixed different kinds of sugar at 10, 13 and 16 cents a pound, so that he could sell the mixture for 12 cts. a pound; how much of each kind did he take?

5. A farmer mixed 10 bushels of wheat at $1.40, with rye at 96 cents, corn at 72 cts., and oats at 60 cts., so that he could sell the mixture at 76 cts. a bushel; how much of each did he take?

6. How much water must be mixed with wine, at 90 cents a gallon, so that there may be 100 gallons worth 60 cts. a gal.?

7. A grocer has different kinds of sugar at 12, 11, 9, and 8 cents a pound; how may he mix them so that he can sell the mixture at 10 cents a pound?

8. How many bushels of corn at $1, and oats at 60 cents a bushel, must be mixed with 20 bushels of rye at $1.30, to make the mixture worth 82 cents?

9. A wine merchant mixed different kinds of wine at 62½, 87½ and 112½ cents a gallcn, with water, so that the mixture was worth 75 cts.; how much of each did he take?

10. A grocer having sugar at 8, 16 and 24 cents a pound, made a mixture of 240 lbs. worth 20 cts. a pound; how many pounds of each did he take?

INVOLUTION.

Art. 191.—Involution is multiplying a number by itself. The product is called its **power.**

$3 \times 3 = 9$; $3 \times 3 \times 3 = 27$; 9 and 27 are powers of 3.

The different powers are distinguished as the *first, second, third, &c.*; and are expressed by small figures above the numbers at the right, called an *index* (sing.) *indices* (plu.,) the index of the first power being usually omitted.

The first power of 3 is written 3
The second power of 3 is written $3^2=3\times3$
The third power of 3 is written $3^3=3\times3\times3$

The second power is usually called the *Square;* the third power the *Cube.*

RULE.—*To involve a number, or find any power of it, multiply the number by itself one less times than the name of the power denotes.*

There is one less multiplication than the times the number is used as a factor. In $3\times3=9$, there is one multiplication, but 3 is used twice as a factor.

Powers already found, when multiplied together, produce the power denoted by the *sum* of their indices; $3^2\times3^3$ or $9\times27=3^5$ or 243.

EXAMPLES.

What are the squares or second powers of 1, 3, 5, 7, 9, 11, 13, 25?

What are the cubes or third powers of 2, 4, 6, 8, 10, 12?

What are the squares, cubes, and fourth powers of 14, 20, 27, 36?

What are the squares and cubes of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{1}{10}$, .5, 1.2, .05, 1.02?

EVOLUTION.

Art. 192.—Evolution, the opposite of Involution, is finding a number which, multiplied into itself, will produce the given number. The number found is called the **Root** of the other, and is one of its equal factors:

$9=3\times3$; $27=3\times3\times3$; 3 is the root of 9 or 27.

The root of a square is called the *square* root; of a cube, the cube root; of a fourth power, the fourth root, &c.

The *Radical Sign*, $\sqrt{\ }$ is used to express roots. When written alone before a number it expresses the square root; with the figure 3 over it, the cube root, &c.

$\sqrt{9}$ expresses the square root of $9=3$.

$\sqrt[3]{27}$ expresses the cube root of $27=3$.

$\sqrt[4]{81}$ expresses the fourth root of $81=3$.

Square Root.

Art. 193.—The square root of a number, is another number, which multiplied by itself once will produce the given number or square.

The square of	$1=$	1	therefore the	$\sqrt{1}=1$
" "	$2=$	4	"	$\sqrt{4}=2$
" "	$3=$	9	"	$\sqrt{9}=3$
" "	$4=$	16	"	$\sqrt{16}=4$
" "	$5=$	25	"	$\sqrt{25}=5$
" "	$6=$	36	"	$\sqrt{36}=6$
" "	$7=$	49	"	$\sqrt{49}=7$
" "	$8=$	64	"	$\sqrt{64}=8$
" "	$9=$	81	"	$\sqrt{81}=9$
" "	$10=$	100	"	$\sqrt{100}=10$
" "	$11=$	121	"	$\sqrt{121}=11$
" "	$12=$	144	"	$\sqrt{144}=12$

To find the square root of any large number we observe—

First—That in squaring a number, each figure is squared and multiplied twice into each of the others.

Secondly—That the square of any number contains as many periods of two figures each, beginning at the right, as there are figures in the root, except the left-hand period may have only one figure.

Thirdly—That the square of a unit's figure is in the corresponding unit's period; of a ten's figure, in the ten's period; of a hundred's figure, in the hundred's period, &c.; also, that the square of the left-hand figure is the greatest square in its corresponding period.

The square of any number from

1	to	9 is from	1	to	'81	and has 1 period.
10	to	99 is from	1'00	to	98'01	and has 2 periods.
100	to	999 is from	1'00'00	to	99'80'01	and has 3 periods.
1000	to	9999 is from	1'00'00'00	to	99'98'00'01	and has 4 periods.

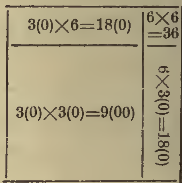
Therefore the square root of any number from

'1	to	'81	is from	1 to 9.	1	figure.
1'00	to	98'01	is from	10 to 99.	2	figures.
1'00'00	to	99'80'01	is from	100 to 999.	3	figures.

Or we may consider a square number as representing some square surface, as a floor or field, &c.

The square of 36 is

36		$6\times6=6^2=$	36	
36		$3\times6=$	18	
216	or	$6\times3=$	18	or
108		$3\times3=3^2=$	9	
12'96			12'96	



To find the square root of 1296 we reverse this process:

```
   12'96(36
    9
66) 396
    396
```

Dividing the number into periods of two figures each, we ascertain that there will be two figures in the root. The greatest square in the left-hand or ten's period being 9 hundreds its corresponding root is 3 tens, which we write on the right and subtract its square (9) from the left-hand period (12.) The remainder with the next period annexed is 396. This must contain the root (30) already found multiplied twice, or 30×2 multiplied once into the next figure, also the square of that figure; therefore 396÷60 allowing for the addition of the remaining square, equals the next figure (6) of the root. This figure we also add to the divisor, that it may be squared as well as multiplied into 60. After multiplying, there being no remainder, 36 is found to be exactly the square root of 1296.

Regarding 1296 as the contents of a square surface, 36 is the length of each side.

RULE FOR EXTRACTING THE SQUARE ROOT.—*Separate the numbers into periods of two figures each; beginning at the right of whole numbers, and the left of decimals.*

Find the greatest square in the left-hand period, and placing its root at the right, subtract it from that period, and bring down the next.

Double the root already found, and regarding its local value, find how many times it is contained in the remainder of the given square. Annex the quotient to the root and to the divisor, then multiply the the divisor by it, subtract the product and bring down the next period.

Proceed thus till the figures in the root are equal in number to the periods. If there is still a remainder, periods of decimal ciphers may be supplied.

To find the root of a common fraction, reduce it to its lowest terms and extract the root of the numerator and denominator separately if they are complete squares; otherwise reduce the fraction to a decimal and extract its root.

MENTAL EXERCISES.

What is the square root of 4, 16, 9, 25, 49, 36, 64, 100, 81, 121, 144, $\frac{1}{4}$, $\frac{4}{16}$, $\frac{9}{36}$, $\frac{25}{100}$, $\frac{49}{121}$, $\frac{36}{144}$, .9, .25?

EXAMPLES.

What is the square root of

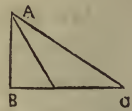
(1.)...........1764	(7.).........53824	(13.)...... 531441
(2.)...........9801	(8.).........12321	(14.)...... 5499025
(3.)...........4489	(9.)........8.1225	(15.)......36372961
(4.)...........6561	(10.)........ 40401	(16.)...... $\frac{225}{1024}$
(5.)...........8649	(11.)........ .0625	(17.)...... $20\frac{1}{4}$
(6.)...........7225	(12.)........ 56644	(18.).......0045369

Art. 194.—Applications of the Square Root.

The areas of circles, squares, and all similar figures, are to each other as the squares of their corresponding dimensions. A circle whose diameter is 4 feet, is to a circle whose diameter is 2 feet, as 4^2 to 2^2, or 16 to 4, four times greater.

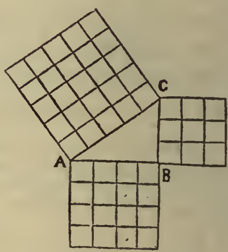
A **Triangle** is a figure bounded by three straight lines. The difference in the direction of two lines which meet, is called an *angle;* and if one line is perpendicular to the other, the angle is a *right* angle.

A Right-angled Triangle is a triangle that has a right angle; as A B C.



The **Hypothenuse** of a right-angled triangle is the side opposite the right angle; as A C. The *Base* is the horizontal line; as B C. The other is the *Perpendicular;* as B A.

It is demonstrated in geometry, that the square of the hypothenuse is equal to the sum of the squares of the other two sides.



This may be illustrated by the opposite figure. The small squares are all square inches or feet, &c.; and those on the hypothenuse are equal to those on the other two sides: 25=16+9.

RULE FOR FINDING THE HYPOTHENUSE.—*Add the squares of the base and perpendicular and extract the square root of the sum.*

RULE FOR FINDING EITHER THE BASE OR PERPENDICULAR.—*Subtract from the square of the hypothenuse the square of the other given side, and extract the square root of the remainder.*

EXAMPLES.

19. How long must a ladder be, to reach to the top of a house 40 feet high, when its foot is placed 30 feet from the house?

20. A ladder 50 feet long, and having its foot 30 feet from a house, just reaches the top; what is the height of the house?

21. A ladder 50 feet long just reaches the top of a house 40 feet high; how far is its foot from the house?

22. A room is 16 feet long and 12 feet wide; what is the diagonal distance between its opposite corners?

23. Two ships, one sailing directly north and the other directly west, are 100 miles apart, both having sailed the same distance from the same place; how far have they sailed?

Art. 195.—*The side of a square equal to any given surface, is found by extracting the square root of the surface.*

Rectangles, whose length is a certain number of times greater than their breadth, may be divided into that number of squares, with sides equal to the breadth of the rectangles.

24. It takes 25 square yards of carpeting to cover a room; how many feet square is it?

25. A rectangular field containing 20 acres is twice as long as it is wide; what is its length and breadth?

Art. 196.—*The areas of similar figures are in proportion to the squares of their similar sides or dimensions.*

26. A man having a grass plot 16 feet square, wishes to make it 4 times larger; how many feet long must each side be?

27. A man having water conducted from a spring to his house by a lead pipe ½-inch in diameter wishes to increase the quantity four times; how large a pipe must he use?

The square root of the product of two numbers is a *mean* proportional between them.

What is the mean proportional between

(28.) 9 and 25	(31.) 3 and 27	(34.) 2 and 8
(29.) 4 and 16	(32.) 1 and 49	(35.) 4 and 9
(30.) 8 and 18	(33.) 5 and 20	(36.) 5 and 125

Promiscuous Examples in Square Root.

37. A man about to build a house 32 feet square, wishes to have the peak 12 feet higher than the plate beams; how long must the rafters be?

38. A man having 1764 peach trees, wishes to set them out

in a square field, so that it shall be exactly filled with the trees; how many must be in row each way?

39. A man having 1152 apple trees, wishes to set them in rows twice as long one way as the other; how many must be in a row each way?

40. A farmer having a ditch 3 feet deep and 2 feet wide, wishes to make it twice as large in the same proportion; how deep and wide must it be?

Cube Root.

Art. 197.—The *cube* root of a number, is another number, which multiplied by itself twice, will produce the given number, or cube.

The cube of	1	is	1	therefore the	$\sqrt[3]{1}$	is	1
" "	2	"	8	"	$\sqrt[3]{8}$	"	2
" "	3	"	27	"	$\sqrt[3]{27}$	"	3
" "	4	"	64	"	$\sqrt[3]{64}$	"	4
" "	5	"	125	"	$\sqrt[3]{125}$	"	5
" "	6	"	216	"	$\sqrt[3]{216}$	"	6
" "	7	"	343	"	$\sqrt[3]{343}$	"	7
" "	8	"	512	"	$\sqrt[3]{512}$	"	8
" "	9	"	729	"	$\sqrt[3]{729}$	"	9
" "	10	"	1000	"	$\sqrt[3]{1000}$	"	10

To find the cube root of any large number, we observe—

First—That in cubing a number, each figure is cubed, and multiplied three times into the squares of each of the others, also its square is multiplied three times into each of them.

Secondly—That the cube of any number contains as many periods of three figures each, beginning at the right, as there are figures in the root, except the left-hand period may have only one or two figures.

Thirdly—That the cube of a unit's figure is contained in the corresponding unit's period; of a ten's figure in the ten's period, &c.; also, that the cube of the left hand figure is the greatest cube in its corresponding period.

The cube of any number from

1 to 9	is from	1 to 729	and has 1 period.
10 to 99	is from	1000 to 970299	and has 2 periods.
100 to 999	is from	100000 to 997′002′999	and has 3 periods.

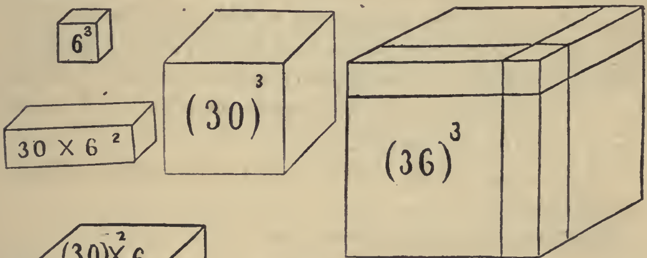
Therefore the cube root of any number from

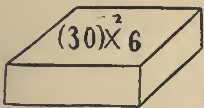
1 to 729	is from 1 to 9,	1 figure.
1000 to 970′299	is from 10 to 99,	2 figures.
1000000 to 997′002′999	is from 100 to 999,	3 figures.

Or we may consider any cubic number as representing the contents of a cube, in cubic inches, feet &c.

The cube of 36 is—

$$\begin{array}{r} 36 \\ 36 \\ \hline 216 \\ 108 \\ \hline 1296 \\ 36 \\ \hline 7776 \\ 3888 \\ \hline 46'656 \end{array} \qquad (36)^2 \text{ or } \left.\begin{array}{r} 6\times6 \\ 30\times6 \\ 6\times30 \\ 30\times30 \end{array}\right\} \times 36 = \left\{\begin{array}{llr} 6\times6\times6=6^3 & = & 216 \\ 30\times6\times6=30\times6^2 & = & 1080 \\ 6\times30\times6=30\times6^2 & = & 1080 \\ 30\times30\times6=(30)^2 & = & 5400 \\ 6\times6\times30=30\times6^2 & = & 1080 \\ 30\times6\times30=(30)^2\times6 & = & 5400 \\ 6\times30\times30=(30)^2\times6 & = & 5400 \\ 30\times30\times30=(30)^2 & = & 27000 \\ & & \overline{46656} \end{array}\right.$$





To find the cube root of 46656, we reverse this process.

Dividing the number into periods of three figures, except the left-hand period, we ascertain that the root has two figures. The greatest cube in the left-hand period being 27 thousands, its corresponding root is 3 tens, which we write on the

$$\begin{array}{rr|l} 30^2\times3 & =2700 & 46'656(36 \\ 30\times6\times3 & =540 & 27 \\ 6^2 & =36 & \\ \hline & 3275 &)19656 \\ & & 19656 \end{array}$$

right, and subtract its cube from the period. The remainder with the next period annexed is 19656. This must contain three times the square of the root (30, already found) multiplied by the next root figure (to be found,) three times the root (found,) multiplied by the square of that figure, and its cube; therefore 19656÷2700, allowing for an increase of the divisor, equals (6) the next root figure. Three times this figure multiplied by the former part of the root, we add to the divisor, also its square that in multiplying, its square thus multiplied and its cube may be included in the product. The divisor, thus increased, is (2700 + 540 + 36) 3276 which multiplied by (6) the last root figure equals 19656 the last remainder. Therefore the cube root of 46656 is exactly 36.

Rule for Extracting the Cube Root.—*Separate the given number into periods of three figures each, beginning at the right of whole numbers, and the left of decimals.*

Find the greatest cube in the left-hand period, and placing its root at the right, subtract it from that period, and bring down the next for a dividend.

Write three times the square of the root already found, with a cipher annexed, for a trial divisor, and, allowing for its increase, find how many times it is contained in the dividend, and annex the quotient to the root. Add three times the product of the last root figure with a cipher annexed, and the former part of the root, to the trial divisor, also the square of the last root figure. Multiply the completed divisor by the last figure in the root, subtract the product from the dividend and bring down the next period.

Proceed thus, till the figures in the root are equal in number to the periods.

Treat fractions as in square root.

EXAMPLES.

What is the cube root of

Ex. (1.)	205379	(5.)	80.763
(2.)	614125	(6.)	29.503629
(3.)	41421736	(7.)	146363.183
(4.)	162771336	(8.)	122615327232

(9.) $\frac{27}{125}$ (10.) $\frac{64}{729}$ (11.) $\frac{1}{1000}$ (12.) $5\frac{104}{125}$ (13.) $\frac{216}{343}$

Art. 198.—*The contents of all similar solids are to each other as the cubes of their corresponding dimensions.*

14. What is the side of a cube containing 1728 cubic feet ?

15. What is the side of a cube equal to a block 36 in. long, 8 in. wide, 6 in. high ?

16. If a ball 3 inches in diameter weighs 24 lbs., what will be weight of a similar ball 6 inches in diameter ?

17. A man wishes to have a cubical cistern made, which will hold 25 hogsh'ds; what must be its depth, &c., allowing 231 cubic inches to a gallon ?

18. A stack of hay 16 feet high, is worth $50; what is the value of a stack of similar shape, 20 feet high ?

19. A farmer wishes to have a cubical box made that will hold 100 bushels of grain; allowing 2150.4 cu. in. to a bushel, what must be its depth, &c. ?

20. A cistern 15 feet deep, holds 1410.048 gals.; how deep must a cistern of similar shape be to hold half the quantity ?

Promiscuous Examples in Square and Cube Root.

1. The pole of a circular tent 96 feet in diameter is 36 feet high; what must be the length of a rope that will reach from the top of the pole to the circumference of the tent ?

2. A speculator has bought 1000 acres of western land, to be laid out in a square ; what must be the length of its sides ?

3. A man wishes to have a cubical ice-house under ground, that will contain 4096 solid feet of ice ; what must be its inside dimensions ?

4. A gentleman wishes to have a new house built with a foundation twice as large, but in the same proportions, as the old one, which is 40 feet long and 30 feet wide ; what must be its length and width ?

5. What is the mean proportional between 45 and 96 ?

6. If a cylindrical cistern 6 feet in diameter will hold 30 hogs-

heads of water, how many gallons will a similar cistern hold, whose diameter is 12 feet?

7. A general has an army of 5184 men; how many must he place in rank and file to form them into a square?

8. If a pipe $2\frac{1}{2}$ in. diameter discharges 10 hogsheads of water in an hour, what must be the diameter of another pipe, to discharge 40 hhds. in the same time?

9. What is the side of a cubical box that will hold just one bushel (1250.4 cu. in.)?

10. A certain square, containing 20736 sq. feet, is paved with stones a foot square; how many are there in a row across one of its sides?

11. If a piece of silver 3 inches in length, is worth \$150; what is a similar piece worth which is 6 inches long?

12. The area of a circle 80 feet in diameter, is 5026.56 sq. feet; what is the area of a circle whose diameter is 60 feet?

13. A sphere 30 in. in diameter contains 8 cu. ft. 313.2 cu. in.; what is the solidity of a sphere 50 feet in diameter?

PROGRESSION.

Art. 199.—Progression is a series of numbers increasing or decreasing uniformly. There are two kinds, *Arithmetical* and *Geometrical.*

The **Terms** are the numbers forming the series; the first and last of which are called the *Extremes*, the others, the *Means.*

When the terms increase the series is *Ascending;* when they decrease, *Descending.*

Arithmetical Progression.

Art. 200.—Arithmetical Progression is a series of numbers increasing or decreasing by a *common difference;* as, 2, 4, 6, 8, 10, 12, 9, 6, 3.

In arithmetical progression it is required to find the *first* term (a), the last term (l), the *common* difference (d), the *number* of terms (n), or the *sum* of the series (s), of which three must be given.

In the series
ascending, 3, 5, 7, 9, 11, 13, 15,
descending, 15, 13, 11, 9, 7, 5, 3, if the common difference (2) be multiplied by the number representing any term, less 1, and added to or subtracted from the first term, it will give the term sought.

Again, half the sum of the first and last terms is the average of all the terms, which, multiplied by the number of terms, equals the series. Hence the following rules:—

Art. 201.—To find the *last* term, the first, the common difference, and the number of terms being given.

Multiply the common difference by the number of terms less 1, *and* (*in an ascending series*) *add the product to the first term, or subtract it* (*in a descending series*).

$$l = a \pm d \times (n - 1)$$

Art. 202.—To find the *common difference*, the extremes and number of terms being given.

Divide the difference of the extremes by the number of terms less 1.

$$d = \frac{l - a \text{ or } a - l}{n - 1}$$

Art. 203.—To find the *number of terms*, the extremes and common difference being given.

Divide the difference of the extremes by the common difference, and add one to the quotient.

$$n = \frac{l - a \text{ or } a - l}{d} + 1$$

Art. 204.—To find the sum of the series, the extremes and number of terms being given.

Multiply half the sum of the extremes by the number of terms.

$$s = \frac{a + l}{2} \times n$$

Many more cases might be added.

Examples.—1. What is the last term of a series whose first term is 2, common difference 3, and the number of terms 25?

2. What is the common difference in a series whose first term is 2, last term 200, number of terms 10?

3. What is the number of terms of a series, whose extremes are 2 and 32; common difference 3?

4. What is the sum of a series whose extremes are 3 and 23; the number of terms 11?

5. I agree to give a man for work 3 cents the first hour, 7 cts. for the second, 11 cts. for the third, &c., for 10 hours; what will he receive for the last hour?

6. A man had nine children at equal intervals, the oldest 35, and the youngest 3 years old; what was the difference in their ages?

7. A man traveled on foot the first day 5 miles, and the last 45, increasing his journey each day 4 miles; how many days did he travel?

8. How many times does a clock strike in twelve hours?

Geometrical Progression.

Art. 205.—Geometrical Progression is a series of numbers increasing by a common multiplier, or decreasing by a common divisor.

In geometrical progression it is required to find the first term (a), the last term (l), the ratio (r), the number of terms (n), and the sum of the series (s).

In the series
ascending, 3, 6, 12, 24, 48, 96, or
descending, 96, 48, 24, 12, 6, 3, if the ratio (2 or $\frac{1}{2}$) be raised to a power one less than the number of any term, and multiplied by the first term, the product will be the other term.

Again, if we multiply the ascending series by the ratio, we shall have another series, 6, 12, 24, 48, 96, 192, twice as great as the other,
3, 6, 12, 24, 48, 96, . The difference between them

(192−3, the other terms being alike in both series), 189, is therefore the sum of the given series. Hence the following rules:—

Art. 206.—To find the *last* term, the first term, the ratio, and number of terms being given :—

Multiply the first term by that power of the ratio whose index is one less than the number of terms.

$$l = a \times r^{n-1}$$

Art. 207.—To find the *sum* of the series, the extremes and number of terms being given :—

Multiply the last term by the ratio, and divide the difference between the product and first term by the difference between the ratio and 1.

The last term is frequently to be first found.

$$s = \frac{(l \times r) - a}{r - 1 \text{ or } 1 - r}$$

Many more cases might be added.

Examples.—1. What is the 12th term of a series whose first term is 3, and the ratio 2?

2. What is the sum of a series whose first term is 2; ratio 3; and number of terms 10?

3. A man gave his son \$100 when he was 20 years old, and promised to give him \$200 the next year, and to double the sum every year for ten years; how much did he give his son when he was 30 years old? and how much in the ten years?

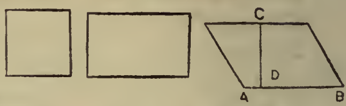
MENSURATION.

Art. 208.—Mensuration is finding the length of lines, and the contents of surfaces and solids.

The rules for Mensuration being derived from Geometry, can not be explained by Arithmetic.

Mensuration has already been applied to squares, cubes, &c. It may also be applied to other surfaces and solids, as

Parallelograms.—A parallelogram is any four-sided figure whose opposite sides are equal and parallel. They include squares and rectangles.

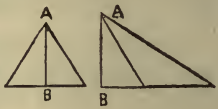


The *Base* is the line (AB) on which the figure appears to stand. The *Altitude* is the length of a perpendicular line (CD) from the base to the opposite side.

Art. 209.—To find the area of a parallelogram.

Rule.—*Multiply the base by the altitude.*

Triangles.—(Art. 194.) The altitude of a triangle is the length of the perpendicular drawn to the base, or the base produced from the opposite angle.



Art. 210.—To find the area of a triangle.

Rule.—*Multiply the base by half the altitude.*

Or, when the three sides are given, from half the sum subtract each side separately; multiply together the remainders and half the sum, and extract the square root of the product.

Circles.—(See Art. 51.)

To find the circumference.

Rule.—*Multiply the diameter by* 3.14159.

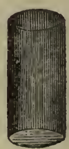
Art. 211.—To find the diameter.

Rule.—*Multiply the circumference by* .3183.

Art. 212.—To find the area.

Rule.—*Multiply the square of the diameter by* .7854; *or the square of the circumference by* .07954.

Cylinders.—A cylinder is a body whose diameter is uniform, and whose ends are parallel circles.

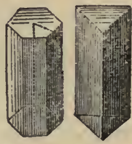


Art. 213.—To find the surface of a cylinder.

Rule.—*Multiply the circumference by its length or height, and add to the product the area of the two ends.* (Art. 212.)

Art. 214.—To find the solidity.

Rule.—*Multiply the area of either end by the length or height.*



The areas and solidity of *prisms* may be found by the same rules.

A **Prism** is a solid whose sides are rectangles, and whose ends are similar and equal.

A **Pyramid** is a solid having triangular sides meeting at a point at its top called its vertex.

A **Cone** is a solid having a circular base and tapering to a point at its top called its vertex.

A **Frustum** of a pyramid or cone is the part which remains after the top is cut off by a plane parallel to the base.

Art. 215.—To find the lateral surface of a pyramid or cone:

Rule.—*Multiply the perimeter or circumference of the base by one-half its slant height.*

To find the solidity of a pyramid or cone:

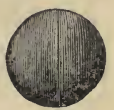
Rule.—*Multiply the area of its base by one-third its height.*

Art. 216.—To find the surface of a frustum:

Rule.—*Add the perimeters or circumference of the two ends together, and multiply the sum by one-half the slant height; to the product add the areas of the ends.*

To find the solidity of a frustum:

Rule.—*Add the areas of the ends to the square root of their product, and multiply the sum by one-third the height.*



Spheres.—A sphere is a body every part of whose surface is equally distant from the center.

Art. 217.—To find the surface of a sphere:

Rule.—*Multiply the square of the diameter by* 3.14159.

Art. 218.—To find the solidity of a sphere.

RULE.—*Multiply the cube of the diameter by* .5236.

Art. 219.—To find the curved surface of a cone.

RULE.—*Multiply the circumference of the base by half the slant height.*

The slant height differs from the perpendicular height, as the hypotenuse of a right-angled triangle from its perpendicular.

Art. 220.—To find the contents of a cask in gallons.

RULE.—*Add two-thirds the difference between the head and bung diameters to the head diameter; or,* .6 *if the staves are little curved, to find the mean diameter; then multiply the product of the square of the mean diameter into the length by* .0034.

EXAMPLES.

1. One side of a triangular field is 16 rods long, and a straight line at right angles with that side from the opposite corner is 20 rods; how many acres are there in the field?

2. The sides of a triangular field are respectively 20, 24, and 32 rods long; what are the contents of the field?

3. In laying out a grass-plot, I fastened a rope, 36 feet long, at one end, and with the other end, extended the whole length of the rope, I described a circle; what are the contents of the plot?

4. How many square feet of sheet-iron will it take to make a stove-pipe 18 feet long and 6 inches in diameter?

5. How many square feet of tin will cover the ball of a spire 3 feet in diameter?

6. How many gallons of oil will a cylindrical tank hold, that is $7\frac{1}{2}$ feet deep, and 4 feet in diameter, allowing 231 cubic inches to a gallon?

7. How many cubic feet of gas will fill a balloon 25 feet in diameter?

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8. How many acres are there in a field which can be divided

into two triangles, one having a side 32 rods long, and distant in a straight line, at right angles to it, 24 rods from the opposite corner; the other having its sides respectively, 36, 28, and 20 rods long?

9. The water-wheel of a mill is 24 feet in diameter; what is its circumference?

10. A circular pond is 300 feet in circumference; how much land does it cover?

11. The diameter of the earth is 8,000 miles (nearly); supposing it to be a perfect sphere, how many square miles are there on its surface?

12. How many sheets of tin, 18 inches long and 1 foot wide, will cover a conical spire, 40 feet high and 18 feet in diameter at its base, allowing $\frac{1}{3}$ for waste in cutting?

13. How many cubic feet in a stick of timber, 40 feet long, 3 feet in diameter at the larger end, and 2 feet 6 inches at the smaller?

14. How many gallons in a cask whose bung diameter is 18 inches, head diameter 12 inches, and length 24 inches?

15. How many square feet of tin will be required to make an oil-can, in the form of a frustum of a cone, 12 inches in diameter at the bottom, and 4 inches at the top, the slant height being $2\frac{1}{2}$ feet, allowing $\frac{1}{4}$ for waste in cutting?

PROMISCUOUS EXAMPLES.

United States Money and Compound Numbers.

1. A lady purchased a shawl for \$16, 2 pairs of gloves at \$1.37½ a pair, and 14 yds, of silk; she gave the merchant three \$20 bills and received back \$16.05; what was the price of the silk per yard?

2. A silversmith had 10 lbs. 3½ oz. of silver, and made it into spoons; these he sold for \$1⅛ apiece, receiving for them all

$42¾; how many spoons did he make, and what was the weight of each?

3. Reduce 3 m. 4 fur. 21 rods, 3 yds. 6 in. to inches and prove the operation.

4. How many shingles will it take to cover the roof of a house 42 feet long, and 30 feet wide, whose rafters are 18 feet long, allowing each shingle to be 6 inches wide and to lie 6 inches to the weather?

5. What will it cost to build a brick house 36 feet long, 32 feet wide, and 24 feet, high (on an average,) the walls 1 foot thick, each brick being 8 inches long, 4 inches wide and 2 inches thick, at $6¼ per 1000 bricks?

6. A man was born at 3¼ o'clock A. M., Sept. 5, 1835, and died at 6 o'clock P. M., April 21, 1863; what was his age?

7. The longitude of New York is 74° 1′, and that of Cincinnati, 84° 24′; what is the difference of time?

8. The difference of time between Washington and St. Petersburg is 7 hours 9 minutes; what is the difference of longitude?

9. A farmer exchanged 3 pairs of oxen at $112 a pair, and 7 cows at $42 each, for 126 sheep; what was the price of the sheep per head?

10. Bought of J. Ayers 7 yds of cloth at $6 a yard, 3 bbls. of flour at $9 barrel, and a cheese for $3; he agreed to take in payment 6 cords of wood at $7 a cord, and 18 bushels of corn at $1; how much cash must I pay him to balance the account?

11. In measuring a portion of a railroad, one man made the distance 43 m. 7 fur. 31 rds. 1 yd. 1¼ft.; and another 42 m. 1 fur. 39 rods, 5 ft. 7 in.; what was the difference in the measurements?

12. In measuring a field, one man made it contain 4 acres, 3 roods, 18 square rods, 6 feet, 64 square inches; but another 4 acres, 2 roods, 39 square rods, 8 yards, 100 square inches; what was the difference in the measurements?

13. A certain railroad is 56½ miles long; in making it, it

was divided equally between 6 contractors; how much of it did each make?

14. The difference in longitude between London and Boston is 71° 8′; what is the difference of time?

15. The difference of time between Jerusalem and Baltimore is 7 h. 24 m. 36 sec.; the long. of Baltimore is 76° 37′ west; what is the long. of Jerusalem?

16. How many blocks of marble 6 inches square, will pave two halls of a hotel, crossing each other in the centre; one 32 ft. long, and 12 ft. wide, the other 64 ft. long and 8 feet wide?

17. In a town 5 miles square, how many farms can there be containing 150 acres each?

18. A man has agreed to deliver 48 cords of wood; the wood is 4 feet long, and he makes the pile 5 feet high; how long must he make it?

19. A man has $1623 which is 4 times as much as he had last year, wanting $121; his brother had last year 3 times as much as he had, and $10.50 more, but has since lost $1000; how much has his brother left?

20. A merchant having $1000, paid $510 for dry goods and the remainder for 20 barrels of molasses; how much was the molasses per barrel?

21. How many suits of clothes, each requiring 4 yds. 2 qrs., can be made from 639 yards?

22. How many bricks will be required to pave a sidewalk 80 yds. long and 3 yds. wide, each brick being 8 in. long, and 4 in. wide?

23. How many yards of muslin, 3 qrs. wide, will line 9 yds, 1 qr. 2 na. of merino cloth, 1 yd. 1 qr. wide?

24. How much will it cost at 2s. 6d. a square yard to plaster a room 24 feet long, 18 feet wide, and 12 feet high, there being two doors 7 ft. high, and 3 ft. 6 in. wide; also two windows 5 feet high, and 3 feet 4 in. wide, and a mop-board 8 inches wide.

25. A man wishes to divide a field of 4 acres into 7 building lots of equal size; how much will each lot contain?

26. The longitude of New York is 74° 1′; a sea captain, sailing thence, finds that his watch has lost 2 hours; what is his longitude?

27. The difference of longitude between New York and New Orleans is 15° 4′. What is the difference of time?

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28. A lady shopping in New York, bought 14 yds. of silk at 15s. 6d. a yard; 5 yds. of linen at 7s. 4d.; 2 yds. of thread-lace at 18s., and a pair of gloves for 10s. 9d. She gave in payment a $100 bill; how much money should she have been paid back?

29. Purchased 24 A. 1 R. 27 rods at $360 an acre. I sold the same for building lots at $3.15 a square rod; what did I gain?

30. A grocer bought 5 barrels of cider at $4 a barrel, and after making it into vinegar, sold it at 10 cts. a quart; how much did he gain?

COMMON AND DECIMAL FRACTIONS.

1. A farmer had $\frac{1}{5}$ of his sheep in one pasture, $\frac{1}{6}$ in another, $\frac{1}{4}$ in another, and the remainder, 46, in a fourth; how many sheep had he?

2. A merchant bought $6\frac{3}{4}$ cords of wood at $\$5\frac{7}{8}$ a cord; and paid for it in cloth at $\$4\frac{1}{2}$ a yard; how many yards did it require?

3. Bought 640 sheep at $\$2\frac{1}{4}$ a head, and afterward 270 at $\$2\frac{3}{8}$; sold $\frac{3}{5}$ of them at $\$3\frac{1}{16}$; $\frac{1}{4}$ of the remainder were killed by dogs, and what still remained I sold for $3 a head; how much did I gain?

4. What will 75 yds. $1\frac{1}{2}$ qrs. of silk cost, at £.375 a yard?

5. A butcher had $351\frac{1}{5}$ lbs. of beef; he sold $\frac{3}{5}$ of it, corned $\frac{2}{3}$ of the remainder, and used what was then left in his family; what was the value of that which his family consumed at $17\frac{1}{2}$ cts. a pound?

6. What part of $\frac{1}{3}$ of a solid yrd., is $\frac{1}{3}$ of a yard solid?

7. How much can a man earn in $\frac{3}{7}$ of a year at $\$1\frac{1}{4}$ a day?

8. A piece of oil-cloth 3 yds. square is worth \$30. What is 3 square yards of it worth?

9. A farmer sold 4 fields containing respectively, $6\frac{7}{12}$, $8\frac{1}{4}$, $5\frac{1}{2}$, and $7\frac{1}{3}$ acres, at \$100 an acre; what did he receive for them?

10. A hogshead of molasses contained 96 gallons; $\frac{5}{12}$ of it has been sold; $\frac{2}{3}$ of the remainder has been used; how much now remains?

11. Two men were 82 miles apart, one of them traveled $\frac{3}{11}$ of this distance, and the other $\frac{5}{8}$ of the remainder; how far were they then apart?

12. A laborer hired to a farmer for a year for \$313, but he was sick $\frac{1}{9}$ of the working time, and was absent $\frac{1}{4}$ of the remaining time; how much wages was due to him at the end of the year?

13. A man had a field $36\frac{7}{12}$ rods long, and $24\frac{3}{4}$ rods wide; he sold $\frac{4}{7}$ of it for \$112.40; what is $\frac{2}{3}$ of the remainder worth at the same rate?

14. A merchant sold $\frac{3}{8}$ of a hogshead of sugar weighing 879 lbs. for \$96. What is $\frac{3}{5}$ of the remainder worth at the same rate?

15. Bought a piece of merino cloth containing $48\frac{3}{8}$ yards, and having cut off $\frac{2}{3}$ of it, sold $\frac{3}{4}$ of the remainder at $\$1\frac{1}{4}$ a yard, and what still remained at $87\frac{1}{2}$ cents a yard; what was the amount sold?

16. A man having $28\frac{1}{4}$ tons of coal, sold $\frac{3}{8}$ of it at $\$10\frac{5}{8}$ a ton, and the remainder at $\$9\frac{3}{4}$. What amount was received for the coal?

17. A man bought $42\frac{5}{16}$ tons of hay, at $\$11\frac{1}{2}$ a ton, and sold $\frac{1}{2}$ of it at $\$11\frac{3}{4}$, and the remainder at $\$13\frac{1}{4}$ a ton; how much did he gain?

18. A grocer bought a cask of vinegar containing $43\frac{3}{4}$ gals. for $\$15\frac{1}{3}$, and has sold 19 gals. 2 qts. $1\frac{1}{2}$ pts. at cost; for what must he sell the remainder in order to gain $\frac{1}{4}$ as much as the whole cost?

19. What cost .778125 ton of buckwheat flour, a 2d. at pound?

20. What cost 1.8875 acre of land at $\$1\frac{1}{4}$ a rod?

21. What cost 112 hhds. 3.35 gals. of molasses, at £3 8s. 9d. a hogshead?

22. If .3125 yard of cloth cost £$\frac{7}{8}$; what is the price per yard?

23. If .0625 bbl. of flour cost \$$\frac{5}{16}$; what is the price per barrel?

24. A man having 185$\frac{3}{4}$ acres, sold $\frac{1}{3}$ of it, at another time $\frac{3}{8}$ of it; what is the value of the remainder at \$115$\frac{3}{5}$ an acre?

25. A man deposited his money in 4 banks, in one $\frac{2}{5}$, in another $\frac{1}{3}$, in the third $\frac{1}{6}$, and in the fourth the remainder, which was \$48 more than $\frac{1}{15}$ of the whole; how much money did he deposit?

26. Bought $\frac{6}{7}$ of a box of starch, sold $\frac{7}{8}$ of it for \$4$\frac{12}{25}$; what was the whole box worth at the same price?

27. A man gave $\frac{1}{2}$ of his money for a horse; $\frac{2}{3}$ of the remainder for a wagon; $\frac{1}{4}$ of what then remained for a saddle; he then had \$24.25 left; how much had he at first?

28. A man bequeathed his property to his five children: to the first $\frac{1}{3}$ of it; to the second $\frac{1}{4}$; to the third $\frac{1}{5}$; to the fourth $\frac{1}{6}$; and the remainder to the fifth, who had \$665 less than the fourth; what was the amount of the property?

29. A person being asked the time of day, said: The time past noon is $\frac{3}{5}$ of the time from now to midnight.

30. Two men, A and B, were playing cards for money; $\frac{5}{8}$ of A's money was equal to $\frac{3}{5}$ of B's; but B lost \$42, and then had only $\frac{1}{10}$ times $\frac{8}{9}$ as much as A then had; how much had each?

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31. A man gave \$1750 to two benevolent societies, and gave one \$150 more than the $\frac{3}{5}$ of what he gave the other; what were the amounts given?

32. A man bequeathed \$5,420 to his son and daughter, so that \$240 more than $\frac{2}{3}$ of what his daughter had was equal to $\frac{3}{4}$ of what his son had; how much had each?

33. A pole 102 feet high stands near a house, and $\frac{3}{4}$ of the part above the house equals $\frac{2}{3}$ the other part; how much higher is the pole than the house?

34. James said to Henry, 2 years added to $\frac{2}{4}$ of my age equals

$\frac{2}{3}$ of yours, and the sum of our ages is 37 years; what was the age of each?

35. A person being asked his age, replied, that $\frac{2}{3}$ of it, $\frac{3}{5}$ of it, $\frac{1}{2}$ of it, and 7 more, would be twice his age; what was his age?

36. The difference between my neighbor's property and my own is \$1000; $\frac{3}{4}$ of mine equals $\frac{3}{5}$ of his; but $\frac{3}{4}$ of his is $1\frac{1}{4}$ times $\frac{3}{4}$ of mine; what is the property of each?

37. I wish to make 3 boxes, each $5\frac{1}{2}$ feet long, $3\frac{7}{12}$ feet wide, and $2\frac{1}{3}$ feet high; how many square feet of boards $1\frac{1}{2}$ inches thick will they require?

38. A man has a garden $10\frac{1}{4}$ rods long and $7\frac{1}{2}$ rods wide; what will it cost to dig a ditch around it 3 feet wide and $4\frac{1}{2}$ feet deep, at 3 cents a cubic foot?

39. A man traveled $28\frac{3}{7}$ miles the first day, $33\frac{11}{14}$ miles the second, and $29\frac{1}{21}$ miles the third; how far did he travel in the three days?

40. The distance from Boston to Worcester is 40 miles; a man having traveled $\frac{2}{17}$ of the distance, and afterward $\frac{2}{11}$ of it; how far was he then from Boston?

41. Bought $23\frac{3}{4}$ bushels of corn at $\$1\frac{3}{8}$ a bushel, and sold $\frac{2}{3}$ of it for $\$1\frac{5}{8}$, and the remainder at $\$1\frac{7}{8}$; what was the whole gain?

42. Bought $\frac{2}{5}$ of a flouring mill for \$1,837.50, and having sold $\frac{5}{7}$ of my share, I gave $\frac{2}{3}$ of the remainder to cancel a mortgage on it, and what still remained I gave the miller for $\frac{1}{2}$ a year's wages; what was the amount of his wages a year?

43. Bought a horse, carriage, and harness for \$350. The harness cost $\frac{5}{12}$ as much as the horse, and the horse $\frac{2}{3}$ as much as the carriage; what did each cost?

44. Bought $\frac{7}{9}$ of a ton of plaster, and sold $\frac{9}{14}$ of it for \$4.50; what was the price per ton?

45. One of my horses usually travels 6 miles in $\frac{7}{8}$ of an hour, and the other 7 miles in $\frac{9}{10}$ of an hour; how much longer will it take one to travel 20 miles than the other?

46. I have a garden $13\frac{2}{7}$ rods long, and $\frac{3}{4}$ as wide, surrounded by a fence $6\frac{1}{2}$ feet high. Next to the fence is a border 1 rod wide, for shrubbery and fruit-trees; then a gravel walk $8\frac{1}{4}$ feet

wide, the rest is for cultivation; how much is there to be cultivated?

47. How many pieces of paper, $9\frac{1}{2}$ yards long and 20 inches wide, will it take to paper a room $22\frac{1}{2}$ feet long, $15\frac{1}{2}$ feet wide, and $11\frac{1}{4}$ feet high?

48. A man invested $\frac{1}{3}$ of his property in his business, $\frac{1}{4}$ of the remainder in stocks, $\frac{1}{6}$ of what still remained, \$454, in a farm; what part of his property was thus invested, and what was the amount of the whole?

49. A man invested $\frac{1}{4}$ of his property in a farm, and $\frac{1}{5}$ of the remainder he spent in building a house; the farm cost \$1000 more than the house; what was the amount of his property?

50. After spending $\frac{2}{3}$ of my money, and $\frac{1}{4}$ of what remained, I had \$62.50 left; what sum had I at first?

51. A cask was $\frac{2}{3}$ full of vinegar; after drawing from it 8 gallons it was $\frac{1}{2}$ full; how many gallons did it hold?

52. If $\frac{3}{5}$ of the time past noon is $\frac{3}{7}$ of the time to midnight, what time is it?

53. If a man draw 350 loads of bricks, and 1,500 bricks at each load; how much will he receive at the rate of $87\frac{1}{2}$ cents a thousand?

54. Sold 3000 feet of boards, at \$9.50 a thousand, and 700 lath at 50 cents a hundred, what was the amount?

55. Bought 160 sheep at $\$2\frac{1}{2}$ a head, and 215 at $\$1.87\frac{1}{2}$ a head, sold $\frac{2}{3}$ of them at $\$2\frac{1}{8}$, and the remainder at \$2; did I gain or lose, and how much?

56. Bought 364 pounds of sugar, at $16\frac{3}{8}$ cents a pound; if the price per pound had been $3\frac{7}{8}$ cents less, how many pounds could have been bought for the same money?

57. My farm is $\frac{3}{4}$ meadow, $\frac{1}{10}$ orchard, and the remainder, 20 acres, more than $\frac{1}{8}$ of the whole, is timber; how large is my farm?

58. The income of a farm, consisting of 125A. 1R. $7\frac{1}{2}$ rods, was £202 11s. $3\frac{3}{8}$d.; how much was it an acre?

59. How much wheat will 32A. 1R. 10 rods yield, at the rate of 25 bu. 3 pks. 1 qt., per acre?

60. I have two small farms, $\frac{2}{3}$ of the acres in one, added to $\frac{3}{4}$ of the acres in the other, make 90 acres; and $\frac{2}{3}$ of the first is $\frac{7}{8}$ of $\frac{3}{4}$ of the second; how much larger is the first farm than the second?

PERCENTAGE AND ITS APPLICATIONS.

1. Sold a quantity of grain for $251.50, for which I received a note dated Oct. 1, 1862, payable in 6 months; what is the value of the note Dec. 27?

2. I have forwarded to my agent in St. Louis, $5000 for the purchase of flour; after deducting $3\frac{1}{4}\%$ commission, what will be the cost of the flour?

3. What must be the face of a note payable in 4 months, for which I may receive from a bank $600 at 6% discount?

4. A woolen factory was insured for $37,500 at $2\frac{1}{2}\%$; after 2 years it was burnt; what was the loss to the company?

5. What is the duty at 30% ad valorem, on 26 barrels of sugar, each weighing 225 pounds; tare 15%, and the sugar costing 16 cents a pound?

6. Bought 25 barrels of flour at $10 a barrel, and sold it immediately at $11.96 a barrel, on 8 months' credit, what per cent. did I gain, allowing 6% interest?

7. Sold 34 tons of coal at $8 a ton, for which I received a note payable in 90 days, and had it discounted at a bank. I then found that I had lost 10% on the coal; what did it cost?

8. If I buy cloth at $7.50 a yard on 9 months' credit, for what must I sell it for cash, to gain 12%?

9. Messrs. Mead, Gage & Storrs, of Chicago, made a consignment of flour to New York; M. furnished $1,400; G. $600, and S. 200 barrels of flour. They gained $270, of which S. had $90; at what was his flour valued per barrel, what was M.'s and G.'s share of the profits?

10. Three men A, B, and C, hired a pasture for $72. A put in 3 horses for 6 weeks, B put 3 pairs of oxen for 5 weeks, and C put in 12 cows for 4 weeks. It was agreed that 5 cows should

be considered equal to 3 oxen, and 4 oxen to 3 horses; what was each one's share of the expense?

11. Messrs. Dudley & Swift contracted to build a section of a railroad for $26,000 a mile; D. furnished 60 men, and S. 40 horses and carts, with boys to drive them. It was agreed that 3 men be considered equal to 2 horses and their drivers. Swift also was to be allowed $100 a mile for overseeing the work. After completing $5\frac{1}{2}$ miles, what was each one's share?

12. A merchant in Philadelphia wishes to remit to Liverpool £1000; what will a bill of exchange for this amount cost him at $9\frac{1}{2}$% premium?

13. When gold is 135, which is the better investment, U. S. 5-20's at 103, or bank stock at 108, paying a semi-annual dividend of 4%?

14. Bought goods amounting to $256.50, and having kept them 6 months, sold them so that I gained 6%; for what were they sold?

15. Borrowed of my neighbor $450 for 6 months; I afterward lent him $300, long enough to compensate him; how long did he keep it?

16. A certain town is taxed $3022.75; the taxable property amounts to $146,637.50; there are 150 polls, each taxed 60 cts.; what per cent. is the tax, and what is a man's tax who pays for two polls and whose property is valued at $1837.50?

17. What is due on the following note, at 7% interest, Jan. 1, 1868?

$500. SING SING, N. Y., Oct. 10, 1862.

On demand, I promise to pay S. Wilbur, or order, five hundred dollars, value received, with interest. H. VAN WYCK.

Indorsed,

Jan. 1, 1863, $60.	April 1, 1865, $75.
June 15, 1864, $150.	Jan. 1, 1866, $100.

18. Sold a lot of lumber for $500, and gained $12\frac{1}{2}$%; what did it cost?

19. Clark & Smith are partners; C. put in $2000, and they

have gained $203, of which Smith's share is $87; what was his capital?

20. Lent $176 for 1 year 6 months; it then amounted to $195.14; what was the per cent.?

21. Sold several shares in an Oil Company for $6875, which was 40 per cent. less than they cost; what did they cost?

22. Bought goods to the amount of $1200, sold them for $1344; what was the gain per cent.?

23. Paid $10,000 for a cargo of cotton, and sold it for $15,000, but invested it in stock, which I sold at 15 per cent. less than it cost; what was the net gain?

24. Sold 36 bushels of corn for $29.70, and lost $17\frac{1}{2}\%$; what per cent. should I have gained if I had sold it for $\$40\frac{1}{2}$?

25. Bought 30 yards of cloth at 5% less than the first cost, and sold it at 5% more than the first cost; I gained $15; what was the first cost per yard?

26. A bank discounted a note payable in 60 days, at 6% discount; and gave for it $2.52 less than the face of the note; what was the amount of the note?

27. Bought a house for $3000; rented it at $350; paid $\frac{1}{2}\%$ for insurance, for taxes $1\frac{4}{5}\%$, for repairs $106; what per cent. did the investment yield?

28. A man sold his farm for $2500, which was $16\frac{2}{3}\%$ less than he paid for it; he then bought another, and sold it for 16% more than he paid for it; he thus gained as much as he had lost; what did he pay for each farm?

29. A merchant bought 120 yards of cloth, at $4 a yard, on 6 months' credit, and sold it immediately for $500, money being at 6%; what did he gain?

30. The tax in a certain town is $1\frac{1}{4}\%$ besides each poll $1. One man's tax is $127, including 2 polls; what is the amount of his property?

31. A collector received $36 for his services, at $2\frac{1}{2}\%$; what was the amount he collected?

32. A man bought a house, and after spending 10% of the

price in repairs, found that the whole cost was $4400; what was the price of the house?

33. Bought a house for $4000; paid for repairing it $1500, it remained unoccupied 3 months, when I sold it for $6000, for which I received a note payable in 90 days, after one month I had the note discounted at a bank at 7%; what per cent. did I gain by the purchase?

34. Lent my neighbor $900 from Jan. 1 to Sept. 1; then borrowed of him $1150 from Sept. 1 to March 1; what is the balance of interest at 6%, and to whom due?

35. What investment at 5% will yield a semi-annual income of $250?

36. I have bought a bill of goods amounting to $420, on 60 days' credit. Is it better for me to pay cash at 2½% discount, or receive 6% interest for the money till the time of credit expires?

37. In order to pay the above bill is it better for me to obtain the money from a bank at 6% discount, or take the goods on credit?

38. Exchanged 100 shares of Erie stock at 40% below par, for stock in a Gold Mining Company at 125; how many shares did I receive?

39. Gained $175 by buying 50 shares of bank stock, at 5% advance, and selling them again: for how much a share were they sold?

40. A man has $4000, invested in U. S. 5-20's; what income will it yield when gold is 130?

41. Bought 7% bonds at 103, amounting to $7210; what annual income will they yield?

42. How much must be invested in 6% bonds at 90, to yield a semi-annual income of $500?

43. If I buy 6% bonds at 90, amounting to $6000, what % will the investment yield?

44. Which is the better investment, U. S. 7-30's at 102, or 6% State bonds at 98?

45. What must be the price of gold that U. S. 5% bonds at 95 may yield 6% interest?

46. A bankrupt settled with his creditors by paying them 70 cents on a dollar; one of them received $507.50; what was the amount of his claim?

47. What will be the premium at $1\frac{1}{2}\%$ for insuring a vessel to cover both its value $10,000 and the premium?

48. At the age of 40 a man insured his life for $5000, at the rate of $36 a 1000, not to be paid after 20 years. What will his family gain or lose if he die at 45, 50, 55, 60, or afterward, money being worth 6%?

49. How much must be paid in currency for duties on 25 barrels of sugar, each weighing 224 lbs., tare 12%, and the duty 5 cts. a pound in gold, when gold is 130?

50. A gentleman in St. Louis owns 50 shares of the Corn Exchange Bank in New York, which has declared a semi-annual dividend of 5%, a draft for which he sells at 1% premium; what does he receive for it?

51. A merchant in New York owes a debt in Liverpool of £250. When gold is 130, and exchange $9\frac{1}{4}$, is it better to buy a bill of exchange, or remit U. S. bonds at 95, which can be sold in Liverpool at 60?

52. A man left $10,000 to be divided between his two sons, 16 and 18 years old, so that at 6% interest they should each have the same amount when 21 years old; what did he leave for each?

53. A man owed $287.70; he paid at one time 40% of the debt; at another time 25% of what he then owed; and afterward $12\frac{1}{2}\%$ of what he still owed; how much of the debt remained to be paid?

54. A merchant having $5000, lost $\frac{1}{4}$ of it in speculation, and $\frac{1}{3}$ of the remainder in bad debts; what per cent. of the whole did he lose?

55. A man spent $487.50 in traveling, which was 15% of his income; what was his income?

56. A man exchanged 14 shares of bank stock, at 7% premium, for 25 shares of railroad stock, at $12\frac{1}{2}\%$ discount, and agreed to pay the difference in cash; how much did he pay?

MISCELLANEOUS RULES.

[These include various rules not used in the last promiscuous examples.]

1. I have 3 rooms, respectively 12, 16, and 20 feet wide, which I wish to cover with oil-cloth that will exactly fit all of them without cutting off the width; how wide must the oil-cloth be?

2. I have 3 pieces of oil-cloth, respectively 3, 6, and 9 feet wide; what must be the width of a room that either of them will exactly fit without cutting off the width?

3. What will it cost to gild a ball 10 in. in diameter, at $10.80 a square foot?

4. What must be the height of a pole which, being broken 30 feet from the top, struck the ground 18 feet from the bottom?

5. Jan. 1. I owe J. Bush $325 due in 4 months; $362.50 due in 8 months; and $250 due in 12 months; at what date should I give my note to settle the account?

6. What is the equated time of payment of the following bill:

1867.	Walter Hickok to H. Walcot.	
June 1.	Mdse, ...	$225
" 12.	" (4 mos.)	250
Aug. 16.	Cash, ...	125

7. What is the equated time of settling the following account:

Dr.		A. Knapp.			Cr.
1867.			1867.		
June 1.	Mdse, ...	$200	July 4.	Cash, ...	$200
" 16.	" (3 mos.)	400	Aug. 20.	Mdse, ...	75
Oct. 20.	Cash, ...	175	Sept. 20.	" . . .	250

8. J. Smith has a horse worth $250, but in trading, values it at $280; W. Read's horse cost $300½; at what should he value it in trading with Smith?

9. If 24 men can build a wall 33¾ feet long, 5⅗ ft. high, and 3½ ft. thick, in 126 days, by working 9h. 20min. each day; how many hours a day must 217 men work to build a wall 23¼ ft. long, 3⅔ ft. high, and 2⅓ ft. thick, in 3⅔ days?

10. Bought 4 tubs of lard, each weighing 50 lbs., at 13 cts. a pound; 10 tubs of 40 lbs. each, at 10 cts.; 24 tubs, 25 lbs, each,

at 7 cts.; sold the whole at an average of $9\frac{1}{2}$ cts. a pound; how much was the gain?

11. A butcher bought lambs worth \$2, \$$2\frac{1}{2}$, \$3, and \$4 a head, for which he gave, on an average, \$$2\frac{3}{4}$; how many at each price did he buy?

12. A butcher bought 12 calves at \$6 a head; how many must he buy at \$9 and \$15 a head that he may sell them all at \$12 a head without loss?

13. A butcher bought 85 sheep at an average price of \$$1\frac{3}{4}$ a head; for some he paid \$$1\frac{1}{8}$, for some \$$1\frac{1}{2}$, for some \$$2\frac{1}{4}$, and for others $2\frac{1}{2}$; how many at each price did he buy?

14. The diameter of a circle is 10 feet; what will be the diameter of another circle twice the area of the first?

15. A farmer wishes to make a bin which will contain 250 bushels of grain; its width to be twice its depth, and its length twice its width; what must be its dimensions?

16. A boy agreed to work 19 days, for which he was to receive 4 cents the first day, and 3 cents more every day than the preceding; how much did he receive the last day?

17. A boy bought ten apples; for the first he agreed to pay 1 mill, for the second 2 mills, and so on; what did he pay for the last?

18. The circumference of a park is 84 rods; what is its area?

19. A cistern is 6 feet deep and $5\frac{1}{2}$ feet in diameter; how many hogsheads of water will it contain?

20. What is the solidity of the largest ball that can be cut out of a cubical block whose sides are five inches square?

21. Two boys are running around a block—the larger boy runs around it every $5\frac{1}{2}$ minutes, and the smaller boy every $6\frac{1}{4}$ minutes; if they started together how many times must each run around the block before they will be together.

22. How much corn must I take to a mill, that there may be 4 bushels left after taking 4 % from each bushel for toll?

23. A liquor dealer has 60 gallons of brandy, worth \$3 a gallon, which he wishes to reduce so that he can sell it at \$2.40 a gallon; how much water must he add to it?

24. A thief started from a place at midnight and traveled 8 miles an hour; the sheriff started in pursuit 3 hours later, and traveled 10 miles an hour; at what time was the thief overtaken?

25. J. Taylor can mow 4 acres in 3 days, and his son can mow 5 acres in 4 days; in how many days can they both mow $12\frac{11}{12}$ acres?

26. I mixed 16 lbs. of tea at 75 cents, 20 lbs. at $87\frac{1}{2}$ cents, and 12 lbs. at $1.25; what is a pound of the mixture worth?

27. If $1\frac{1}{2}$ lbs. of tea be worth $8\frac{1}{4}$ lbs. of coffee, $3\frac{1}{6}$ lbs. of coffee be worth $5\frac{2}{3}$ lbs. of sugar, and 3 lbs. of sugar be worth 40 cents, what is the price of the tea?

28. What is the equated time of settling the following account:

W. JOHNSON.

Dr.					Cr.
1868.			1868.		
Jan. 1,	Mdse.	$224	Jan. 20,	Cash	$280
Feb. 1,	Draft (3 days) .	182	Feb. 6,	Draft (10 days).	132
" 20,	Cash	116	" 25,	Consm't. . . .	450

29. A grocer has spices at 9d., 1s., 2s., 2s. 6d.; how must he mix them so that he can sell the mixture at 1s. 8d. a pound?

30. What is the greatest number of hills of corn that can be planted on a square acre, the centers of the hills to be $3\frac{1}{2}$ feet apart?

31. If 10 barrels of water flow through a pipe $2\frac{1}{2}$ inches in diameter, what must be the diameter of a pipe that will discharge four times as much in the same time?

32. If a silver ball 3 inches in diameter be worth $270, what is another 6 inches in diameter worth?

33. A man bought 19 yards of linen; for the first he gave 1s., and for the last £1 17s.; what did the whole cost?

34. What is the area of a circular plot, 11 rods in diameter?

35. What are the solid contents of a column whose average diameter is 18 inches, and whose length is 20 feet?

QUESTIONS.

Article 1.—What is **arithmetic?** Number? Abstract numbers? Concrete?

[2] What is **notation?** How many kinds of notation are in common use? [3] What is the **Roman** method? How many letters does it use? What is the effect of repeating the letters? of writing a letter before another of greater value? after it? [Examples.]

[4] What is the **Arabic** notation? How many and what figures does it employ? Which are called digits? What is the simple value of a figure? [Examples.] What is its value when written before another? [Examples.] When written before two others? three, four, etc.? How do figures increase in value? Repeat the French Notation and Numeration table. How is it divided? Name the periods in order. How may numbers consisting of several figures be written, or what is a rule for Notation?

[Instead of learning the rules in the book, it is better that the pupil should thoroughly understand them, and express the ideas in his own words.]

[5] What is **numeration?** How may large numbers be read?

[6] What are the **fundamental rules** of arithmetic?

[7] What is **addition?** What is the number found, or answer called? What is simple addition? [Illustration.] What is the sign of addition? Give a rule for adding numbers, consisting of one figure or units; of units, tens, etc., or several figures.

[8] What is **subtraction?** What is the answer called? the number to be subtracted? the number from which another is subtracted? What is simple subtraction? [Illustration.] What is the sign of subtraction? [Illustration.] Give a rule for subtraction.

[9] What is **multiplication?** What is the number to be multiplied called? the number by which another is multiplied? the answer? What are the factors? What is simple multiplication? the sign? What two numbers in multiplication are properly of the same name? What kind of a number is the multiplier? [Illustration.]

[10] Give a rule when the multiplier consists of one figure or a number less than 12: [12] when it is greater than 12: [13] for the

multiplication of numbers having ciphers on the right; [14] What is a composite number? [Illustration.] Give a rule for multiplying by composite numbers.

[15] What is **division?** What is the number to be divided called? the dividing number? the answer? the number that is sometimes left? What is the sign of division? In what other way is division sometimes expressed? What is such an expression called? With what do the divisor and quotient correspond? the dividend? With what must the name of one of the factors in division correspond? What is the other factor?

[16] How many kinds of division are there? What is short division? long division? Give a rule for short division; [17] long division. [18] When there are ciphers on the right hand of the divisor; [19] when the divisor is a composite number.

[20] How is the quotient affected when the dividend is multiplied by any number? when the divisor is divided? when the dividend is divided? when the divisor is multiplied? when the dividend and divisor are both multiplied or both divided by the same number?

United States Money. [22] What is United States Money? Of what does it consist? What are its coins in gold? silver? Repeat the table. [23] What are aliquot parts? What part of $1 are 10 cents? $12\frac{1}{2}$? $16\frac{2}{3}$? 20? 25? $33\frac{1}{3}$? $37\frac{1}{2}$? 50? $62\frac{1}{2}$? 75? $87\frac{1}{2}$? [24] Give a rule for writing United States Money. [25] for reading United States Money. [26] for reducing dollars to cents; cents to mills; dollars to mills; dollars and cents to cents; mills to cents; cents to dollars; mills to dollars.

[27] In what respect is United States Money like simple numbers? [28] Rule for addition of United States Money; [29] subtraction; [30] multiplication; [31] division; [32] when the price of anything is an aliquot part of $1. [34) when the price is per hundred or thousand?

Compound Numbers. [36] What are compound numbers? [Illustration.] What are the general names? [37] What is English or Sterling Money? Repeat the table. [38] For what is Troy Weight used? Table. [39] Avoirdupois Weight? Table. [40] Apothecaries Weight? Table. [41] How many pounds in a barrel of flour? of beef, pork, or fish? firkin of butter? bushel of wheat? rye or corn? barley? oats? [42] What is Cloth Measure? Table. [43] Long Measure? Table. [44] Surveyor's Measure? Table.

[45] Square Measure? Table. What is a square? How may the contents of a square be found? What is the difference between 3 square feet and 3 feet square, etc., etc.? What is a rectangle? How may the contents of a rectangle be found? [46] Cubic Measure? Table. What is a cube? How may the contents of a cube be found? How may one of the dimensions be found? [47] Wine Measure? Table. [48] Beer Measure? Table. [49] Dry Measure? Table. [50] Time Measure? Table. [51] Circular Measure? Table. What is a circle? the circumference? diameter? radius? [52] Miscellaneous Table of units, etc., paper and books?

Reduction of Compound Numbers. [53] What is reduction of compound numbers? What two kinds are there? [54] What is reduction descending? ascending? Give a rule for reduction descending; ascending.

[68] Give a rule for addition of Compound Numbers. [69] Subtraction. [71] Multiplication. [72] Division.

[74] Give a rule for finding the difference of time when the difference of longitude is known, [75] for finding the difference of longitude when the difference of time is known.

Cancellation. [77] What is *cancellation?* Why may we cancel? Give a rule for cancellation.

Properties of Numbers. [78] What are even numbers? odd? prime? composite? [79] Give a rule for resolving composite numbers into prime factors.

[81] What is the greatest common divisor of two or more numbers? [82] How may it be found? [83] What is the multiple of a number? the common multiple of two or more numbers? the least common multiple? How may it be found? Rule I., II., III., IV.

Fractions. [86] What are fractions? What is meant by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$? [87] How many kinds of fractions are there? What are common fractions? decimal fractions? [88] How are common fractions written? What is the number above the line called? the number below the line? What are the terms of a fraction? What does the denominator show? With what does it correspond in division? What does the numerator show? With what does it correspond in division?

[89] How are common fractions divided? What is a simple fraction? when is it proper? when improper? What is a compound fraction? complex? mixed number? What are like fractions? unlike! [90] What is the value of a fraction? [91] How is the value

of a fraction affected by multiplying the numerator by any number? dividing the denominator? dividing the numerator? multiplying the denominator? multiplying both the numerator and denominator? dividing both?

[92] What is reduction of fractions? What are their simplest forms? [93] When is a fraction in its lowest terms? How may it be reduced to its lowest terms? [94] To what may an improper fraction be reduced? How? [95] To what may a mixed number be reduced? How? How may a whole number be reduced to a fraction? [96, 97] How may unlike fractions be reduced to like fractions? [Rules I., II.] [98] In what is reduction of compound fractions included.

[100] Give a rule for addition of fractions? [101] subtraction? [102, 103, 104] multiplication? [106, 107] division? [108] In what is reduction of complex fractions included? In what respects?

Decimal Fractions. [109] What are decimal fractions? How are they distinguished from whole numbers? [110] Repeat the table. [111] What is the denominator of a decimal fraction? What is the effect of prefixing a cipher? why? annexing a cipher? why? Read the examples. [112] Rule for writing decimals, examples. [113] Rule for addition of decimals? [114] subtraction? [115] What is the *general principle* of multiplication of decimals? Rule? Rule for multiplying by 10, 100 etc.? [116] What is the general principle of division of decimals? Rule? Rule for dividing by 10, 100 etc.? [118] How may common fractions be reduced to decimals? [119] decimal to common fractions?

[120] **Fractional Compound Numbers,** what are they? Examples. General Rule. [121] How may compound whole numbers be reduced to fractions? [122] How may fractional compound numbers be reduced to whole numbers? [123] to other denominations?

[127] **Duodecimals,** what are they? Whence arise? From what is the name derived? How added, subtracted etc.? What denomination is the product of feet by feet? square feet by feet? feet by inches? square feet by inches? inches by inches? square inches by inches? inches by seconds? square inches by seconds? seconds by seconds?

[128] What is **Analysis**? General process?

[129] What is Percentage? From what is the term per cent. derived? [130] What three things are chiefly considered in percentage? What is the principal? rate? percentage? How is the rate expressed?

examples. [131] How may the percentage be found? [132] the rate per cent.? [133, 134] the principal?

[136] **Applications of Percentage.** To what is percentage applicable? [137] What is *Commission?* A consignment? gross proceeds? net proceeds? [138] an account of sales? [139] *brokerage?* a broker? [140] *stocks?* common value of a share? when are stocks at par? above par, at a premium or an advance? below par or at a discount? stockholders? dividend? bonds?

[141] How is *gold* bought and sold? [142] What is *insurance?* fire insurance? marine insurance? life insurance? a policy? the premium? [144] What is *profit* and *loss?* how estimated?

[145] **Interest,** what is it? What is the principal when interest is to be found? rate? amount? simple interest? legal interest? the legal interest in the different States? How is interest found for one year? two or more years? months? days? [146] Another method? [147] Exact interest? [148] How is interest on Sterling Money calculated?

[149] **Partial Payments,** what are they? What is a note? Write a note in the usual form. Who is the maker or drawer? the payee? What is the face of a note? How is the amount due on a note found when partial payments have been made? [150] Merchants' Rule? [151] Connecticut rule?

[152] How is the rate found from the principal, interest and time? [153] The time from the principal, interest and rate? [154] The principal from the interest, rate and time?

[155] **Compound interest,** what is it? how found?

[156] **Discount,** what is it? What is the present worth of a sum or debt? How is the present worth found? How is the discount found?

[157] **Bank discount,** what is it? How does it differ from true discount? How may it be found? [158] How may the amount of a note be found, which will be worth a given sum at bank discount?

[159] **Taxes.** What is a tax? poll tax? income tax? What is real estate? personal property? an inventory or list? explain by an example the process of finding a person's tax.

[160] **Duties,** what are they? What is a port of entry? a custom house? ad valorem duty? specific duty? an invoice? tare? draft? leakage? gross weight? net weight? how is the duty on goods found?

[161] **Exchange,** what is it? domestic? foreign? write a draft. Who is the drawer? drawee? payee? How may a draft be accepted?

What is an acceptance? How may a draft be indorsed? What is the use of an indorsement?

[162] Why is the exchange on England always at a premium? [163] how may it be found? how may we find the amount of a bill of exchange which can be bought with U. S. money?

[165] **Partnership,** what is it? a firm or house? [166] How may each partner's share of profit and loss be found? [167] when the times are unequal?

[169] **Equation of Payments** is what? equated time? an account current? [170] How may the equated time be found? [171] when partial payments have been made? [172] of an account current? [173] of an account bearing interest?

[174] **Reduction of Currencies,** what is it? Why have State currencies been used? What is New England Currency? its value? New York Currency? its value? Pennsylvania Currency? its value? Georgia Currency? its value? [175] How may U. S. money be reduced to State currencies? [176] How may State currencies be reduced to U. S. money?

[178] **Ratio,** what is it? arithmetical? geometrical? To what is the ratio of two numbers equal? how expressed? What are the *terms* of a ratio? What are they both called? What is the first term called? the last?

[179] **Compound ratio,** what is it?

[180] **Proportion,** what is it? how expressed? what are the *extremes?* the *means?* what is a mean proportional? [181] a direct proportion? an indirect or inverse proportion? How is the product of the extremes compared with that of the means? How may the terms be arranged? How may the required term be found?

[183] **Compound Proportion,** what is it? How may the terms be arranged? How may the required term be found?

[184] **Conjoined Proportion,** what is it? How may the terms be arranged? How may the required term be found?

[186] **Alligation,** what is it? [187] What is alligation medial? How is the price of a mixture found? [188] What is alligation alternate? How may the quantities of different ingredients be found, that will form a mixture at a certain price?

[191] **Involution,** what is it? What is a power? How are different powers distinguished? What is the second power usually called? the third? How are powers found?

[192] **Evolution,** what is it? What is a *root?* a square root? cube root? the radical sign? How are the different roots expressed?

[193] What is the *square* root of a number? of 1? 4? 9? &c.? How may we find the square root of any number?

[194] How is square root applicable to circles, squares, or other similar figures? What is a triangle? a right angle? a right angled triangle? the hypothenuse? base? perpendicular? How may the hypothenuse be found? the base or perpendicular? [195] the side of a square? [196] the areas of similar figures?

[197] What is the *cube* root of a number? of 1? 8? 27? 64? 125, &c.? How may the cube root of any number be found? [198] How is cube root applicable to solids?

[199] **Progression,** what is it? what kinds are there? What are the *terms?* the *extremes?* the means? an ascending series? descending?

[200] **Arithmetical Progression,** what is it? What are to be found? [201] How may the last term be found? [202] the common difference? [203] the number of terms? [204] the sum of the series?

[205] **Geometrical Progression,** what is it? What are to be found? [206] How may the last term be found? [207] the sum of the series?

[208] **Mensuration,** what is it? What is a parallelogram? cylinder? prism? pyramid? cone? frustum? sphere? How is the surface of each found? How are the solid contents found?

THIS

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